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THE ELEMENTS OF APPLIED PHYSICS

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THE ELEMENTS
OF
APPLIED PHYSICS

BY

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PREFACE

The preparation of this volume was undertaken to meet the needs of a group of students who are primarily interested in the practical applications of physics. The training and habits of thought of such students have been kept clearly in mind and the aim has been to make the book include those topics which these students can thoroughly assimilate. For this reason some time-honored subjects have been omitted and others have been inserted in appendices.

There is presented in the book a limited number of fundamental physical principles stated intimately and concretely in a way to encourage the student to form physical images and to discourage the habit of students to be content with verbal knowledge. A large number of illustrations of the applications of physics to agriculture, engineering and everyday life have been included in an effort to stimulate the student to recognize the universality of physical laws and to find in them an explanation of everyday experiences and observations. The attempt has been made to lay a basis for correct scientific thinking rather than to stress the accumulation of facts or to trace the historical development of the subject.

To encourage accuracy and concreteness in thinking, many solved problems have been included. By inserting these simple problems immediately after the statements of definitions, laws or principles and using them as illustrations of these verbal statements, it is believed that the opportunity for vague and loose thinking is lessened. This is especially true with students who find it very difficult to think clearly except in terms of quantities which are expressed numerically.

It is a pleasure to acknowledge my indebtedness to the following authors for permission to use their illustrations: Dean A. A. Potter, "Farm Motors"; Dean E. S. Keene, "Mechanics of the Household"; Professor W. B. Anderson, "Physics for Technical Students." I am also indebted to Professor Arthur L. Foley of Indiana University for kindly supplying me with the photographs of sound waves and to Dr. Charles Sheard of the American Optical Co. for the astigmatic chart.

The author also wishes to express his appreciation for the kindness of the following firms in supplying illustrative material: Western Electric Co., New York, N. Y.; The Weston Electrical Instrument Co., Newark, N. J.; Leeds and Northrup Co., Philadelphia, Pa.; Adam Hilger, Ltd., London; W. G. Pye & Co., Cambridge, Eng.; Bausch and Lomb Optical Co., Rochester, N. Y.; L. E. Knott, Boston, Mass.; General Electric Co., Schenectady, N. Y.; The Hercules Electric Co., Indianapolis, Ind.; The C. S. Bell Co. Hillsboro, O.; The Pressure Cooker Co., Denver, Col.; The Domestic Engineering Co., Dayton, O.; The Moline Plow Co., Moline, Ill.; The Edison Storage Battery Co., Orange, N. J.; Case Threshing Machine Co., Racine, Wis.; The Taylor Instrument Co., Rochester, N. Y.; The Burnette-Larsh Mfg. Co., Dayton, O.; and The DeLaval Separator Co., Chicago, Ill.

A. W. S.

Columbus, Ohio,
January, 1923.

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INTRODUCTION

1. Subject Matter of Physics.—Physics is a broad science having a wide application to most aspects of modern life. It is, therefore, a study about familiar things and an attempt to find satisfactory explanations for them. The object of physics is to determine exact relations between physical phenomena. It tries to give as simple explanations as possible for the causes and effects which are observed in the physical universe. For such explanations exact measurements are very necessary. In order then to proceed with the study of physics it is necessary to have an agreement with respect to the units in which such measurements are to be made.

2. Units of Length.—The unit of length in the metric system is taken as the centimeter which is one one-hundredth part of the standard meter. The standard meter is defined as the distance at the temperature of melting ice between two marks on a certain platinum-iridium bar which is kept at the International Bureau of Weights and Measures, near Paris. Two copies of this meter are kept at the Bureau of Standards, Washington. It is a familiar fact that a bar of metal changes in length when heated or cooled. In order then to have this unit of length accurate it is necessary to keep the bar at a fixed temperature.

The unit of length employed by the English-speaking people for ordinary purposes is the yard. By an act of Congress the yard is defined as $\frac{3600}{3937}$ of a meter. Hence,

$$1 \text{ meter} = 39.37 \text{ inches}$$
$$1 \text{ inch} = 2.54 \text{ centimeters}$$

3. Units of Mass.—The unit of mass in the metric system is called the gram. It is the one-thousandth part of a mass of metal called a kilogram and is kept at the International Bureau of

Weights and Measures, near Paris. Two copies of this standard are kept at the Bureau of Standards at Washington.

It was the original intention of those who chose this standard that one gram should be the mass of one cubic centimeter of water at 4°C. More exact determinations have shown that this relation is not strictly true. The error is so small that it may be neglected for all practical purposes.

Among the English-speaking peoples the pound is ordinarily used as the unit of mass. The pound is defined as the mass of a certain piece of platinum in the possession of the British Government. By Act of Congress, the kilogram was declared to be equivalent to 2.2 pounds. Hence, the relation between these units of mass is as follows:

1 kilogram = 2.2 pounds
1 pound = 453.6 grams

4. Units of Time.—For most scientific purposes the second is chosen as the unit of time. It is defined as the 1/86400 part of the mean solar day. The mean solar day, as used here, means the average throughout the year between the successive passages of the sun across the meridian. The minute, which is equivalent to 60 seconds and the hour, which is equal to 60 minutes, are also used as units of time.

5. Density.—The mass per unit volume is called the density of a body. In the English system of units, densities are usually expressed in pounds per cubic foot. In the metric system densities are measured in grams per cubic centimeter.

Let V = the volume of a body.
M = the mass of the body.
d = the density.

Then

$$d = \frac{M}{V}$$

Example.—A mass of mercury weighs 136.5 g. It has a volume of 10 cu. cm. What is its density?

$$\text{Density} = \frac{\text{mass}}{\text{volume}} = \frac{136.5}{10} = 13.65 \text{ g. per cu. cm.}$$

6. Trigonometrical Formulae.—In a right triangle ABC (Fig. 1) it is convenient to define the relations between the sides in the following way. It is to be remembered that this is purely a matter of definition suggested by convenience and that by making these definitions much time is saved in finding the sides of such a triangle. Consider the angle at A. Divide the side BC, opposite A, by the hypotenuse AB and call the ratio, for brevity, **Sine A.** Now divide the side AC, adjacent to A, by the hypotenuse AB and call this ratio **Cosine A.** Then divide the side BC opposite A by the side AC adjacent to A and call this ratio **Tangent A.**

$$\text{Sin } A = \frac{\text{side opposite}}{\text{hypotenuse}} = \frac{BC}{AB}$$

$$\text{Cos } A = \frac{\text{side adjacent}}{\text{hypotenuse}} = \frac{AC}{AB}$$

$$\text{Tan } A = \frac{\text{side opposite}}{\text{side adjacent}} = \frac{BC}{AC}$$

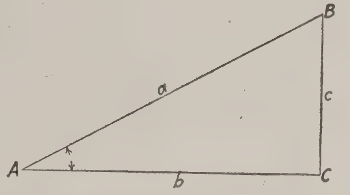


Fig. 1.—The definition of sine, cosine and tangent.

Let $a=$ the hypotenuse, $b=$ the adjacent side and $c=$ the opposite side.

$$\text{Sin } A = c/a \qquad c = a \text{ Sin } A$$

$$\text{Cos } A = b/a \qquad b = a \text{ Cos } A$$

$$\text{Tan } A = c/b \qquad c = b \text{ Tan } A$$

Tables are prepared giving the values of Sin A, Cos A and Tan A for all values of A. With such a table at hand it is easy to find any side of a right triangle when an acute angle and one of the sides are given.

Example.—In a right triangle ABC (Fig. 1) the angle A is 30° and the side BC is 2 ft. Find the hypotenuse.

$$\text{Sin } A = \frac{BC}{AB}$$

$$\tfrac{1}{2} = \frac{2}{AB}$$

$$AB = 4 \text{ ft.}$$

7. Universal Gravitation.—It has been found that there is a tendency for every body in the universe to move toward every other body. This tendency arises out of the fact that every particle of matter attracts every other particle of matter with a force whose direction is that of the line joining the particles. When the masses of these bodies are small this attractive force is small, but the attraction between two bodies like the sun and the earth is very large. It is this force of attraction between the sun and the earth which causes the earth to describe its orbit about the sun. The weight of a body is the attractive force of the earth on the body. This force pulls the body toward the center of the earth. The greater the mass of the body, the greater is this pull and the greater the weight. The further the body from the center of the earth the less is this pull. Hence, the weight of a body varies with the altitude and also the latitude at which the observations are made.

Newton found that **the force of attraction between two small bodies or between two spherical bodies of any size (Fig. 2) is proportional to the product of their masses and inversely proportional to the square of the distance between their centers.** This law may be written as,

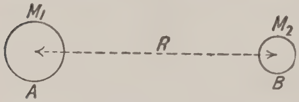


FIG. 2.—The law of universal gravitation.

$$F = k\frac{M_1M_2}{R^2}$$

where F is the force of attraction, M_1 and M_2 the masses of the bodies, R, the distance between their centers, and k, a constant known as the constant of gravitation.

8. Force.—Our first ideas of force come from muscular effort exerted to produce change in the motion of a body. If a ball is thrown into the air or a heavy stone lifted into a wagon a certain muscular effort is required. This muscular effort is greater for large bodies than for small ones. When bodies are once set in motion they require muscular effort to stop them and the more rapid their motion the greater is the force to bring them to rest in a given time.

A force is an action exerted by one body on another tending to change the state of motion of the body acted upon. Thus a loaded wagon is drawn by a team of horses or an automobile is driven forward by its engine. When a man lifts a bucket of water his hand exerts a force on the bucket which changes the state of motion of the bucket. It may happen that the force exerted on the bucket is not sufficient to lift it. In that case the force only tends to change the state of motion of the body.

9. Units of Force.—The most familiar force in nature is that arising from the pull of the earth on bodies which are on or near its surface. Thus if a mass of one pound is held in the outstretched hand, a downward pull is felt which requires an exertion to overcome it. If the mass is 100 lbs. instead of one pound, the pull is so great that the hand cannot hold the mass in place. So important is this force and so easily can it be observed that it is convenient to make it a unit in which to measure other forces.

A force of one pound is defined as a force as large as the force with which the earth pulls on a mass of one pound. Since the pull of the earth on a mass of one pound is not the same at all points on the surface of the earth this unit of force will not be exactly the same at all places on the earth surface.

In like manner a kilogram of force is defined as a force equal to the attraction of the earth for a mass of one kilogram. Also a force of one gram is a force as large as the attraction of the earth on a mass of one gram. These units are called gravitational units of force.

PART I.—MECHANICS

CHAPTER I

MOTIONS OF TRANSLATION

10. Types of Motion.—The motions of bodies may be divided into three classes: (1) **translation,** (2) **rotation,** (3) **vibration or oscillation.** A body is said to have a motion of translation when it moves on continuously in the same direction. A ball thrown from the hand of the pitcher and an automobile running on a straight piece of road are illustrations of **motions of translation.** If the body instead of travelling forward turns on a fixed axis, the body has a **motion of rotation.** Thus the fly wheel of a stationary engine turns continuously around its axis without ever moving forward. Any point on the wheel returns again and again to its original position. This is a motion of rotation. The drive wheels of a locomotive are moving forward and are at the same time rotating. They will therefore have two motions, one of rotation and the other of translation. Some bodies reverse their motions from time to time and return at regular intervals to their original positions. Such bodies are said to have a motion of vibration or oscillation. The pendulum of an ordinary clock swings back and forth at regular intervals and after a certain time repeats its former motion. It has a motion of vibration.

11. Velocity.—**The velocity of a body is defined to be the rate at which it is passing through space or the space passed over in unit time.** It is determined by dividing the space through which a body has passed by the time required to pass over that space.

$$\text{Velocity} = \text{space per unit time} = \frac{\text{distance}}{\text{time}}.$$

Example.—A train which is moving at a uniform rate passes over 75 miles in 3 hrs. What is its velocity?

$$\text{Velocity} = \frac{\text{distance}}{\text{time}} = \frac{75}{3} = 25 \text{ mi. per hr.}$$

12. Constant and Variable Velocities.—The velocity of a body is constant or uniform when the body passes over equal distances in equal intervals of time. The magnitude of such a velocity is measured by the space passed over in unit time. A body has a variable velocity when it pasess over unequal distances in equal intervals of time. Where the velocity of a body changes from time to time, it is convenient to consider what is known as the **average velocity.** The average velocity is that constant velocity which would cause the body to move from one point to the other in the same time which is required when the velocity is variable. Thus the average velocity of a train which runs 600 mi. in 20 hrs. is $600 \div 20 = 30$ mi. per hr. This means that the train would travel this same distance in 20 hrs. if it had a constant velocity of 30 mi. per hr. instead of the variable velocity with which it actually travels. In case the velocity changes uniformly, the average velocity over any interval of time can be found by taking the velocity at the beginning and at the end of the time, adding them together and dividing the sum by two. This gives the velocity with which the body on the average has been moving. Part of the time it has moved with a velocity greater than the average velocity and the remainder of the time it has moved with a velocity less than the average velocity. The distance covered is the same as if the body had always moved with a velocity as great as the average velocity. This is equivalent to replacing the varying velocity with a constant velocity such that the body would move in a given time the same distance it moves with the uniformly changing velocity.

Example.—At the beginning of a certain time the velocity of a body is 30 ft. per sec. The velocity changes uniformly for 5 min. and is then 80 ft. per sec. What is the average velocity over this time?

$$\text{Average velocity} = \frac{\text{initial velocity} + \text{final velocity}}{2}$$

$$= \frac{30 + 80}{2} = 55 \text{ ft. per sec.}$$

13. Acceleration.—**The rate at which the velocity of a body changes is called the acceleration.** It is found by dividing the change in velocity by the time in which the change takes place. It is, therefore, the change in velocity per unit of time.

Example.—A ball which has a velocity of 30 ft. per sec. moves for 5 sec. and at the end of that time is found to have a velocity of 70 ft. per sec. Find the acceleration

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time}}$$

$$\text{Change in velocity} = 70 - 30 = 40 \text{ ft. per sec.}$$

$$\text{Acceleration} = \frac{70 - 30}{5} = \frac{40}{5} = 8 \text{ ft. per sec. per sec.}$$

Hence, the velocity of the ball is increasing 8 ft. per sec. for each second during which it moves. It is important to observe that two units of time must be stated in order to express an acceleration. One of these units expresses the unit of time in which the original velocity is measured. The other of these units expresses the unit of time used to measure the interval of time over which the velocity is allowed to change. For example, a velocity of 60 mi. per hr. might be allowed to change for one second, one hour or one week, and the acceleration would be different in each case.

An acceleration may be either **uniform** or **variable.** A uniform or constant acceleration is one in which equal changes of velocity take place in equal intervals of time. To calculate a constant acceleration it is only necessary to divide the total change in velocity by the time in which it took place. Where the acceleration is variable the total change in velocity divided by the time gives the average acceleration.

14. Illustrations of Uniform Acceleration.—When a train is stopping at a station it is losing velocity. When it is leaving the station it is gaining velocity. In the former case the acceleration is negative, in the latter it is positive. If it gains velocity at the same rate; for example, 2 mi. per min., the acceleration is constant. A falling body is a good illustration of uniformly accelerated motion. The body gains 32 ft. per sec. each second that it falls. A loaded sled on the side of a hill will slide to the bottom and its velocity will increase uniformly as it goes. When it reaches the bottom of the hill the velocity will decrease. The acceleration has now become negative.

15. Motion of Bodies with Constant Velocity.—The velocity of a body which is moving uniformly has already been defined as the space passed over in unit time. This relation may be written as,

$$v = \frac{s}{t}$$

in which v represents the velocity, s the distance and t the time during which the body was moving. This equation may be written in the form,

$$s = vt$$

In this form the equation gives the space passed over in the time t by a body moving with constant velocity v. If it happens that the body is moving with variable velocity, the space passed over may be found by multiplying the average velocity by the time.

Example.—The velocity of an automobile is 15 mi. per hr. How far will it move in 3 hrs.?

$$\begin{aligned}\text{Space passed over} &= \text{velocity} \times \text{time}\\ &= 15 \times 3 = 45 \text{ mi.}\end{aligned}$$

Example.—The average velocity of a falling body for 5 sec. is 80 ft. per sec. How far does the body fall in that time?

$$\begin{aligned}\text{Space passed over} &= \text{average velocity} \times \text{time}\\ &= 80 \times 5 = 400 \text{ ft.}\end{aligned}$$

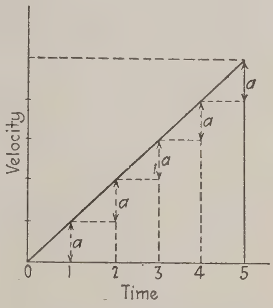


FIG. 3.—Relation between time and velocity when the initial velocity is zero.

16. Motion of Bodies with Constant Acceleration Starting from Rest.—Suppose that a body starts from rest with an acceleration a. At the end of the first second its velocity is a (Fig. 3), at the end of the second second it is $2a$, at the end of the third second it is $3a$, and a velocity equal to a will be added during each second the body moves. At the end of t seconds its velocity will be, at

$$\begin{aligned}\text{Final velocity} &= \text{rate of change of velocity} \times \text{time}\\ &= at\end{aligned}$$

The average velocity over this time is,

$$\frac{\text{Initial velocity} + \text{final velocity}}{2} = \frac{0 + at}{2} = \tfrac{1}{2}at$$

$$\begin{aligned}\text{The space passed over} &= \text{average velocity} \times \text{time}\\ &= \tfrac{1}{2}at \times t = \tfrac{1}{2}at^2\end{aligned}$$

In the case of bodies falling freely under the action of gravity the acceleration is 980 cm. per sec. per sec. or 32.2 ft. per sec. per sec. For this special case these relations then become,

$$\begin{aligned}v &= 32.2\, t \text{ ft. per sec.}\\ &= 980\, t \text{ cm. per sec.}\\ s &= \tfrac{1}{2}\, 32.2\, t^2 \text{ ft.}\\ &= \tfrac{1}{2}\, 980\, t^2 \text{ cm.}\end{aligned}$$

Example.—A ball which is thrown upward leaves the hand of the thrower with a velocity of 80 ft. per sec. How long before it comes to rest?

Time to come to rest

$$= \frac{\text{initial velocity}}{\text{rate of losing velocity}} = \frac{\text{initial velocity}}{\text{acceleration}} = \frac{80}{32.2} = 2.5 \text{ sec.}$$

Example.—A body starts from rest and falls freely for 10 sec. Find the space passed over in this time.

Final velocity = acceleration × time = 980 × 10 = 9,800 cm. per sec.

$$\text{Average velocity} = \frac{\text{initial velocity} + \text{final velocity}}{2}$$

$$= \frac{0 + 9{,}800}{2} = \begin{array}{c}4{,}900 \text{ cm.}\\ \text{per sec.}\end{array}$$

Space passed over = average velocity × time = 4,900 × 10 = 49,000 cm.

17. Motion of Bodies with Constant Acceleration and Initial Velocity.—In the preceding discussion it was assumed that the body started from rest and moved with a constant acceleration. In such cases the velocity of the body at the end of a given time is the velocity with which it started plus or minus its change in velocity during this time. To find the final velocity in these cases (Fig. 4) it is only necessary to take the initial velocity and add to it or subtract from it the change in velocity. For example a body is thrown directly downward from the top of a tower with a velocity of 80 ft. per sec. What is its velocity at the end of 10 seconds? During each second the velocity increases 32.2 ft. per sec. In 10 seconds the increase will be 322 ft. per sec. Hence the final velocity will be 80 + 322 = 402 ft. per sec.

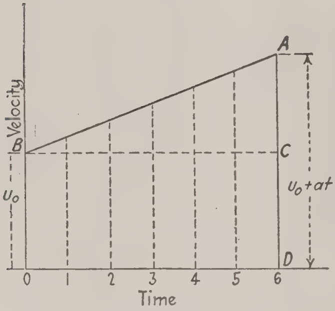


FIG. 4.—Relation between time and velocity when there is an initial velocity.

Let u = the initial velocity
v = the final velocity
t = the time
a = the acceleration
$v = u + at$, when the velocity is increasing
$v = u - at$, when the velocity is decreasing.

The average velocity during the time t is,

$$\frac{\text{Initial velocity} + \text{final velocity}}{2} = \frac{u + v}{2}$$

Space passed over = average velocity × time

$$= \frac{u + u + at}{2} = ut + \tfrac{1}{2}\,at^2$$

In the case of freely falling bodies, a may be replaced by g, where g is 32.2 ft. per sec. per sec. in the English system and 980 cm. per sec. per sec. in the C. S. S. system. The equations for freely falling bodies then become

$$v = u + gt$$
$$s = ut + \tfrac{1}{2}gt^2$$

Example.—A body is thrown on ice with a velocity of 60 ft. per sec. It has a negative acceleration of 6 ft. per sec. per sec. Find the velocity at the end of 5 sec. and the space passed over in that time.

Final velocity = initial velocity − loss in velocity
= initial velocity − acceleration × time
= 60 − 6 × 5 = 30 ft. per sec.

Space passed over = average velocity × time

$$= \frac{\text{initial velocity} + \text{final velocity}}{2} \times \text{time}$$

$$= \frac{60 + 30}{2} \times 5 = 45 \times 5 = 225 \text{ ft.}$$

Problems

1. A body at a certain time has a speed of 16 ft. per sec. and 15 sec. later it has a speed of 46 ft. per sec. Find the acceleration.

2. A body is thrown vertically upward with a velocity of 128 ft. per sec. How high will it rise and how soon will it reach the ground?

3. What initial velocity must a body have in order to rise 640 ft. against gravity?

4. A stone is thrown from the top of a cliff which is 260 ft. high with a downward velocity of 50 ft. per second. How long will it take the stone to reach the ground?

5. A body is thrown vertically upward with a velocity of 180 ft. per sec. At what height will its velocity be 60 ft. per sec.?

6. A tractor traveling at the rate of 6 miles per hour is brought to rest in 30 seconds. How far does it go and what is the acceleration?

7. A sled sliding down a frictionless inclined plane has a velocity of 40 ft. per sec. at the end of 4 sec. How far will it slide in 15 sec.?

8. A lump of coal drops from a bucket in the shaft of a mine. The bucket, which is 250 ft. from the bottom of the shaft, is rising with a velocity of 15 ft. per sec. In what time will the coal reach the bottom of the shaft?

9. A train running at the rate of 40 miles per hour comes to rest in 15 sec. Find the acceleration in ft. per sec. per sec.

CHAPTER II

LAWS OF MOTION

In the preceding paragraphs, various cases of motion have been discussed without considering the influences which affect these motions. It is necessary now to consider the way in which the motions of bodies may be changed and the laws which govern these changes.

18. Inertia.—One often observes that in stepping off from a moving car it requires effort to stop at the desired place. In other words there is a marked tendency for the man to keep on moving after he has left the car. On the other hand, a man finds it easier to jump onto a moving car if he is running forward at about the rate at which the car is running. This is an illustration of an important law discovered by Newton. This law states that there is a tendency of matter to keep moving when it has once been set in motion and a tendency of matter to remain at rest unless it is forced to move by the application of a force. In like manner, it is found that if a heavy fly wheel is standing still it requires a considerable effort to set it in rotation about its axis. When, however, it is once in rotation it tends to keep on rotating about its axis until some force, suitably applied, brings it to rest.

That property of matter by virtue of which a body tends to remain at rest or to keep in motion in a straight line is called inertia. There will be two kinds of inertia as there are two types of motion,—translation and rotation. Bodies show opposition to being translated and also opposition to being rotated. The former opposition is called **inertia**; the latter is called **rotary inertia.** The inertia of a body for motions of translation is proportional to the mass of the body, but rotary inertia depends also on the distribution of the mass about the axis of rotation.

19. Newton's First Law of Motion.—**Every body continues in a state of rest or uniform motion in a straight line, unless it is compelled to change that state by the application of some external force.** This form of statement applies to motions of translation. For motions of rotation the law states **that every body continues**

in a state of uniform motion of rotation around a fixed axis unless acted upon by some impressed force applied at some point not on the axis of rotation. This law states that a stone lying on the ground will remain there until some outside force compels it to move. When a ball is thrown into the air it would move on indefinitely unless some outside force offered a resistance to its motion and caused it to stop. A fly-wheel which is standing still will not start unless a force is applied to it in such a way that it will produce rotation and when it once starts it will continue indefinitely unless some force operates to stop it. Even after the force has been applied, it requires time to put the mass in motion or to bring it to rest. The change in motion is not immediate.

20. Momentum.—Certain properties of moving bodies depend jointly on the mass and the velocity. Thus the time for a locomotive to start a train depends on the mass of the train and the velocity which is given to it. It is more difficult to give a freight train a speed of 10 mi. per hr. than it is to give a passenger train that speed. It is convenient to have a name for this property and to define it accurately. This property is called the **momentum** of the body and **it is defined as the product of the mass and the velocity.** Since the velocity has direction as well as magnitude and the mass has only magnitude the momentum will be a directed quantity and due regard must be paid to its sign. Whenever either the velocity or the mass of a body increases the momentum also increases.

Example.—A tractor weighing 7,500 lb. has a velocity of 8 ft. per sec. What is its momentum?

$$\text{Momentum} = \text{mass} \times \text{velocity}$$
$$= 7{,}500 \times 8 = 60{,}000 \text{ (lb. ft. sec.)}$$

21. Newton's Second Law of Motion.—We have been discussing the motion of a body in which the velocity is changing the same amount each second. To produce such a change a constant force must act on the body. This force produces an acceleration which is always in the direction in which the force is acting. The acceleration produced by the force is proportional to the force; that is, the greater the force the greater is the acceleration. For example, let a force of 32 lb. act on a load of 64 lb. The load will have an acceleration in the direction of the force and if the force is increased to 64 lb. the acceleration will be doubled. The force effective in producing this acceleration is not the entire force acting on the body, but the unbalanced force. By an unbalanced force is meant the amount by which the pull or push in one direction exceeds the pull or push in the opposite direction. When the engine in a tractor exerts a force on the tractor, there is besides this force on the tractor the forces due to the resistance which must be overcome in pulling the plows through the earth. This resistance acts in the opposite direction to the pull of the engine on the plows. The unbalanced or net force is the difference between the force pulling the tractor forward and the forces tending to stop it. When this net force is zero the tractor moves forward with constant velocity. If the forward force exceeds the backward force the tractor moves forward with constant acceleration. If the resistance exceeds the forward force, the motion of the tractor slows down at a constant rate.

Experiments also have shown that if equal accelerations are imparted to bodies of unequal masses, the forces required to produce these accelerations are proportional to the masses of the bodies. Thus if an acceleration of 2 ft. per sec. per sec. is imparted to a mass of 5 lb. and to another of 15 lb. it requires three times as much force to produce this acceleration in the 15 lb. mass as it does in the 5 lb. mass. The larger the mass the more difficult it is to produce a given acceleration.

Combining these statements we get Newton's second law of motion. **The net force acting on a body is proportional jointly to the mass and the acceleration produced by this force.** The product of the mass and the acceleration is therefore a measure of the force acting on the body. This law can be more conveniently expressed as follows:

Let $F =$ the net force expressed in pounds acting on the body
$a =$ the acceleration of the body in ft. per sec. per sec.
$M =$ the mass of the body in pounds
$g = 32.2 =$ the acceleration of a freely falling body

Then $F = \frac{M}{g} \times a = \frac{M}{32.2} \times a$

If the mass is measured in grams, the acceleration in cm. per sec. per sec. and the force in grams weight, the law becomes,

$$F = \frac{Ma}{980}$$

where 980 is the acceleration of gravity in cm. per sec. per sec.

Example.—What pull must be applied by a locomotive to give a train of 250 tons an acceleration of 2.5 ft. per sec. per sec.? Neglect friction.

$$\text{Net force} = \frac{\text{mass}}{32.2} \times \text{acceleration}$$

$$F = \frac{Ma}{32.2}$$

$$M = 250 \times 2{,}000 = 500{,}000 \text{ lb.}$$

$$a = 2.5 \text{ ft. per sec. per sec.}$$

$$F \text{ in pounds} = \frac{500{,}000}{32.2} \times 2.5 = 38{,}800 \text{ lb.}$$

Example.—A force equal to the weight of 30 g. acts on a body which has a mass of 98 g.; find the acceleration.

$$F \text{ in g.} = \frac{M \text{ in g.} \times \text{acceleration in cm. per sec. per sec.}}{980}$$

$$F = \frac{Ma}{980}$$

$$F = 30 \text{ g.}$$

$$M = 98 \text{ g.}$$

$$30 = \frac{98a}{980} \text{ or } 30 = a/10$$

$$a = 300 \text{ cm. per sec. per sec.}$$

Problems

1. Find the force required to give a mass of 25 lb. an acceleration of 75 ft. per sec. per sec.

2. A force of 10 g. weight acts on a mass of 3.5 kg. Find the acceleration.

3. A force of 1 g. weight is observed to produce an acceleration 9.8 cm. per sec. per sec. in a body. What is the mass of the body?

4. A loaded farm truck is started from rest with an acceleration of 2 ft. per sec. per sec. If the truck weighs 5 tons, what is the force exerted by the engine?

5. A horizontal force equal to the weight of 10 lb. is applied to a mass of 50 lb. on a horizontal smooth plane. What velocity in feet per second will be acquired in 5 sec.?

6. A mass of 5 kg. moving with a velocity of 100 m. per sec. is opposed by a force equal to the weight of 20 g. How long before this force will bring the body to rest?

22. Newton's Third Law of Motion.—The third law of motion like the first and second laws is the result of experience. The hand of a person sustaining a load is subjected to the downward force of gravity acting on the body. The hand must apply an equal upward force in order to keep the body in position. The wheels of an automobile push backward on the road but the road pushes forward on the wheels with an equal force. When the road is icy so that the wheels can not grip it, the road can not push forward on the wheels and the car does not move forward although the motor is at full speed. If a horse pulls a load along the road the horse pushes back on the road with a certain force and the road pushes forward on the horse with an equal force. This fact is made use of in a tread mill where the track on which the horse stands is movable. The horse attempts to walk forward but in reality the track moves back under his feet. The horse remains in one position while the track on which he stands moves and thus develops the power which turns the mill.

Wherever one force is found in nature it is possible to find another force which has the same magnitude but is always in the opposite direction. If for example a man pulls on a rope with a force of 40 lb., the rope pulls on the man's hands with a force of 40 lb. The train pulls back on the locomotive with a force which is just as great as the force with which the locomotive pulls forward on the train. The sun pulls on the earth and the earth pulls on the sun with an equal and opposite force.

These facts may be stated as the third law of motion in the following form:

To every force or action there is always an equal and opposite reaction.

CHAPTER III

ROTARY MOTIONS

23. Circular Measure of Angles.—Instead of measuring angles in degrees it is convenient in science to define a new unit of angular measure. **This unit is called a radian.** On the circumference of a circle Fig. 5 lay off an arc AC which is equal in length to the radius OA. The angle AOC subtended by the arc AC is defined as one radian. **Or a radian is an angle whose arc is equal to its radius.** Since the radius can be applied to the circumference 2π times, it follows that

$$360^\circ = 2\pi \text{ radians}$$
$$1 \text{ radian} = 57^\circ.3 \text{ approximately.}$$

From this definition of a radian we have the relation

$$x = s/R$$

where $x =$ the angle in radians,

$s =$ the arc and $R =$ the radius.

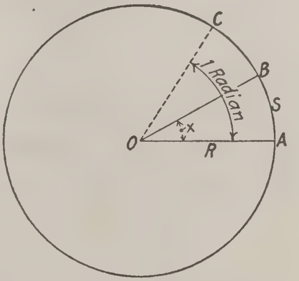


FIG. 5.—Definition of a radian.

Example.—Find the number of radians in an angle of 45°.

$$\text{Radians} = \frac{\text{number of degrees}}{57.3}$$
$$= \frac{45}{57.3} = 0.78 \text{ radian}$$

24. Angular Velocity.—Let one end of a line OA of Fig. 6 be fixed and then let the line revolve in a plane about this fixed end. The rate at which the line rotates is called its **angular velocity.** The number of radians swept out by the line per unit of time gives the angular velocity. If the rate of rotation of the line is constant the angular velocity is constant and is equal to the angle through which the line turns in unit time. The angular velocity may also be measured in revolutions per sec. or per min. The angular velocity in radians is then given by multiplying the number of revolutions per unit of time by 2π, the number of radians in one revolution.

Example.—A wheel makes 5 revolutions per min. What is its angular velocity?

$$\text{Angular velocity} = 2\pi \times \text{number of revolutions} = \omega$$

$$\omega = 2\pi \times 5 = \frac{220}{7} \text{ radians per min.}$$

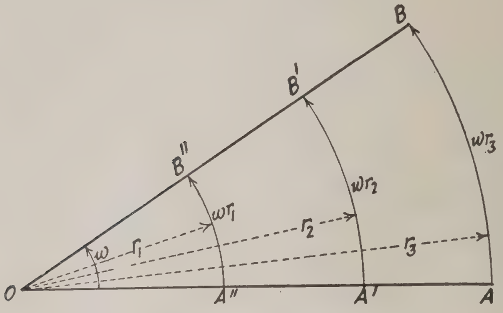

B
B'
B''
ωr3
ωr2
ωr1
ω
r1
r2
r3
O
A''
A'
A

Fig. 6.—Definition of angular velocity.

The linear velocity of a point on the line OA increases as the distance from the axis of rotation increases. Thus the velocity of A is largest and equal to ωr_3 while that of A' is less and equal to ωr_2. This fact is made use of in a number of devices for the regulation of speeds in machines. Fig. 7 shows a device used on a grain drill for regulating the rate at which the grain is discharged into the furrow. It is an application of the fact that the linear speed increases as the distance from the axis of rotation increases.

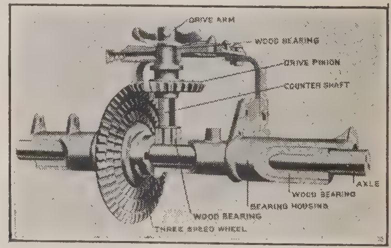

DRIVE ARM
WOOD BEARING
DRIVE PINION
COUNTER SHAFT
AXLE
WOOD BEARING
BEARING HOUSING
WOOD BEARING
THREE SPEED WHEEL

Fig. 7.—Change of linear speed along a radius.

25. Angular Velocities in Pulleys and Wheels.—When a belt is stretched around two pulleys as in Fig. 8 or Fig. 9 the angular velocities of the pulleys are not the same unless the pulleys have the same radii. In Fig. 8 the directions of rotation of the pulleys are the same but in Fig. 9 their directions of rotation are opposite. The linear velocities of points on the circumferences of both pulleys are equal to the linear velocity of the belt.

If V = the linear velocity of the belt,
ω_A = angular velocity of A,
ω_B = angular velocity of B,
R_A = radius of A
R_B = radius of B

$$\omega_A = \frac{V}{R_A}$$

$$\omega_B = \frac{V}{R_B}$$

$$\frac{\omega_A}{\omega_B} = \frac{R_B}{R_A}$$

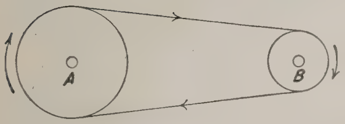


Fig. 8.—Angular velocity of pulleys.

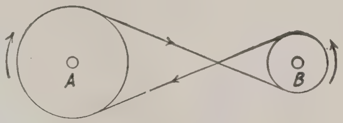


Fig. 9.—Reversal of direction of rotation of pulleys.

Hence the angular velocities of the pulleys are inversely proportional to their radii. In this way it is possible to increase or decrease the angular speed at which one of the pulleys is driven.

By means of toothed wheels (Fig. 10) the same kind of alteration of angular velocities may be produced. In this case there can be no slipping and the angular velocities of the wheels are inversely proportional to the number of teeth.

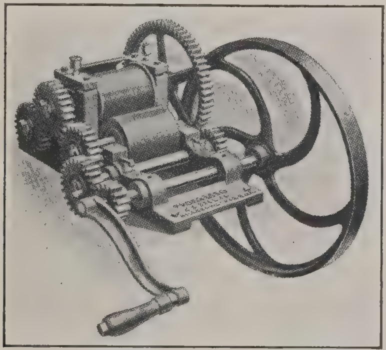


Fig. 10.—Angular velocities of toothed wheels.

$$\frac{\omega_A}{\omega_B} = \frac{N_B}{N_A}$$

26. Angular Acceleration.—The rate at which the angular velocity changes is called the angular acceleration. It is the increase or decrease in angular velocity per unit of time. It is related to the angular velocity in the same way in which the linear acceleration is related to linear velocity. In an angular acceleration it is necessary to specify two units of time. One of these units gives the unit of time in which the angular velocity is measured and the other gives the unit of time used to measure the change in the angular velocity. Ordinarily the same unit of time is used in both cases.

Example.—At a certain instant the angular velocity of a wheel is 30 radians per sec. In 10 sec. the angular velocity has become 50 radians per sec. What is the angular acceleration?

$$\text{Angular acceleration} = \frac{\text{change in angular velocity}}{\text{time}}$$

$$= \frac{50 - 30}{10} = 2 \text{ radians per sec. per sec.}$$

27. Torque.—The effect of a force in producing rotation depends on the magnitude of the force and on the perpendicular distance between the line of action of the force and the axis about which the rotation takes place (Fig. 11). It is found by experiment that the effect of a force in producing rotation is proportional to the magnitude of the force and also proportional to the distance between the line of action of the force and the axis of rotation. **It is convenient to define torque or moment of force as the product of the force and the perpendicular distance between the line of action of the force and the axis of rotation.**

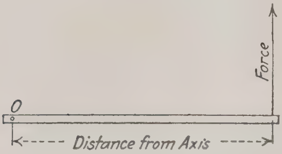


FIG. 11.—Torque = force × lever arm.

Torque = force × distance from axis.

Example.—A man pulls at right angles to the spoke of a wheel with a force of 75 lb. If the distance from the axis to the hand is 3 ft., what moment of force does he apply to the wheel?

Torque = force × distance to axis
= 75 × 3 = 225 (lb.-ft.)

28. Relation of Torque to Angular Acceleration.—In order to produce linear acceleration it is necessary to apply a force to the body, and the greater the force the greater the acceleration. To produce angular acceleration it is necessary to apply a torque to the body. The greater the torque the greater is the angular acceleration which is produced. For a particular body rotating about a fixed axis it is found that the angular acceleration is proportional to the torque which produces it. This factor of proportionality by which the angular acceleration must be multiplied in order to give the torque is a measure of the opposition of the body, to being set in rotation. It is analogous to the mass of a body which is a measure of the opposition of a body to being set in translation. The relation between the torque and the angular acceleration can be expressed by the following equation,

$$Tg = IA$$

where A = angular acceleration in radians per sec. per sec.
T = the torque (force in pounds and distance in ft., or force in grams and distance in cm.)
I = the factor of proportionality called the rotary inertia.
g = the acceleration of gravity, 32.2 ft. per sec. per sec. or 980 cm. per sec. per sec.

Example.—A grindstone has a rotary of inertia of 300 (lb.-ft.). Find the torque necessary to produce an angular acceleration of 3 radians per sec. per sec.

Torque = rotary inertia × angular acceleration

$$T \times g = I \times A$$
$$T \times 32.2 = 300 \times 3 = 900$$
$$T = \frac{900}{32.2} = 27.9 \text{ (lb.-ft.)}$$

29. Rotary Inertia.—The opposition of a body to being set in translation is determined completely by the mass of the body. Experiment has shown that the opposition of a body to being set in translation is proportional to the mass of the body and does not depend on anything else. It is, however, found that the opposition which a body offers to being rotated about a fixed axis depends not only on the mass but also the way in which this mass is distributed about the axis. This opposition which is called the **rotary inertia or the moment of inertia is proportional to the mass and to the square of the distance of the mass from the axis of rotation.** For this reason when it is desired to make the rotary inertia of a flywheel of given mass as large as possible the mass of the wheel is concentrated in the rim. When the mass is concentrated near the axis the tendency of the wheel to continue in motion or its resistance to being set in motion is much less. In order to calculate the rotary inertia multiply each mass by the square of its distance from the axis of rotation and then add together all of these products.

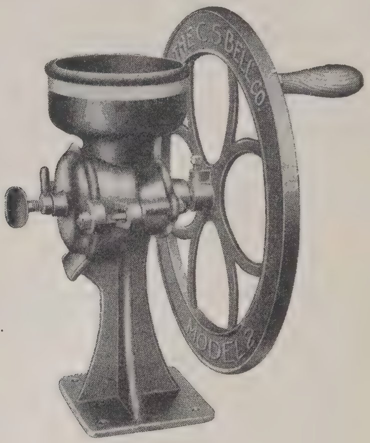


FIG. 12.—Rotary inertia of a wheel.

Thus the rotary inertia of a sphere A of mass m about 0 (Fig. 13a) is, $I = mr^2$, where r is the distance of 0, the axis of rotation from the center of the sphere. If the axis of rotation is moved to 0′, a distance a, the rotary inertia of the sphere about 0′ is

$$I' = (a + r)^2m$$

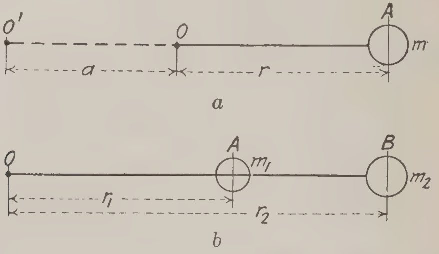


FIG. 13.—Rotary inertia of spheres about an axis.

Since $(a + r)^2$ is greater than r^2 the rotary inertia of the sphere is larger about the axis through 0′ than it is about the axis through 0. Hence the further the axis is from the mass, the greater is the rotary inertia.

If two masses A and B (Fig. 13b) are fastened together by a weightless rod the rotary inertia of these masses about an axis through 0 is

$$I = m_1r_1^2 + m_2r_2^2$$

where r_1 and r_2 are the respective distances of A and B from 0. In like manner the rotary inertia of any number of masses is,

$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \text{etc.}$$

where r_1, r_2, r_3, etc., are the distances of the masses from the axis of rotations.

Example.—Find the rotary inertia of a mass of 10 lb. and another mass of 15 lb. about an axis of rotation which is 3 ft. from the 10 lb. mass and 4 ft. from the 15 lb. mass.

Rotary inertia = sum of products of the masses by the square of the respective distances from the axis.

$$\begin{aligned} I &= m_1r_1^2 + m_2r_2^2 \\ m_1 &= 10, \quad r_1 = 3 \text{ ft.} \\ m_2 &= 15, \quad r_2 = 4 \text{ ft.} \\ I &= 10 \times (3)^2 + 15 \times (4)^2 \\ &= 90 + 240 = 330 \text{ (lb.-ft.)} \end{aligned}$$

30. Uniform Circular Motion.—If a body moves in a circular path (Fig. 14) in such a way that it always passes over equal distance in equal intervals of time it is said to have uniform circular motion. The magnitude of its velocity is constant, but its direction of motion is always changing. Such a body has an acceleration but this acceleration consists in a change in the direction whereas the other accelerations we have considered

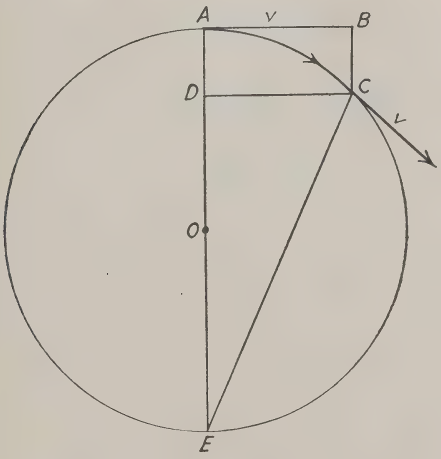


Fig. 14.—Uniform circular motion.

have consisted in a change in the magnitude of the velocity. A force is necessary to produce a change in the direction of the motion of a body just as a force is necessary to produce a change in its rate of motion. Since in circular motion this force produces no change in the magnitude of the velocity but only a change in the direction of motion, the force must act at right angles to the direction of motion. Otherwise it would act partly in the direction of motion and thus produce either an increase or a decrease in the magnitude of the velocity. Since the force is at right angles to the direction of motion and the motion is in a circular path, the force is along the radius of the circle and directed toward the center of the circle. The rate at which the direction of the motion is changing is constant and the force which produces it must therefore be constant. This force produces an acceleration which is found to be (Appendix B)

$$a = V^2/R \text{ cm. per sec. per sec.}$$

where $V =$ the velocity of the body in the circular path
$R =$ the radius of the circular path.
$a =$ the acceleration toward the center

This expression may be written in another form. If N denotes the number of revolutions per sec.,

$$V = 2\pi RN$$

$$a = \frac{4\pi^2R^2N^2}{R} = 4\pi^2RN^2$$

From this last expression it is evident that the acceleration increases when the radius of the path is increased, and it also increases when the number of revolutions is increased.

Example.—A machine is running on a circular speedway with a velocity of 120 ft. per sec. The radius of the speedway is 1,000 ft. What is the acceleration toward the center?

$$\text{Acceleration} = \frac{V^2}{R}$$

$$= \frac{(120)^2}{1{,}000} = \frac{14{,}400}{1{,}000} = 14.4 \text{ ft. per sec. per sec.}$$

31. Centrifugal Force.—Newton's second law of motion states that wherever an acceleration is produced a force must be applied. It has been seen that a body moving in a circular path with uniform velocity has an acceleration toward the center of the circle. To produce this acceleration and keep the body from flying off tangent to the circle a force must be applied at right angle to the direction of motion. This force which is directed toward the center of the circle is called the centrifugal force. To find its magnitude, recall that Newton's second law of motion is,

$$F = Ma/g$$

Now the acceleration in the case of uniform circular motion is,

$$a = \frac{V^2}{R}$$

Whence

$$F = M\frac{a}{g} = \frac{MV^2/R}{g}$$

or

$$= \frac{4\pi^2 RN^2 M}{g}.$$

Example.—A mass of 10 lb. at the end of a string is being whirled in a circle of radius 3 ft. with a constant velocity of 10 ft. per sec. What is the pull in the string?

$$\text{Centrifugal force} = \frac{M}{g}\frac{V^2}{R}\text{ lb.}$$

$$F = \frac{10}{32.2}\frac{10^2}{3} = \frac{1{,}000}{96.6} = 10.3\text{ lb.}$$

This property of a body to move on in a straight line is of much importance in nature. The mud on a rotating carriage wheel or the water on a grind stone tends to fly off along the tangent. When a horse runs on a circular race track he has some difficulty in keeping on the track. Where a train runs on a circular piece of track the outside rail is elevated in order to overcome the tendency of the train to jump the track. The greater the curvature of the track the greater is this tendency for a given velocity of the train. As the velocity of the train increases, the centrifugal force increases and for this reason the train slows down when running around a curve.

32. Cream Separator.—In a cream separator (Figs. 15a and 15b) the difference between the centrifugal force on the milk and on the cream makes it possible to separate the milk from the cream by rotating them about an axis. The heavier milk moves out further than the lighter cream because the centrifugal force on the milk is greater than it is on the cream. This leaves the cream near the axis of the separator and the milk finds a place nearer the outer edge of the separator.

33. Washing Machine.—This same principle is used in one type of washing machine. When the clothes in the washer have been washed, the inner perforated tub is lifted above the water line and the tub is then made to revolve rapidly about its axis. The water tends to fly off the clothes

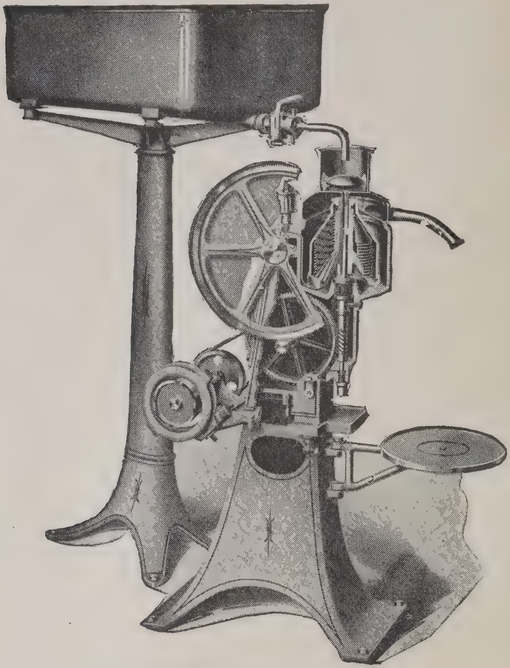


FIG 15*a*.—Cream separator.

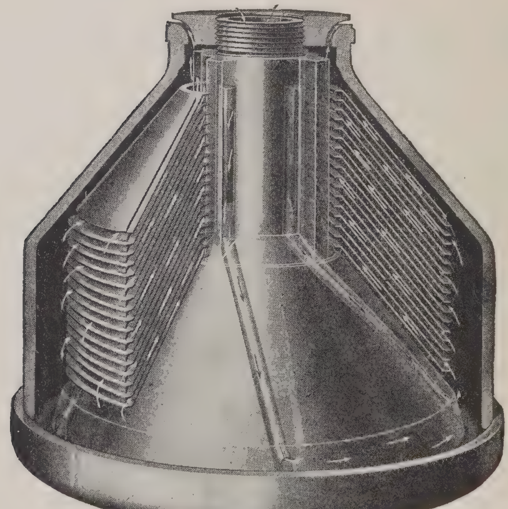


FIG. 15*b*.—Bowl of a cream separator.

as the water flies off a rapidly turning grindstone. In a short time the clothes are as dry as if they had been run through a wringer.

34. Centrifuge.—The centrifuge, which is widely used in the separation of liquids of unequal densities, depends for its operation on centrifugal force. One type of this machine (Fig. 16) consists of a wheel which rotates in a horizontal plane. To this wheel are attached buckets which are vertical when the wheel is at rest, but when the wheel is revolving rapidly the buckets assume a position so that their axes are horizontal. If a mixture of liquids of unequal densities is introduced into the buckets and the wheel is revolved rapidly, the heavier liquids go as far away as possible from the axis of rotation while the lighter liquids come in near the axis of rotation. This means that the heavier liquids will be at the bottom of the buckets while the lighter liquids are at the top. When the centrifuge is stopped, the buckets return to their normal positions with their axes vertical. The lighter liquids are found on the top of the bucket and the heavier liquids at the bottom. A separation of the liquids has thus been brought about. Such a centrifuge may also be used for the separation of soils.

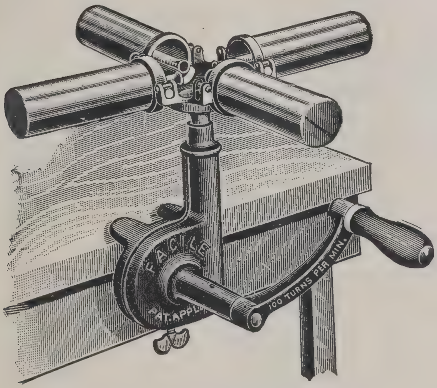

FACILE
PAT. APPL.
100 TURNS PER MIN.

FIG. 16.—Babcock milk tester.

A modification of the centrifuge known as a Babcock milk tester is widely used in the examination of milks. The heavier and lighter constituents of the milk are separated by this method.

35. Centrifugal Switch.—Automatic switches are provided for charging the battery of an automobile when the speed of the engine reaches a certain value. One of these switches depends for its action on centrifugal force. The terminals *A* and *B* of the battery (Fig. 17) are not joined together while the engine is not running. As the speed of the engine increases the mercury which rested on the bottom of the cylindrical cavity is uniformly distributed by the action of centrifugal force over the inner surface of the cavity. The mercury then connects the insulated segments of the rotating cavity and thus connects the terminal *A* with the terminal *B*, so that the battery charges while the engine has a speed sufficient to keep the mercury against the surface of the cavity. When the speed of the engine becomes low enough for the mercury to collect in a pool at the bottom of the cavity the battery circuit is opened.

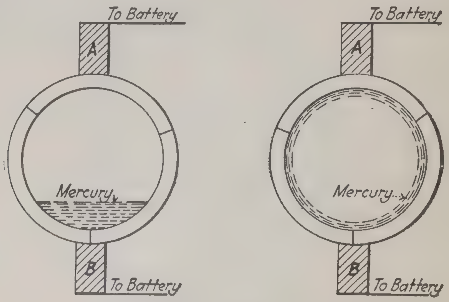


Fig. 17.—Centrifugal switch.

36. Steam Engine Governor.—The governor of a steam engine depends for its action on centrifugal force. The spheres *C* (Fig. 18) revolving about a vertical axis are acted on by the force of gravity pulling them into a vertical position and the centrifugal force tending to carry them away from the axis of rotation. As the speed of rotation of the governor is increased, the centrifugal force increases and the spheres are lifted further away from the axis. When this happens they partially shut off the supply of steam from the engine. This tends to make the engine slow down.

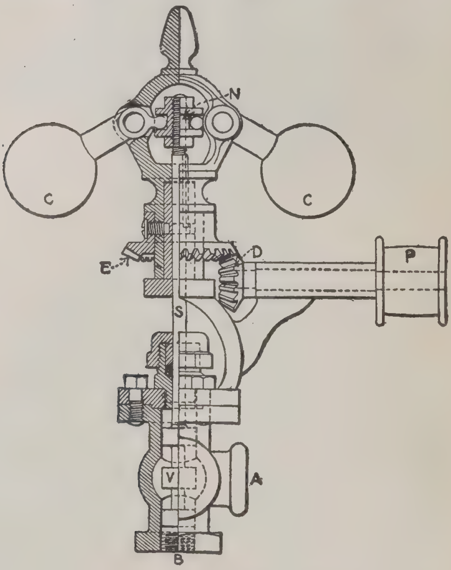


Fig. 18.—Steam engine governor.

The spheres C and C then descend again, opening the steam valve somewhat and increasing the supply of steam. The speed of the engine again increases. In this way the speed of the engine is kept constant by regulating the supply of steam.

Problems

1. A flywheel starting from rest acquires an angular velocity of 300 radians per minute in 30 min. Find the angular acceleration in radians per sec. per sec.

2. A wheel which is revolving about an axis through its center gains in angular speed at the rate of one-half of a revolution per sec. Find its angular velocity in radians per sec. at the end of two minutes.

3. Find the torque necessary to give a body, the rotary inertia of which is 200 (lb.-ft.), an angular acceleration of 3 radians per sec. per sec.

4. The rotary inertia of a body about a certain axis is 120 (lb.-ft.). There is applied to it a torque of 10 (lb.-ft.). Find the angular velocity of the body at the end of 3 min.

5. Calculate the rotary inertia of two spheres of masses 30 and 40 lb. when they are at a distance of 4 and 5 ft. respectively from the axis of rotation.

6. How far must the axis of rotation be from the center of a sphere of mass 12 lb. in order that the rotary inertia about this axis be 192 (lb.-ft.)?

7. A stone weighing 5 lb. is whirled in a circle by means of a string which is 4 ft. long. The string breaks and the stone flies off with a velocity of 25 ft. per sec. Find the pull in the string when it broke.

8. The force required to hold a mass of 10 g. on the rim of a fly wheel which is 50 cm. in radius is found to be 50 g. Find the number of revolutions which the wheel makes each second.

9. A mass moving in a circular path of 3 ft. radius makes 5 revolutions per min. It requires a force of 2 lb. to keep the mass from leaving the circular path. Find the number of pounds in the mass.

CHAPTER IV

COMPOSITION AND RESOLUTION OF FORCES

37. Representation of Forces.—In order to completely define a force it is necessary to give its magnitude, its direction and its point of application. It is easier to form a picture of the force if it is represented by an arrow. The length of the arrow represents the magnitude of the force, the direction of the arrow represents its direction and the tail of the arrow shows the point at which the force is applied.

It may be that two forces act on the body at the same time. Suppose a force of 40 lb. is acting north and a force of 30 lb. is acting east. Two arrows are then drawn, one pointing north and the other pointing east. The arrows begin at the same point, showing that the forces are applied at the same point. If the arrow pointing north is made 4 in. long, then each inch of its length represents a force of 10 lb. The arrow pointing east must now be made 3 in. long in order to represent that force.

38. Resultant.—If a force acts on a body which is free to move, the body moves in the direction of the force. When two horses pull in opposite directions on a load, the load moves in the direction of the stronger pull. The force tending to displace the load in this case is the difference between the two pulls. When the forces act in the same direction the effective force is the sum of the two pulls. This effective or equivalent force is known as the **resultant** of the forces. In the case of two oppositely directed forces acting at the same point this resultant is found by taking the difference between the applied forces and its direction is the direction of the greater force. When the forces act in the same direction the resultant is found by adding the applied forces.

39. Resultant of Forces Acting at Right Angles.—When two forces acting at right angles are applied to a body the effect is the same as if some single force acted on the body. For example, if two ropes are fastened to a sled and two men pull on them at right angles to each other with a force of 50 and 75 lb. respectively the sled will move along the diagonal of a parallelogram of which the forces form the sides. Lay off as in Fig. 19 a line *OB*, 3 in. long. Each inch represents a force of 25 lb. Now at right angles to this line lay off another line which is 2 in. long. This line represents the other force in magnitude and direction. Under the simultaneous action of these forces the body will move along the diagonal *OC*. This diagonal represents the direction and magnitude of the resultant or equivalent force.

To calculate the magnitude of the resultant force when the forces are acting at right angles, it is only necessary to make use of the familiar relation between the sides and the hypotenuse of a right triangle. Thus,

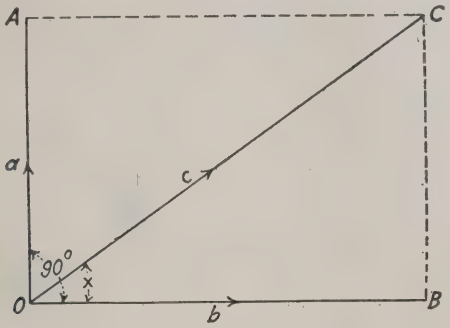


FIG. 19.—Resultant of forces acting at right angles.

$$\overline{OC}^2 = \overline{OA}^2 + \overline{OB}^2 \text{ or } R = \sqrt{X^2 + Y^2}$$

The direction of the resultant is,

$$\tan x = \frac{BC}{OB}$$

Example.—A body lying on a horizontal table is acted on by two forces which are at right angles to each other. One of these forces has a magnitude of 25 lb. and points north. The other has a magnitude of 50 lb. and points east. Find the magnitude and direction of the resultant force.

$$R = \sqrt{X^2 + Y^2}$$

$$\tan x = \frac{Y}{X}$$

$$R^2 = 25^2 + 50^2$$

$$R = 55.9 \text{ lb.}$$

$$\tan x = \frac{50}{25} = 2$$

40. Resultant of Forces not at Right Angles.—Suppose that two mules on opposite sides of a canal are pulling on a boat by means of two ropes. When the boat is some distance behind the mules the angle between the forces will be less than a right angle. In order to find the resultant of these two forces the arrows are drawn to represent the forces as in the preceding case but in this case a parallelogram (Fig. 20) instead of a rectangle is constructed. The resultant is then represented by the diagonal of the parallelogram. By this graphical method the magnitude and direction of the resultant can be obtained. If it is desired to calculate either the direction or the magnitude of the resultant it is necessary to use the well-known trigonometric relation between the sides of a triangle. Thus,

$$c^2 = a^2 + b^2 + 2ab \cos x$$

The correctness of this equation can be tested by suspending three weights as in Fig. 21 and measuring the angle ACB for different magnitudes of the wieghts.

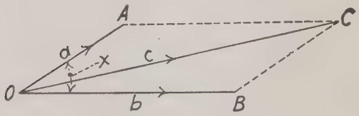


Fig. 20.—Resultant of forces not at right angles.

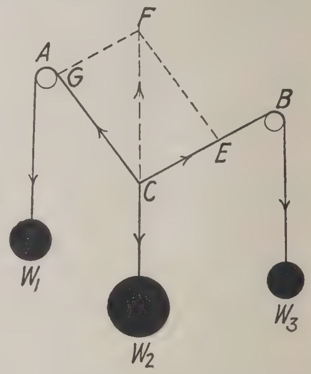


Fig. 21.—Experimental proof of resultant of forces.

41. Illustrations of Composition of Forces.—In Fig. 22 is shown a pendulum consisting of a heavy bob A suspended by a cord attached at O and having a spring balance at C. By means of a second spring balance at D the bob is pulled aside from its vertical position. This pull is exerted in the horizontal direction and is read on the spring balance at D. The spring balance at C gives the tension in the string for any position of the bob. When the bob is in the vertical position the reading of the spring balance C is the weight of the bob. When the bob is pulled aside from this position there are three forces acting on the bob,—the horizontal pull, the tension in the string and the weight of the bob. The reading of the spring balance C is the resultant of the horizontal and vertical forces on the bob.

A simple crane (Fig. 23) illustrates the composition of the two forces. Let the top of the beam *BC* be connected to the wall at *A* by means of a cord in such a way that when the beam is in equilibrium the cord *AB* is at right angles to the wall. A spring balance inserted in this cord indicates the tension in it. From *B* is hung a weight *W* and under the action of these two forces the beam exerts a thrust which is just enough to overcome their combined action. This thrust is equal in magnitude and opposite in direction to the resultant of these two forces. To get the force diagram lay off on a vertical line a distance which represents the weight *W* and a right angle to this line lay off a second line which represents the magnitude and direction of the tension *T* in the cord *AB*. Now draw the rectangle of which *T* and *W* are the two sides. The diagonal of this rectangle represents the resultant of the horizontal and vertical forces, and is equal to the force exerted on the beam. If a weight of 300 lb. is hung from *B* and if the tension in the cord *AB* is 200 lb. then the resultant force on the beam is

$$R = \sqrt{300^2 + 200^2} = 3606 \text{ lb.}$$

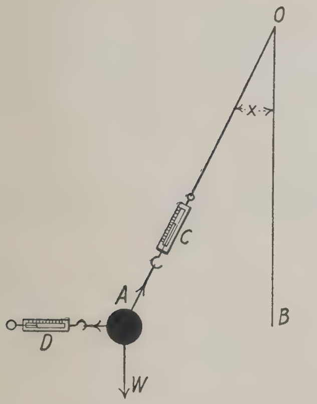


Fig. 22.—Forces acting on a pendulum bob.

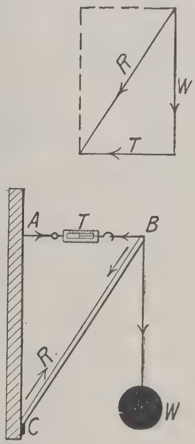


Fig. 23.—The simple crane.

42. Derrick Crane.—A derrick crane (Fig. 24) is a modification of a simple crane. It differs from the simple crane in the fact that the force in the rope *AB* is not always perpendicular to the post to which it is fastened. Instead of obtaining the resultant by drawing the rectangle of forces it is now necessary to draw a parallelogram. As the boom is elevated more and more, the force which it must sustain is more and more increased.

43. Stump Puller.—In pulling stumps farmers sometimes fasten a rope or chain around the base of the stump and then pass this chain over the top of a heavy board as shown in Fig. 25. There is a pull in the rope tending to uproot the stump and a force on the board tending to push it into the ground. At the point at which the rope passes over the board there are three forces acting, the tension in the rope between the stump and the board, the tension in the rope between the team and the board and the thrust of the board on the rope. The thrust of the board on the rope is equal in magnitude and opposite in direction to the resultant of the tensions in the two parts of the rope.

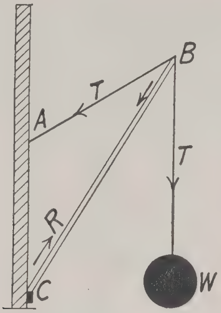


FIG. 24.—The derrick crane.

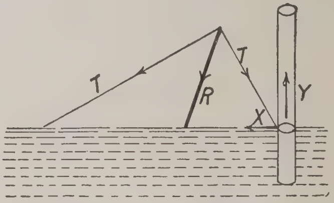


FIG. 25.—The stump puller.

44. Resolution of Forces.—It has been seen that two forces can be combined into a single force. On the other hand a single force may be broken up into two or more forces to which it is equivalent. When one force is given it is possible to find two other forces which when applied simultaneously will produce the same effect as the single force. This process of splitting up a single force into two or more parts is known as **the resolution of forces** and the parts into which the force are split up are called the **components of the force.**

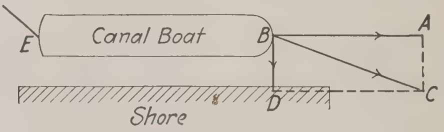


FIG. 26.—Components of the force on a canal boat.

Consider a canal boat as indicated in Fig. 26 and let a force be applied to it in the direction BC. This force produces two distinct effects on the boat. It moves the boat forward and also pulls it to the bank. Two separate forces might have been applied with the same result. The single force *BC* is therefore equivalent to two forces, one urging the boat forward and the other pulling it to the shore. These forces which produce the same effect as *BC* are the components of *BC*. The components of this force are represented by the sides of a parallelogram of which the original force is the diagonal. To find the components of a force it is necessary to know the direction in which the components are desired since there are any number of parallelograms which may be constructed on a given diagonal. When the directions of the components have been given, the magnitudes of the components are found by constructing a parallelogram whose diagonal is long enough to represent the original force and whose sides have the directions of the desired components.

45. Rectangular Components of a Force.—Most frequently the force is resolved in such a way that the components are at right angles to each other. In Fig. 27 *OR* represents a given force and the components are desired along *OX* and *OY* two directions which are at right angles to each other. By completing the rectangle *OARB* the magnitudes of the components are found to be *OA* and *OB*. The relation between the components and the original force is given by the trigonometric formulæ,

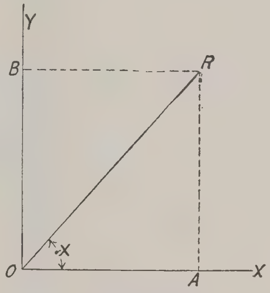


FIG. 27.—Rectangular components of a force.

$$OA = OR \cos x.$$
$$OB = AR = OR \sin x.$$

Example.—A mass weighing 10 lb. is kept at rest on a smooth plane which is inclined 30° to the horizontal, by a force which acts parallel to the plane. Find the force parallel to the plane and the force at right angles to the plane.

Force parallel to plane = weight × sin 30°
Force at right angles to plane = weight × cos 30°
Force parallel to plane = $10 \times \frac{1}{2} = 5$ lb.
Force perpendicular to plane = $10 \times \frac{1}{2}\sqrt{3} = 8.6$ lb.

When a weight *W* hangs from a horizontal bar *AC* as indicated in Fig. 28, the tension *T* in the string *BC* can be resolved into two components, one horizontal and the other vertical. The horizontal component,

$$R = T \cos x,$$

causes a pressure against the vertical wall. This pressure is just balanced by the push of the wall against the bar. The vertical component,

$$Y = T \sin x,$$

tends to lift the end C of the bar AC. This tendency to lift the bar AC is just balanced by the pull of gravity on the weight W. Hence

$$Y = T \sin x = W.$$

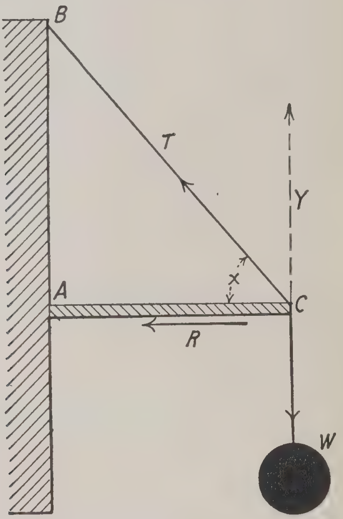


Fig. 28.—Resolution of tension in a string.

Example.—A mass of 100 lb. hangs from the bar at C (Fig. 28). Find the tension in the string BC which makes an angle of 45° with the bar. Assume AC is without weight.

$$\begin{aligned} \text{Weight} &= \text{tension} \times \sin 45^\circ \\ W &= T \times \sin 45^\circ \\ 100 &= T \times 0.707 \\ T &= \frac{100}{0.707} = 141 \text{ lb.} \end{aligned}$$

If a mass rests on an inclined plane (Fig. 29a) the force of gravity acting on the mass can be resolved into two components, one parallel to the plane and the other perpendicular to the plane. The former is $M \sin x$ and the latter is $M \cos x$. The component parallel to the plane causes the mass to slide down the plane and the component perpendicular to the plane presses the mass against the plane.

Example.—A mass of 300 lb. rests on a plane making an angle of 30° with the horizontal. What is the force down the plane and the force at right angles to the plane?

Force down the plane = $M \sin 30° = 300 \times 0.5 = 150$ lb.

Force perpendicular to plane = $M \cos 30° = 300 \times 0.866 = 260$ lb.

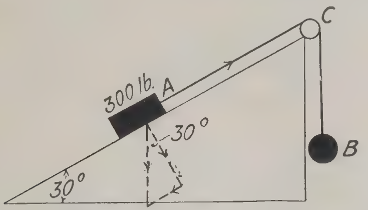


FIG. 29*a*.—Resolution of forces on an inclined plane.

46. Draft of a Horse.—The direction of the traces exerts an important influence on the amount a horse can draw. If the line of draft is horizontal there is a tendency to lift the fore feet of the horse from the ground. If the traces are moved up further this tendency to tip the horse up on his hind feet is increased and the amount which the horse can pull is decreased. If, however, the traces are lowered below the horizontal position the tendency to raise the horse from his fore feet is lessened and the amount the horse can pull is increased. If the traces make an angle of about 20° with the horizontal the horse can draw about double as much load as when the traces are in the horizontal position. The weight of a rider on the horse still further increases the draft. The rider is more effective when he sits forward than when he sits near the rear of the horse.

FIG. 29*b*. Grain elevator—an inclined plane.

47. The Line of Draft in a Plow.—In a walking plow as shown in Fig. 30 the pull of the horses is exerted in the direction of the line AD. This force is equivalent to two forces one tending to lift the plow directly out of the ground and the other tending to drag the plow forward. The less inclined the line of draft to the horizontal the greater is the horizontal force and the less is the vertical force.

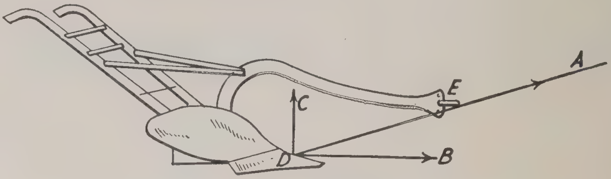


Fig. 30.—Resolution of force acting on a plow.

48. Side Draft in a Plow.—If four horses are hitched abreast as shown in Fig. 31 the true line of draft does not coincide with the line of load. This gives a side draft which presses the plow against the furrow. This is a case of the resolution of forces. The pull of the team on the plow is resolved into two forces, one in the direction of the furrow and the other at right angles to it. The first force is effective in moving the plow forward but the second one produces pressure against the furrow and thus causes it to be more difficult to pull the plow.

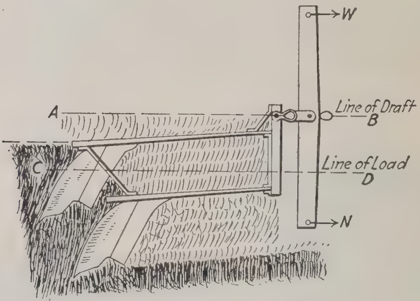


Fig. 31.—Side draft in a plow.

49. A Road Machine.—In a road machine (Fig. 32), used for grading the pull of the horses on the machine is resolved into two forces, one at right angle to the scraper and the other along it. The effect of the first force is to push the dirt forward. The other force pushes the dirt toward the side of the road. If the inclination of the scraper to the road is changed, the force tending to push the dirt to one side of the road is changed. This force is greatest when the scraper is nearly parallel to the road and it is least when the scraper is nearly perpendicular to the road. The forward force is greatest when the scraper is at right angles to the road and least when it is nearly parallel to the road. For this reason whenever the ground is very hard and the force to pull the scraper becomes excessive, the operator changes the inclination of the scraper in order to lessen the load.

Problems

1. A horse is hitched to a sleigh so that the traces make an angle of 30° with the surface of the snow. Suppose the sleigh offers a resistance of 500 lb. to the horizontal motion, how much must the horse pull to move it?

2. A sled weighing 250 lb. rests on the side of a hill which rises 1 ft. in 30 ft. along the incline. Neglecting friction, find the force necessary to keep the sled from sliding down the hill.

3. A man drags a weight along the surface of the ground by means of a rope which passes over his shoulder. If the rope makes an angle of 45° with the ground, what force is acting parallel to the ground when the tension in the rope is 150 lb.? Neglect friction.

4. A pair of horses pulling at an angle of 90° to each other exert a forward force of 1,800 lb. If one horse pulls twice as much as the other, what does each horse pull?

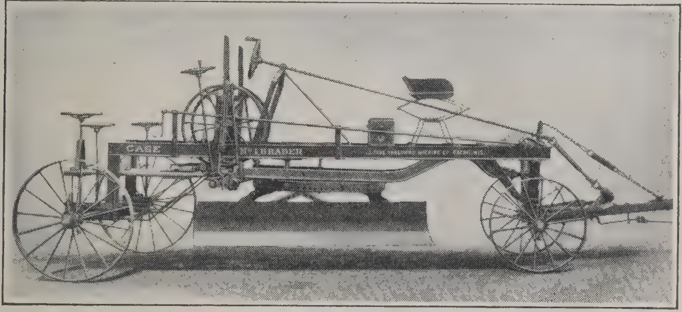


Fig. 32.—Road machine.

5. A fish caught by a pole and line pulls with a force of 4 lb. The angle between the pole and the line is 45. Find the pull at right angles to the length of the pole.

6. A train of 300 tons is drawn up an incline of 1 in 60 and the frictional resistance to motion is 1.5 per cent of its weight. Find the force required to pull the train.

7. A mass of 30 kg. hung by a cord 2 meters long is drawn 30° from the vertical by a horizontal force. Find the tension in the cord and the horizontal force.

8. Three forces of 4 lb., 6 lb., and 8 lb. act simultaneously on a body without producing any motion. Find the angles the forces make with each other.

9. Find the resultant of three forces of 10 lb., 15 lb., and 20 lb. which make angles of 30°, 0° and 45° respectively with the horizontal axis.

10. The resultant of two forces has a magnitude of 600 lb. and makes an angle of 30° with one of the forces which has a value of 300 lb. What is the magnitude of the other force?

CHAPTER V

EQUILIBRIUM OF FORCES

50. Conditions of Equilibrium.—A body is said to be in equilibrium when the forces acting on it cause neither linear nor angular acceleration in the body. In order that the body be in equilibrium under the action of any number of forces two conditions must be satisfied. **The sum of the forces acting on the body in any direction must be equal to zero.** When this condition is satisfied the body will have no linear acceleration in any direction, since there is no net force acting on it. In order that there be no angular acceleration **the moments of force tending to produce clockwise rotation about an axis must be equal to the sum of the moments of force tending to produce counterclockwise rotation about this same axis.** Where this second condition is fulfilled there is no net torque acting on the body and its motion of rotation will not change from time to time. That is, if the body is already at rest it will not start into rotation and if it is already in rotation its rate of rotation will not change.

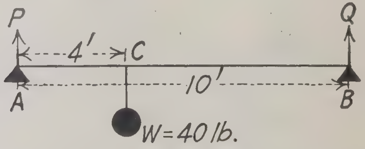


Fig. 33.—Equilibrium of a weight on two supports.

Example.—A weightless rod AB (Fig. 33) has its ends resting on supports. The rod is 10 ft. long and carries a weight W of 40 lb. at a point 4 ft. from A. Find the force exerted by each support.

Forces downward = forces upward.

$$40 = P + Q$$

Moments of force clockwise = moments of force counterclockwise
Taking moments about C,

$$4 \times P = 6 \times Q$$

whence $$P = 24 \text{ lb., and } Q = 16 \text{ lb.}$$

51. Center of Gravity.—Every particle of a body possesses weight so that the pull of the earth on the body is made up of a large number of forces directed toward the center of the earth. For bodies of ordinary size these forces are essentially parallel to each other. Now if two or more parallel forces act on a body they may be replaced by a single force which will produce the same translation and rotation as were produced by all of the forces. The magnitude of this single force is obtained by adding together all the parallel forces and the point at which this resultant force must be applied is found by requiring that the sum of the moments tending to produce clockwise rotation about this point is equal to the sum of the moments tending to produce counterclockwise rotation.

Suppose that there are two particles of mass m and M (Fig. 34) at A and B respectively and that these particles are connected with a light rod. These particles are attracted to the earth with forces which are nearly parallel to each other. If a point C in the rod is so chosen that,

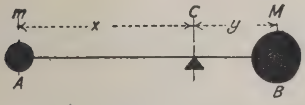


Fig. 34.—Center of gravity of two spheres connected by a light rod.

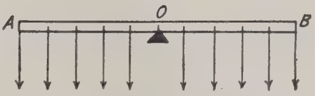


Fig. 35.—Center of gravity of a uniform bar.

$$m \times AC = M \times BC$$

then the moments of force tending to turn the rod clockwise is just equal to the moments tending to turn it counterclockwise. If the rod is turned into some other position the forces will be inclined at different angles to the rod but the moments of force about C will still balance each other. Hence it is possible to regard the two masses as concentrated at C since the action of gravity on these two masses concentrated at C is the same as its action when the masses are at the ends of the rod. Such a point at which it is possible to collect the masses without changing the action of gravity on them is called **the center of gravity** of the masses.

If instead of these two masses we take a long thin rod (Fig. 35), then every particle of matter in the rod is acted on by the force of gravity and the direction of these forces are essentially parallel to each other. By an extension of the reasoning by which two masses were considered, it is possible to find a point O in the rod such that the moments of force tending to turn the rod clockwise about O is equal to the moments tending to turn it counterclockwise. This point is the center of gravity of the rod and at it we may consider all the mass of the rod to be concentrated.

Whatever the shape and size of the body, it is always possible to find one point at which a force equal and opposite to the weight of the body can be applied so that the body will remain at rest. About this point the body has no tendency to rotate under the action of gravity and at this point we may consider all the mass of the body to be concentrated. If the body is balanced on a knife edge this point will lie directly above the knife edge. The center of gravity need not necessarily lie in the substance of the body. Thus the center of gravity of a uniform ring lies outside the material of the ring at its center.

Example.—Two weights, one of 5 lb., and the other of 15 lb., are connected by light stiff rod 3 ft. long. Find the distance of the center of gravity from the 5-lb. weight.

Let x = the distance of the center of gravity from the 5 lb. weight.

$3 - x$ = the distance of the center of gravity from the 15 lb. weight.

Taking the center of gravity as the point about which to take moments,

$$\begin{aligned} 5x &= 15\,(3 - x) \\ 5x - 45 &= -15x \\ 20x &= 45, \\ x &= 2.25 \text{ ft. distance from center of gravity to 5 lb. weight.} \end{aligned}$$

52. Method of Finding the Center of Gravity.—If a body be freely suspended from a point near one of its extremities it will come to rest in such a position that the center of gravity is in the vertical line passing through the support. If this line be indicated by the position of a plumb line and marked on the body the center of gravity lies in this line. If another point of suspension be selected and a vertical line through it marked in the same way, it will also contain the center of gravity. Since the center of gravity lies in each of these lines it must lie in their intersection.

53. Simple Cases of Center of Gravity.—The center of gravity of a rod of uniform density and of the same breadth and thickness throughout its length lies at the center of the rod. If such a rod is divided into particles, for every particle on one side of the middle point of the rod there is an equally heavy particle on the other side at an equal distance from the middle. In a rectangular plate of uniform thickness and density the center of gravity lies at the intersection of the diagonals of the rectangle. This fact can be seen by dividing the plate up into narrow strips which may be considered as uniform rods in the preceding case. In the same way it can be shown that the center of gravity of a triangular lamina of uniform thickness and density lies at the point of intersection of the medians drawn from the angles of the triangles to the opposite sides. The centers of gravity of a sphere and a solid cube lie at their geometric centers.

54. Types of Equilibrium.—The equilibrium of a body may be **stable, unstable or neutral.** When a body returns to its original position after being slightly disturbed, the equilibrium is said to be stable. A cone standing on its base is an illustration of this type of equilibrium. When a cone (Fig. 36) on its base is raised slightly from the table on which it rests it returns to its original position on being released. It is in **stable** equilibrium. If, however, the cone rests on its vertex and is then slightly displaced it tends to fall into a new position rather than return to its original position. In this case the cone is in unstable equilibrium. Any body which tends to get as far as possible from its original position when disturbed is in **unstable** equilibrium. A sphere resting on a horizontal table when slightly displaced tends neither to return to its former position nor to go still further away from it but it remains in any position in which it finds itself. Such a body is in **neutral** equilibrium.

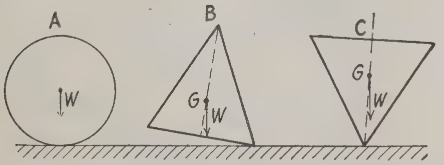


Fig. 36.—Stable, unstable and neutral equilibrium.

55. Stability of a Body.—The position of the center of gravity is of much importance in determining the stability of a body. The lower the center of gravity the greater the stability of the body and the more difficult it is to overturn. A body becomes unstable as soon as the vertical line through the center of gravity falls outside its base. The body which must be lifted the greater amount in order to make the vertical through its center of gravity fall outside the base is the more stable. In case of a wagon, Fig. 37, in which the load is some distance above the ground, the center of gravity of the load is high. If the vertical line through the center of gravity of the load fall inside the wheels of the wagon the load will not overturn. If, however, the wagon is driven over ground which is inclined so that one wheel is raised above the other, the vertical line through the center of gravity approaches the point at which the wheel is in contact with the ground. The load is now in unstable equilibrium. A further elevation of one wheel above the other causes the line of action of gravity on the load to fall outside the wheels and the load will overturn.

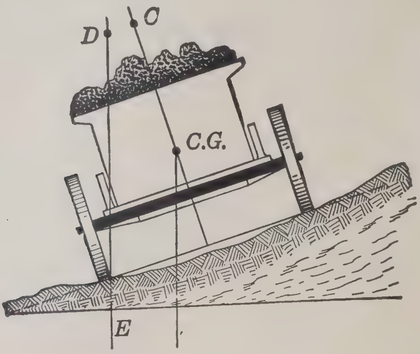


Fig. 37.—Equilibrium in a loaded wagon.

In order to avoid upsetting it is, therefore, desirable to have the load as low as possible in order that the center of gravity of the load may be as near as possible to the ground.

Problems

1. A mass of 400 g. is connected by means of a very light rod 36 cm. long to another mass of 750 g. Find the center of gravity of these two masses.

2. Three masses of 5 lb., 12 lb. and 30 lb. are mounted on a very light rod which is 3 ft. long. The 5 lb. mass is at one end; the 30 lb. mass is at the other end and the 12 lb. mass is in the middle. Find the position of the center of gravity.

3. Find the center of gravity of three masses located at the vertices of an equilateral triangle. The masses are 450 g., 240 g., and 150 g.

4. A uniform bar 36 in. long is bent to an angle of 90° at its middle point. What is the position of its center of gravity?

5. A board of varying thickness is 30 ft. long and weighs 150 lb. When a boy weighing 40 lb. stands on one end, the balancing point is 6 ft. from the end on which the boy stands. Find the position of the center of gravity of the board.

6. A weight of 2 tons is supported at *B*, Fig. 38. Find the horizontal and vertical forces on *A* and *C*.

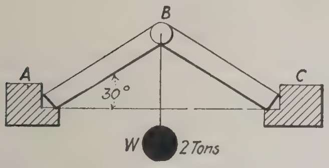


Fig. 38.—Forces exerted by trusses.

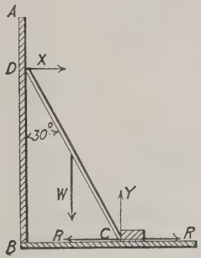


Fig. 39.—Forces exerted by a ladder.

7. A ladder resting against a smooth wall, Fig. 39, makes an angle of 30° with the vertical wall. If the ladder weighs 100 lb. and is 20 ft. long find the force it exerts on the vertical wall and the horizontal and vertical force on the floor at *C*.

CHAPTER VI

WORK, POWER, ENERGY

56. Work.—Work is done when a force is exerted through a certain distance. When a window is raised work is done since a force is exerted to overcome the pull of gravity on the window. This force acts through the distance through which the window is raised. When a bucket of water is carried from the cellar to the attic, work is done which is equal to the weight of the bucket times the distance through which it is carried. **The measure of the amount of work is found by taking the product of the force and the distance through which it acts.**

$$\text{Work} = \text{force} \times \text{distance}$$
$$W = F \times S$$

If the force does not produce a displacement of the body, no work is done in the sense in which work is used in physics. A man holding a ten pound weight does no work in this sense because there is no displacement or movement of the mass. He exerts a force sufficient to overcome the pull of gravity on the mass. This of course requires an effort but it is not work in the sense in which it is now being used.

57. Units of Work.—Since work is measured by the product of the force times the distance through which it acts, in order to measure work it is necessary to measure two quantities,—force and distance. In the English system the force is measured in terms of a unit of force, which is equal to the pull of gravity on a mass of one pound, and the distance is measured in feet. In this system the unit of work is called the **foot-pound.**

A foot pound is defined to be the amount of work which is done when a mass of one pound is raised a distance of one foot against the action of gravity.

In the metric system the unit of work may be chosen as the gram-centimeter or the kilogram-meter. **The gram-centimeter is the amount of work done when a mass of one gram is**

lifted a distance of one centimeter against the action of gravity. The kilogram-meter is the work done when a mass of one kilogram is lifted a distance of one meter against the action of gravity.

Example.—A ton of hay is lifted from the floor into the loft of a barn. The height of the loft above the floor is 10 ft. Find the work done.

$$\text{Work} = \text{force} \times \text{distance}$$
$$2{,}000 \text{ lb.} \times 10 \text{ ft.} = 20{,}000 \text{ ft.-lb.}$$

Problems

1. How much work is done in raising 250 cu. ft. of water to the top of a building which is 75 ft. high, if each cubic foot of water weighs 62.5 lb.?

2. A well was dug 30 ft. deep and 5 ft. in diameter. How much work was done in lifting the dirt to the level of the surface, if each cubic foot of dirt weighed 175 lb.?

3. From clouds which were 1 mile above the surface of the earth enough rain fell to cover the earth to a depth of 1 in. over an area of 2 sq. mi. How much work was done in raising the water to the clouds?

4. On the floor of a barn there is a bale of hay weighing 100 lb. How much work is done in raising it to the loft, which is 25 ft. high by means of a rope which weighs $\frac{1}{2}$ lb. to the foot?

5. A ladder 20 ft. long rests against a wall and makes an angle of 45° with the surface of the ground. How much work is done in carrying 100 lb. of bricks up the ladder and placing them upon the wall 4 ft. above the end of the ladder?

58. Power.—In defining work as the force multiplied by the distance through which it acts, it is to be observed that the element of time does not enter. The same work is done in lifting a mass of 300 lb. a distance of 100 ft. whether the work done in a day or in a minute. The same work is done whether the mass is carried at a single load or in two or more loads. The amount of work done is measured by the end result, and it does not in any way ask concerning the time to do the work. In the consideration of a machine it is necessary to know more than the total amount of work which the machine can do. It is desirable to know the rate at which the machine can work. **The rate of doing work is called power.** Hence,

$$\text{Power} = \frac{\text{work}}{\text{time}} = \frac{\text{force} \times \text{distance}}{\text{time}} = \frac{F \times s}{t} = \text{work per unit of time.}$$

Example.—A horse hitched to a load of hay hauls it from the ground to the loft of the barn in 1 minute. If the load weighs 500 lb. and if the height through which it is lifted is 20 ft. find the rate at which the horse is working.

$$\text{Work} = \text{force} \times \text{distance} = 500 \text{ lb.} \times 20 \text{ ft.} = 10{,}000 \text{ ft.-lb.}$$

$$\text{Power} = \frac{\text{force} \times \text{distance}}{\text{time}} = \frac{500 \text{ lb.} \times 20 \text{ ft.}}{60 \text{ seconds}} = 166.7 \text{ ft.-lb. per sec.}$$

Horsepower.—The English unit of power is called the horsepower. A horsepower denotes the power or ability of a machine to do 33,000 foot-pounds of work in one minute or 550 foot-pounds in one second.

Example.—The maximum walking draft of a horse is about one-half of his weight. If the horse weighs 1,600 lb. and walks at the rate of 2.5 miles per hour, he is working at the rate of

$$\frac{2.5 \times 5{,}280 \times 1{,}600}{60 \times 60 \times 550 \times 2} = 5.3 \text{ horsepower}$$

This, however, is too great a strain for the horse to maintain for any length of time. For continuous work during 10 hours of the day a horse which travels at the rate of 2.5 miles per hour may be expected to pull with a force which is about one-tenth of his weight. At this rate a horse which weighs 1,600 lb. would work at the rate of

$$\frac{1{,}600 \times 5{,}280 \times 2.5}{10 \times 60 \times 60 \times 550} = 1.06 \text{ horsepower.}$$

Problems

1. How high can a 6-horsepower engine raise a 10 ton mass of stone in 10 minutes?

2. A tractor drawing a plow exerts an average force of 1,500 lb. If the tractor travels at the rate of 3 miles per hour what horsepower is developed by the engine?

3. In a well-drilling machine, the drill, which weighs 700 lb., is lifted 30 ft. by the engine in 30 seconds. Find the work done and the power developed by the engine.

4. A horse draws a sulky and the driver at the rate of $\frac{1}{2}$ mile per minute. If the horse exerts a pull of 10 lb. on the sulky, what horsepower is he developing?

59.—Definition of Energy.—The energy of a body may be defined as its capacity for doing work. The flywheel of an engine will keep the machinery running for some time after the power has been shut off. The flywheel, therefore, has energy and can do work. Inanimate bodies possess energy and can do work because work has been done on them at some previous time. Suppose that an elevator has lifted 100 lb. mass from the ground to a height of 20 ft. The operation of lifting this mass has required the expenditure of work which is equal to the work done in lifting a weight of 100 lb. a distance of 20 ft. or 2,000 ft.-lb. of work. The mass in this new position possesses a capacity for doing work. If it is now allowed to slide down an incline in a suitable way it may be made to return this work to some sort of a machine. When a base ball is thrown work must be done in overcoming the inertia of the ball and imparting a velocity to it. This energy remains in the ball as it passes from the pitcher to the catcher.

In order to set a wagon wheel spinning about its axis it is necessary to exert a push or a pull on the spokes or on the tire. During this effort work is stored up in the wheel. When the wheel is once in motion it continues to spin unless a force is applied to stop it. The work done on the wheel is stored up as energy of rotary motion. The energy which is thus stored up may be made available for doing work on other bodies. In all cases energy is required to set bodies in motion and this energy is returned when the bodies are stopped. Hence moving bodies possess energy by virtue of their motion.

60. Potential and Kinetic Energy.—It is necessary to distinguish between two types of energy: (1) potential energy and (2) kinetic energy. The energy which a body has by virtue of its position is called **potential energy.** When a mass has been lifted above the surface of the earth it has energy on account of its position. When a spring has been compressed or a bow has been bent, energy has been stored up. This energy can be again recovered and is potential energy. Any body which is in motion is able to set other bodies in motion by colliding with them. A bullet leaving the muzzle of a gun with a velocity of 300 meters per second has energy stored up in it. This is seen by the fact that when it strikes a body it can overcome a considerable resistance. Common experience teaches that it is difficult to stop or change the direction of motion of a body moving with considerable velocity. It is because the body possesses energy. That energy which a body has on account of its motion is called **kinetic energy.** Every body in motion will have kinetic energy and the faster the body is moving the greater is this kinetic energy. The heavier the body the greater the kinetic energy.

61. Conservation of Energy.—The study of the various forms in which energy may occur and the transformation of one kind of energy into another has led to the statement of a very important principle, known as the **conservation of energy.** This principle may be stated as follows: **In any body or system of bodies**

which is not receiving or giving up energy, the total amount of energy is unchanged. This principle states that energy can never be created or destroyed. It can be transformed from one form into another but the total amount in the end is unchanged.

A bullet leaves the muzzle of the gun with kinetic energy which it received on account of the work done on it by the expanding gases. As it passes through the air it loses some of this kinetic energy because of the heat developed by the friction in the air. When it strikes the target, sound waves are sent out which carry away some of the energy. There may also be a flash of light which uses up some energy Heat will be developed in the target and fragments of the bullet may carry away some of the energy. If all these energies are added together they will be found just equal to the energy with which the bullet left the muzzle of the gun.

62. The Measure of Potential Energy.—The measure of the potential energy which a lifted body such as a pile driver has on account of its position is equal to the work which has been spent in lifting the body. If the height in feet through which the body has been lifted is h and its weight in pounds is W, then the potential energy of the lifted body is

$$\text{Potential energy} = Wh \text{ ft.-lb.}$$

Example.—A clock weight weighing 3 lb. is lifted 6 ft. against gravity. What is the potential energy thus stored up?

$$\begin{aligned}\text{Potential energy} &= Wh\\ &= 3 \times 6 = 18 \text{ ft.-lb.}\end{aligned}$$

Other examples of potential energy are found in the work done in winding a watch and in the work required to stretch an india rubber band. In these cases the material of the spring or the rubber band is in a state of strain and on this account the body possesses potential energy. We have seen that when a body is at a height h and is then allowed to fall freely, all the potential energy is changed into kinetic energy. For example, if a pile driver weighing 2,000 lb. falls 15 ft., the potential energy is all changed into kinetic energy.

63. Measure of Kinetic Energy.—To find the measure of the kinetic energy which a body possesses by virtue of its motion, consider the work which must be done on it in order to give it a certain speed. When the body is stopped it will give up an amount of energy which is equal to the work done in starting it. By definition this latter is its kinetic energy.

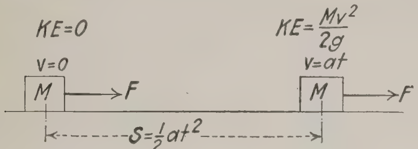


FIG. 40.—Kinetic energy=$\frac{MV^2}{2g}$.

From Newton's second law of motion the force necessary to make a body move with an acceleration a is,

$$F = \frac{Ma}{g}$$

Let s (Fig. 40) be the distance through which the body moves. Then the work done on the body in this distance is,

$$W = Fs$$

If the body starts from rest, its velocity at the end of t seconds is,

$$v = at$$

The space passed over in t seconds is,

$$s = \tfrac{1}{2}\,at^2 \text{ whence } v^2 = 2\,as$$

and

$$as = \tfrac{1}{2}\,v^2.$$

Substituting this value of as in the expression for the work done on the body we have,

$$W = Fs = \frac{Mv^2}{2g} \text{ (ft.-lb. or gram-cent.)}$$

This expression for W gives the work necessary to cause a mass M to acquire a velocity v. This work does not depend on the distance covered or on the acceleration. It is determined solely by the mass of the body and its speed.

If there is now applied to this body a retarding force so that it is brought to rest, the moving body will do work on this retarding force. When the body has come to rest, an amount of work will be done against the retarding force which is just equal to the work done in starting the body. According to the law of conservation of energy, the energy spent in starting the body must be just equal to that derived from the body when it is stopped. Hence, the energy which the body possesses when in motion is,

$$\text{Kinetic energy} = \frac{Mv^2}{2g}$$

This equation states that when a body is increasing in velocity, the gain in kinetic energy is the work done on the body by the accelerating force.

$$\text{Work} = \text{accelerating force} \times \text{distance.}$$
$$= \frac{M}{2g}(v^2 - v_0^2)$$

If the body is decreasing in velocity loss of kinetic energy = work done by the body against the retarding force

$$= \text{retarding force} \times \text{distance.}$$
$$= \frac{M}{2g}(v_0^2 - v^2)$$

If the English system of units is used in this equation, W is measured in ft.-lb., F in pounds weight and distance in feet. The kinetic energy is then expressed in ft.-lb. If the metric system of units is used the work is measured in gram-centimeters, the force in grams weight and the distance in centimeters. In this case the kinetic energy is expressed in gram-centimeters.

Example.—If an automobile weighing 3 tons is moving with a velocity of 30 ft. per second what is its kinetic energy?

$$\text{Kinetic energy} = \frac{M}{32.2}\frac{v^2}{2}$$
$$= \frac{6{,}000 \times (30)^2}{2 \times 32.2} \text{ ft.-lb.}$$
$$= \frac{3{,}000 \times 900}{32.2} = 83{,}800 \text{ ft.-lb.}$$

Example.—A boy weighing 75 lb. starts to slide on ice with a speed of 25 ft. per second. The retarding force due to the friction is 30 lb. How far will he go before coming to rest?

Loss of kinetic energy = retarding force × distance

$$\frac{Mv^2}{2g} = F \times d$$
$$\frac{75 \times (25)^2}{32.2 \times 2} = 30 \times d$$
$$d = \frac{75 \times 225}{32.2 \times 60} = 8.8 \text{ ft.}$$

64. Transformations of Potential and Kinetic Energy.—The energy of a body is capable of changing its form from potential to kinetic or from kinetic to potential. Thus suppose that a stone of mass M is supported on the edge of a cliff at the height h above the base of the cliff. The potential energy is the work done in lifting the stone a height h. Since the force with which the earth attracts the stone is M lb., the potential energy is Mh ft.-lb. If the stone is allowed to fall freely it loses its potential energy but gains in kinetic energy. When the stone has reached the base of the cliff all the potential energy has been changed into kinetic energy. At any point in the path of the stone the sum of the potential and the kinetic energy is the same as at every other point.

The swinging of a pendulum or the oscillation of a weight attached to a spring illustrates the way in which potential energy may be transformed back and forth. When the spring is elongated to the greatest extent, the mass has no velocity and no kinetic energy. As the mass is released it moves upward and acquires kinetic energy. Meanwhile the potential energy stored in the spring is being transformed into kinetic energy. When the mass has risen to its highest point, the spring is compressed to the greatest extent and all of the energy is now stored in the spring as potential energy. The mass now begins to move downward and again acquires velocity and kinetic energy. This interchange of kinetic energy continues as long as the oscillations of the spring persist.

65. Illustrations.—A windmill furnishes a good illustration of the transformation of kinetic into potential energy. The motion of the air imparts a certain motion of rotation to the wheel. The kinetic energy which the wheel possesses on this account is used to pump water into a tank. The work or energy required to lift this water comes from the energy of rotation in the wheel. The work done in lifting the water is stored up as potential energy. This energy may be again transformed into kinetic energy by allowing the water to flow out of the tank.

Another illustration of the transformation of kinetic into potential energy is seen in a sprayer. A cubic foot of water emerging from the nozzle of the spray possesses kinetic energy. On account of the velocity with which it emerges it rises to a certain height before it stops. When the water comes to rest all the kinetic energy has been transformed into potential energy. As the water falls back to the earth, the velocity increases and the kinetic energy increases while the potential energy decreases.

Problems

1. A stone weighing 50 g. is thrown with a velocity of 15 meters per sec. Find its kinetic energy.

2. What is the kinetic energy of an engine weighing 150 tons when it is moving at the rate of 35 miles per hour?

3. If 100 cu. ft. of water pass over a dam which is 20 ft. high in 2 min. how much energy could be derived from it, if none of the energy were lost?

4. What is the potential energy of 100 cu. ft. of water in a water tank which stands 30 ft. above the level of the earth?

5. How many foot-pounds of work will be required to give a truck which weighs 10 tons a velocity of 15 miles per hour?

66. Kinetic Energy of Rotation.—The work required to set a body in translation equals the kinetic energy of translation of the body. In like manner work is required to set a body in rotation and this work is stored up in the body as kinetic energy of rotation. To find the expression for the kinetic energy of rotation in terms of the angular velocity and the rotary inertia of the body, consider Fig. 41, which represents a body rotating with an angular velocity ω about an axis through O perpendicular to the plane of the paper. Consider the particle at A having the mass m_1 and the linear velocity v_1.

$$\text{Kinetic energy of the particle } A = \frac{m_1 v_1^2}{2g}$$

$$\text{Now } v_1 = \omega r_1$$

$$v_1^2 = \omega^2 r_1^2$$

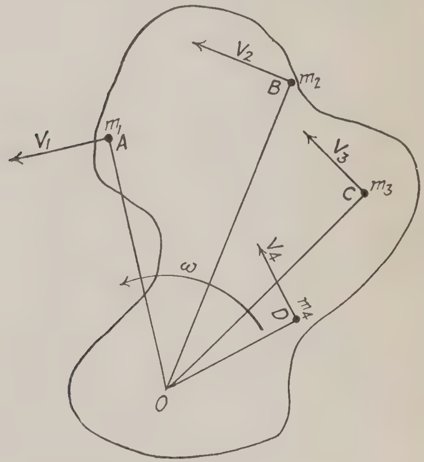


Fig. 41.—Kinetic energy of rotation $= \dfrac{I\omega^2}{2g}$.

Kinetic energy of the particle $A = \frac{m_1(\omega r_1)^2}{2g} = \frac{\omega^2}{2g} \times mr_1^2$

For other particles of m_2, m_3, m_4, etc., moving with velocities v_2, v_3, v_4 similar expressions are found. Hence the total kinetic energy is:

Total kinetic energy

$$= \frac{\omega^2}{2g} m_1r_1^2 + \frac{\omega^2}{2g} m_2r_2^2 + \frac{\omega}{2g} m_3r_3^2 + \ldots$$

$$= \frac{\omega^2}{2g} (m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots)$$

Now $I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \ldots$ is the rotary inertia of the body, so that

Kinetic energy of rotation $= \frac{\omega^2}{2g} I$. (ft.-lbs. or gram-cms.)

Example.—Find the kinetic energy of rotation of a wheel which has a moment of inertia of 10,000 (lb.-ft.) when it makes 240 revolutions per minute.

Kinetic energy of rotation $= \frac{I\omega^2}{2g}$.

$I = 10{,}000$ (lb.- ft. units)

$g = 32.2$ ft. per sec. per sec.

$$\text{K. E.} = \frac{10{,}000 \left(\frac{240}{60} \times 2\pi\right)^2}{2 \times 32.2} = 9512 \text{ ft.-lb.}$$

Problems

1. What is the moment of inertia of a wheel which has a kinetic energy of 7,000 ft.-lb. when it is making 120 revolutions per minute?

2. In a wheel all the mass is concentrated in the rim. The radius of the wheel is 5 ft. and it has a mass of 25 lb. What is its kinetic energy of rotation when it is making 90 revolutions per minute?

CHAPTER VII

FRICTION

67. Nature of Friction.—When one body rests on another body there is a tendency for the inequalities of one body to fit into those of the other body. After such an interlocking has occurred it is possible to move one surface over the other only by separating the two surfaces slightly or by tearing off the interlocking parts. For this reason when a heavy block of wood is pushed along the top of a table a certain resistance is encountered. By making the surface of the table and the surface of the block very smooth the amount of this resistance can be much decreased. Because of the molecular structure of the bodies no matter how smooth the surfaces there will remain slight elevations or depressions which will cause some resistance to motion. There is also a slight adhesion between the surfaces so that no two bodies can slide over each other without developing some resistance to motion. This resistance, the amount of which depends on the characteristics of the rubbing surfaces, is called **friction.**

This friction always opposes the motion whatever its direction. It never tends to push the body either forward or backward. It merely tends to stop the motion and make it more difficult to move the body. When the bodies come to rest the irregularities of the upper surface sink into those of the lower surface and the interlocking becomes greater. In consequence of this fact it is more difficult to start the body than it is to keep it in motion when started. There are for this reason two kinds of friction, **kinetic friction** and **static friction.** Kinetic friction is less than static friction.

68. Laws of Friction.—The laws which determine the amount of friction between the surfaces of solids are as follows:

1. **The friction between two sliding surfaces is nearly independent of the velocity.**

2. **If the pressure perpendicular to the surface remains the same the friction does not depend on the area of the rubbing surfaces.**

3. **The force to overcome friction is proportional to the total pressure of one surface against another.**

4. **The force of friction is greater at the start than after motion has begun.**

69. Coefficient of Friction.—In order to describe with some accuracy the characteristics of the rubbing surfaces, it is customary to define what is known as the coefficient of friction. If the pressure at right angles to the rubbing surfaces (Fig. 42) is P and the force required to just draw one surface over the other is F, **then F/P is called the coefficient of friction.** It states that the quantity obtained by dividing the force of friction by the pressure perpendicular to the rubbing surfaces is a constant which is characteristic of the rubbing surfaces

$$\text{Coefficient of friction} = \frac{\text{force of friction}}{\text{pressure}} = F/P$$

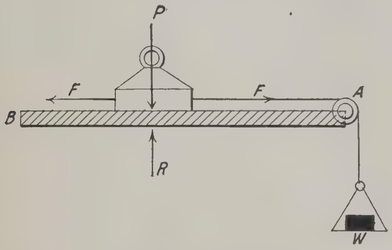


FIG. 42.—Definition of coefficient of friction.

An approximate idea of the magnitude of this coefficient is found by observing by means of a spring balance the force necessary to draw the body along a horizontal surface with uniform velocity and then dividing this force which is equal to the force of friction by the weight of the body.

Example.—A mass of 25 lb. rests on a rough horizontal plane. The force required to pull this mass along with uniform velocity is 2.5 lb. What is the coefficient of friction?

$$\text{Coefficient of friction} = \frac{\text{force to overcome friction}}{\text{force pressing bodies together}}$$
$$= \frac{\text{friction}}{\text{pressure}} = F/P = \frac{2.5}{25} = 0.10$$

70. Rolling Friction.—The friction of a solid rolling on a surface is less than the friction of a solid sliding over a surface. For this reason instead of hauling on a sled a farmer uses a wagon.

In this case there is still some friction between the wheels of the wagon and the road bed and between the surface of the axle and the hub sliding over it. This latter source of friction may be reduced by using what is known as roller or ball bearings (Fig. 43 and Fig. 44). Then the sliding friction in the hub is replaced by rolling friction which is much smaller. When a car wheel rolls on a smooth track (Fig. 45) it is flattened a little where it rests on the track and also makes a slight depression in the track. As the wheel rolls it is forced to continually climb up out of this depression. This requires an additional force which is **rolling friction.** If the road bed were made of steel this depression would be very slight and difficult to detect. On soft dirt road it is considerable. It is less where the wheels of the wagon are provided with wide tires. On steep inclined railways running up the sides of mountains it is necessary to provide the drive wheels of the engine with cogs which fit into a cogged rail. This is because rolling friction is not large enough to prevent the

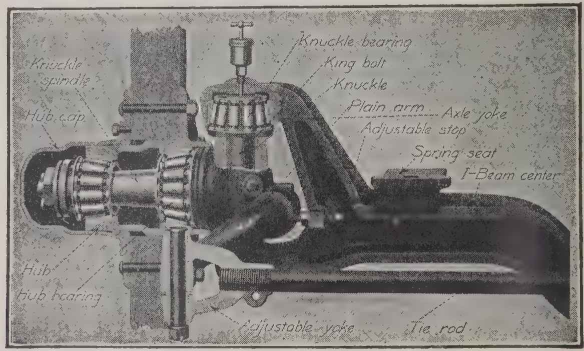

Knuckle spindle
Hub cap
Hub
Hub bearing
Knuckle bearing
King bolt
Knuckle
Plain arm
Axle yoke
Adjustable stop
Spring seat
I-Beam center
Adjustable yoke
Tie rod

Fig. 43.—Roller bearings to reduce friction.

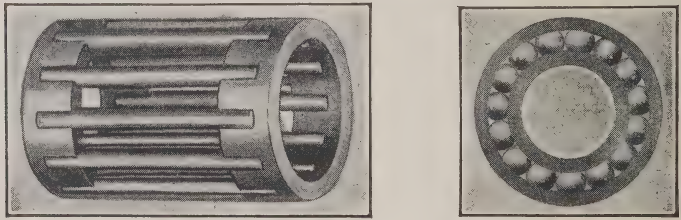


Fig. 44.—Roller bearings and ball bearings.

wheels from sliding. When the rails of an ordinary railroad are wet or covered with ice the friction between the rails and the wheels is small and the wheels slip on the track making it difficult for the engine to pull the load. For this reason the engine is provided with means of supplying sand to the track by which the friction is increased and slipping is prevented.

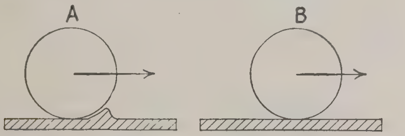


Fig. 45.—Deformation of surface by roller bearing.

71. Advantages of Friction.—Friction has many advantages as well as many disadvantages in machines. Except for friction between the shoes and the pavement a person could not walk. When the pavement is covered with ice friction is small and walking is difficult. On account of friction belts cling to the pulleys and drive the machinery. The brakes on a wagon or on an automobile stop the wagon or automobile by means of friction. Screws and nails hold their places in the objects into which they are driven by means of friction. A friction drive is represented in Fig. 46.

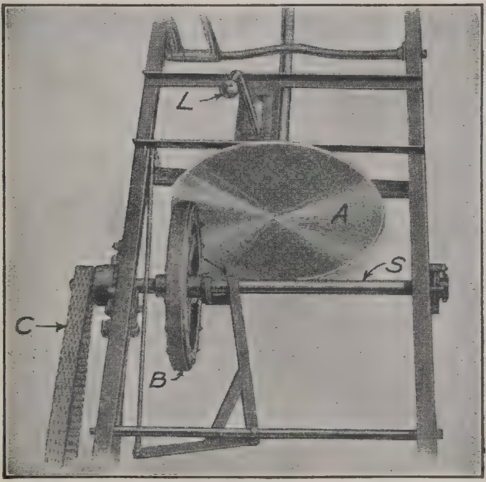


Fig. 46.—Friction drive.

72. Efficiency of Machines.—Every machine wastes energy on account of friction. Consequently the work put into the machine is always less than that obtained from it. This loss decreases the efficiency of the machine. **The efficiency of a machine is defined to be the ratio of the work done by the machine to the work done on the machine.** It is therefore the output of the machine divided by the input.

$$\text{Efficiency} = \frac{\text{work done}}{\text{energy supplied}} = \frac{\text{output of energy}}{\text{input of energy}}$$

Example.—With a block and tackle a horse exerting a force of 500 lb. lifts 3,000 lb. a distance of 10 ft. If the horse moves a distance of 42 ft. in moving this load, what is the efficiency of the block and tackle?

$$\text{Efficiency} = \frac{\text{work done by the machine}}{\text{work done on the machine}} = \frac{2{,}000 \times 10}{500 \times 42} = 95 \text{ per cent}$$

73. Action of Lubricants.—When liquids are brought into contact with solids to which they adhere, a thin layer of liquid is held rigidly to the surface of the solid. If the liquid flows over the surface of the solid, this layer of liquid does not move but remains in contact with the solid. In such cases instead of having friction between the liquid and the solid there is friction between the layers of the liquid. When oil is poured into the bearings of a machine it forms two layers over the metal surfaces. The sliding then takes places for the most part between these layers of oil. Since the friction between the layers of the oil is much less than it is between the surfaces of the metal, the energy lost by friction is much decreased.

74. Friction on an Inclined Plane.—Let *AB* (Fig. 47) be a board on which rests a block *M* and let the board be inclined at such an angle that the block will just slide down the plane with uniform velocity. If *CD* represents the weight of the block, its components parallel to the plane and at right angles to the plane are *DE* and *CE*. The force of friction between the block and the plane is parallel to the plane and in the opposite direction to *DE;* i.e., up the plane. Since the block experiences no acceleration the force up the plane is equal to the force down the plane.

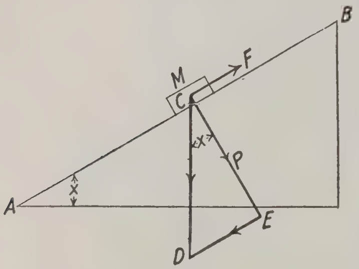


FIG. 47.—Coefficient of friction = tangent of angle of repose.

$$F = DE = M \sin x$$

The pressure perpendicular to the plane = *CE*

$$P = CE = DC \cos x = \mathrm{M} \cos x$$

$$\text{Coefficient of friction} = \frac{F}{P} = \frac{DE}{CE} = \frac{\mathrm{M} \sin x}{M \cos x} = \tan x.$$

Hence, the tangent of the angle at which the block slides down the plane with uniform velocity is numerically equal to the coefficient of friction. This angle is known as the **angle of repose.**

75. Power transmitted by a Belt.—Where a pulley is being driven by a belt (Fig. 48) the tension in the straight parts of the belt are not equal. They differ by the friction which is exerted between the belt and the pulley. Let T_2 be the tension in that part of the belt which is moving toward the pulley and T_1 the tension in that part which is moving away from the pulley and let V the velocity with which the belt is moving. The force of friction between the belt and the pulley is $T_1 - T_2$.

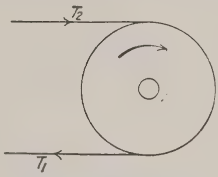


FIG. 48.—Friction between belt and pulley.

The work done per minute by the belt $= (T_1 - T_2)\,V$ ft.-lb.

The horsepower delivered to the pulley $= \dfrac{(T_1 - T_2)V}{33{,}000}$

Example.—The tension on one side of a belt is 300 lb. and that on the other side is 100 lb. The belt is moving 300 ft. per min. Find the horsepower delivered to the pulley.

$$\text{Horsepower} = \frac{(300 - 100)\ 300}{33{,}000} = 1.8 \text{ Horsepower}$$

Problems

1. In order to pull a sled and its load along a piece of horizontal ice a boy exerts a force of 15 lb. If the sled and its load weighs 200 lb., what is the coefficient of friction?

2. A weight of 5 lb. hanging over the edge of a table drags by means of a light cord a weight of 50 lb. along the horizontal surface of the table with uniform velocity. What is the coefficient of friction?

3. A farmer grinding a chisel presses on the grindstone with a force of 25 lb. The coefficient of friction between the stone and the chisel is 0.35. What is the backward pull of friction on the chisel?

4. A pair of horses, which are working at the rate of two horsepower draws a wagon along a level highway. The coefficient of friction between the wagon and the road is 0.15. If the horses travel at the rate of 3 mi. per hr., what load can they pull?

5. In lifting a mass of 1,500 lb. to a height of 25 ft., the hoisting engine did 40,000 ft.-lb. of work. What is the efficiency of the hoist?

6. An engine delivering 12 horsepower runs a pump which delivers 320 cu. ft. of water per min. to a height of 18 ft. If each cu. ft. of water weighs 62.5 lb., what is the efficiency of the pump?

CHAPTER VIII

SIMPLE MACHINES

76. Levers.—Levers may be divided into three classes according to the position of the fulcrum or point of support with respect to the forces. In Fig. 49 the three possible cases are shown. In one of these cases the fulcrum *F* is between the forces and in the other cases it is at the end of the lever.

The simplest kind of lever is one in which the arms are of equal length. The scale beam on a pair of ordinary balances (Fig. 50) is such a lever. In this case equal forces or weights at the ends of the lever just balance each other. Usually the distances of the forces from the point at which the lever is supported are not equal, and for equilibrium in such cases the forces must also be unequal. The larger force has a greater tendency to tip the lever one way than the smaller has to tip it the other way.

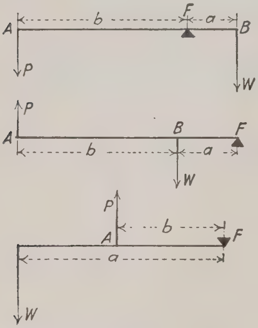


Fig. 49.—Types of levers.

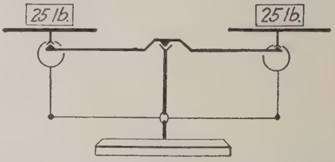


Fig. 50.—Platform balance,—an equal arm lever.

77. Law of the Lever.—**Any lever is balanced when the sum of the moments of force on one side of the lever is equal to the sum of the moments of force on the other side of the lever.** If only one force is applied and one pressure overcome, this law may be stated as follows: A lever is balanced when,

$$\text{Weight} \times \text{weight-arm} = \text{force} \times \text{force-arm}.$$

This law states that when a lever is balanced under the action of two forces, the forces applied to the lever are inversely proportional to their distances from the fulcrum.

78. Simple Lever Appliances.—A pair of scissors is an illustration of this type of lever. If the distance from the screw which acts as fulcrum to the handles at which the force is applied is three times as much as the distance from the screw to the object on which the scissors are cutting, then a force of 10 lb. applied to the handles is capable of exerting a force of 30 lb. on the material which is being cut.

Example.—The length of a pump handle from the point at which the force is applied to the bolt which is acting as fulcrum is 30 in. The length of the weight-arm is 3 in., and a force of 50 lb. is applied to the handle. Find the weight which can be lifted.

$$\text{Weight} \times \text{weight-arm} = \text{force} \times \text{force-arm}$$
$$\text{Weight} \times 3 = 50 \times 30$$
$$\text{Weight} = \frac{1{,}500}{3} = 500 \text{ lb.}$$

79. Mechanical Advantage.—In all simple machines like levers a certain advantage is obtained by the use of the machine. This advantage does not consist in an increase or in a decrease of the work performed on the machine. **Neglecting friction the work done on the machine must always be the same as the work done by the machine.** This law follows from the law of conservation of energy. By means of a suitable lever or other machine it is possible to exert a large force by the application of a small force. The large force will act through a small distance and the small force through a large distance so that the work done in the two cases will be the same.

The ratio of the resistance or the force overcome to the applied force is called the mechanical advantage.

Example.—By means of a crowbar which is 6 ft. long a man lifts a stone which weighs 330 lb. The distance from the fulcrum to the stone is 0.5 ft. and the distance from the fulcrum to the point at which the man pushes is 5.5 ft. The man finds it necessary to exert a force of 30 lb. What is the mechanical advantage of the lever?

$$\text{Mechanical advantage} = \frac{\text{force overcome}}{\text{force applied}} = \frac{330}{30} = 11$$

Also applying the principle of moments,

Force overcome × lever arm = force applied × lever arm

$$R \times a = F \times b$$
$$R/F = b/a = 5.5/0.5 = 11$$

80. Eveners.—The principle of moments is well illustrated in the common two-horse evener or double-tree, Fig. 51. If the three holes are in the same straight line, and if the distance between the holes is the same, then no matter in what position the horses are in relation to each other, the load will be evenly divided between them. For every position of the evener the length of the lever arm on the one side will be the same as that on the other side.

If the middle hole is near the front of the evener and the end holes are placed to the rear of the middle hole, the load will not be evenly divided between the horses except when they are in the normal position. With such an evener the lever arm for the horse which is in the rear is shorter than that for the other horse. Hence the horse which is in the rear must pull the greater portion of the whole load. If one horse falls in the rear this evener tends to keep him always behind.

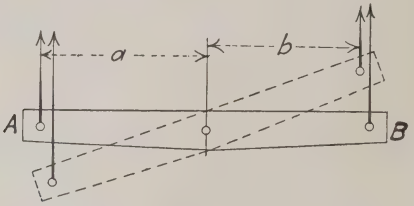


Fig. 51.—Evener—a type of lever.

If one horse is stronger than the other, an advantage may be given the weaker horse by lengthening his end of the evener and shortening the other end. If the total pull is 400 lb. and it is desired that one horse pull 250 lb. and the other horse pull 150 lb., the lever arm of the stronger horse should be only 3/5 of that of the weaker horse. The pull of the stronger horse multiplied by his lever arm must equal to the pull of the weaker horse times his lever arm.

81. A Roller.—A roller used in crushing clods on the farm (Fig. 52) serves also to show the application of the important principle of moments.

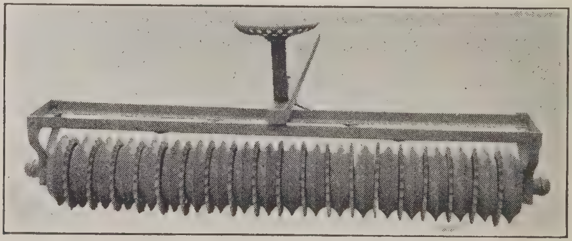


Fig. 52.—Roller for crushing clods.

In this case the axle of the roller acts as fulcrum about which the frame work mounted above the roller tends to rotate. If the driver is in his position on the seat the pull of gravity on him tends to rotate the tongue of the roller upward. On the other hand the pull of gravity on the tongue tends to rotate the tongue downward. When the moments of force tending to rotate the tongue of the roller one way are just equal to those tending to rotate it in the opposite direction the tongue will remain in the horizontal position and exert neither upward nor downward pressure on the necks of the horses.

Example.—The tongue in a roller weighs 25 lb. and may be considered as if all the weight were concentrated at a point in the tongue which is 6 ft. from that point on the tongue which lies directly above the axle. How far must a man who weighs 165 lb. be behind this point on the tongue which lies directly above the axle in order that his weight may just balance the tongue so that there is no pressure on the necks of the horses? For the purpose of this problem consider the mass of the man sitting in the seat, to be concentrated at a single point.

Let b = the distance of this mass behind the fulcrum.

Then,

$$25 \times 6 = 165b.$$

$$b = 165/150 = 1.1 \text{ ft.}$$

Hence the man must sit about 1.1 ft. behind the axle.

Problems

1. A plank which is to be used as a seesaw by two boys is 15 ft. long. If one of the boys weighs 75 lb. and the other 100 lb., where must the support for the plank be placed?

2. A piece of wire, which is being cut with a pair of shears, is placed 1 cm. from the rivet. If a force of 12 kg. is applied on the handles at a distance of 30 cm. from the rivet, find the force which is exerted on the wire.

3. If the total pull on a 42-in. evener is 500 lb., how long should each arm of the evener be in order that one horse may pull 45 per cent. and the other horse 55 per cent. of the total load? How many pounds will each horse pull?

4. The short arm of a steelyard is 1 in. The weight on the long arm weighs 15 lb. What must be the length of the long arm in order to be able to weigh on these steelyards a hog whose weight is 250 lb.?

5. Two horses are hitched to a plow by means of a 44-in. evener. One arm of the evener is 2 in. shorter than the other. What part of the load will each horse pull?

82. One Arm Lever.—Sometimes the fulcrum of the lever is located at one end as shown in Fig. 49. The principle of moments applies in this case as in the preceding case. The moment of force tending to tip the lever down must be equal to the moment of force tending to tip it upward when the lever is balanced. One of these forces tends to rotate the lever in the direction in which the hands of a clock moves, the other tends to rotate it in the opposite direction. The moment of force tending to rotate the lever clockwise is found by multiplying the force W by its perpendicular distance from the fulcrum. For equilibrium,

$$P \times b = W \times a$$

This type of lever is found in the ordinary wheelbarrow. The axle of the wheel is the fulcrum, the load is the weight and the force applied to the handles corresponds to P in Fig. 53. When the force W multiplied by its distance from the axle is just equal to P multiplied by its distance from the axle, the legs of the wheelbarrow can be just lifted from the ground.

The damper regulator of a hot water boiler (Fig. 54) is also a lever of this kind. By moving the weight w outward the force at g necessary to operate the damper is increased.

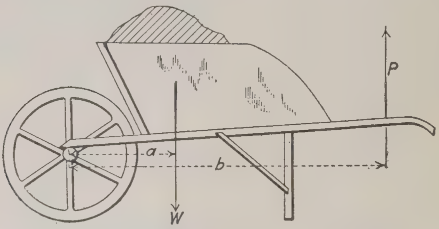


FIG. 53.—Wheelbarrow—a one arm lever.

The brake on an ordinary farm wagon (Fig. 55) gives an illustration of a combination of this type of lever with the one in which the fulcrum lies between the forces. The handle bar AB is a lever in which the fulcrum lies at one end A. The distance from the point B at which the hand is applied to the other end A of the bar is the force arm, and the distance from the rod C which actuates the brake to the end A of the bar which is fastened to the bed is the weight arm. This rod fastened to the handle bar extends to the rear of the wagon where its other end is again fastened to another bar DOE which acts as a lever of the first class in the application of the brake to the wheel. The force which can be exerted by the hand on the wheel is magnified by each of these levers so that the two together are more effective than either alone.

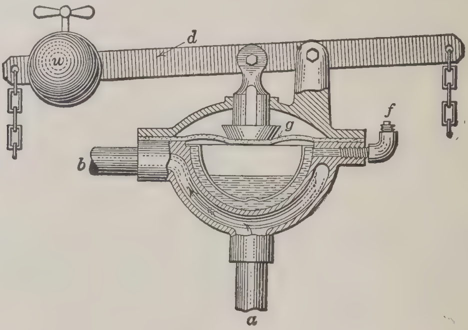


FIG. 54.—Damper regulator for hot-water boiler.

Example.—In a wagon brake the length of the lever bar is 3 ft. and the distance from the fixed end to the rod which actuates the brake is 4 in.

The length of the bar OD is 9 in. and the length of the part OE is 3 in. (Fig. 55.) If a man exerts a force of 25 lb. with his hand on the handle of the lever bar find the force exerted on the wheel.

$AB = 36$ in., $AC = 3$ in., $OD = 9$ in., $OE = 3$ in.
$F = 25$ lb. Find P
$F \times 36 = W \times 3$
Hence $W = 12 \times F = 12 \times 25 = 300$ lb.
$W \times 9 = P \times 3$
Hence $P = 3 \times 300 = 900$ lb.

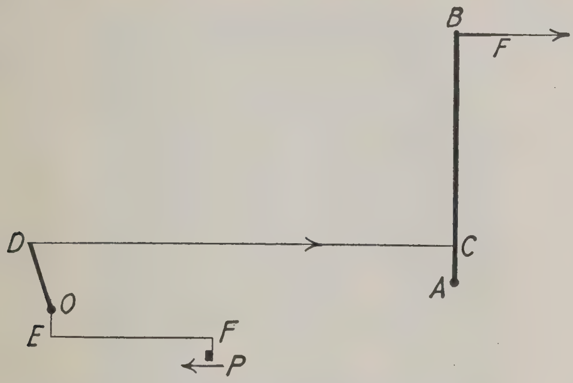


FIG. 55.—Wagon brake.

83. Levers of the Third Class.—Sometimes appliances involving the one-arm lever instead of having the force at the end of the lever and the pressure between the force and the fulcrum, exert the pressure at the end of the lever by applying the force at some point between the pressure and the fulcrum. Such a lever is said to be a lever of the third class. A pair of fire tongs or a pair of shears for cutting grass belong to this type of lever. The treadle of a sewing machine and the bones of the fore arm together with the muscles operating them are levers of this class.

Problems

1. The handle of a lemon squeezer is 20 cm. long, and the distance from the pivot acting as fulcrum to the point at which the force may be considered as applied to the lemon is 5 cm. If a woman exerts a force of 3 kg. on the end of the handle what pressure is applied to the lemon?

2. A nut cracker is 6 in. long and the distance from the center of the nut to the fulcrum is 1 in. If it requires a force of 18 lb. to crack the nut what force must be applied to the end of the nut cracker?

3. On a mule cart the load of $\frac{1}{4}$ ton is centered $\frac{1}{4}$ in. in front of the axle.

The length of the shafts from the axle to the saddle straps is 6 ft. How many pounds does the mule support on his back?

4. A load of hay weighing 2 tons stands on a pair of wagon scales. (Fig. 56.) The distance from A to B is $\frac{1}{2}$ ft., the distance from B to C is 5 ft., the distance from D to K is 1.2 in. and the distance from K to E is 24 in. How much weight must be hung at E in order to just balance the load?

84. The Balance.—The balance (Fig. 57) which is an instrument for comparing masses is a lever of the first class with equal arms. It consists of a light strong beam supported at its center by a knife-edge which rests on an agate plate. From either end of this beam is suspended a pan which holds the weights to be compared. These pans are also suspended from the end of the beam by knife-edges. When a mass is placed in one of the pans, there is moment of force tending to rotate the beam about the central knife-edge. When an equal mass is placed in the other pan there is an equal moment tending to rotate the beam in the opposite direction. If the lengths of the arms of the balance are equal and the masses are equal to each other the moment of force tending to rotate the beam in one direction is just equal to the moment of force tending to rotate it in the opposite direction. When the center of gravity of the balance lies below the knife edge, the beam of the balance will return to its original position of equilibrium. The pointer which projects downward from the center of the beam will oscillate back and forth about its equilibrium position.

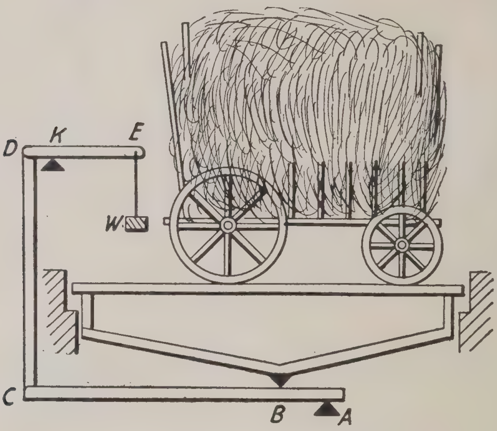


FIG. 56.—Wagon scales—combination of levers.

85. Double Weighing.—In case the arms of the balance do not have the same length, the true weight of the body may be determined by weighing it first on one side and then on the other side of the balance. Let l_1 and l_2 denote the lengths of the arms, and W be the true weight of the body. Suppose the body when hanging from the arm l_1 is balanced by a mass M_2 in the other pan. Suppose also that when the body hangs from the arm l_2 it requires a mass M_1 in the pan hanging from the arm l_1 to balance it. Then for the first case by the principle of moments we may write,

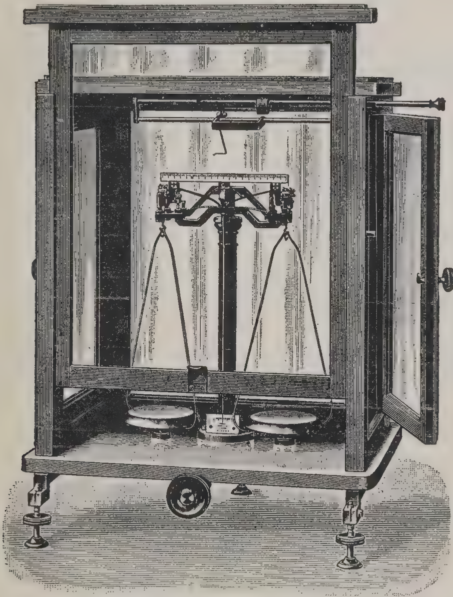


FIG. 57.—A balance.

$$W \times l_1 = M_2 \times l_2$$

and for the second case

$$W \times l_2 = M_1 \times l_1$$

By dividing the first of the equations by the second, we get

$$\frac{W}{M_1} = \frac{M_2}{W}$$

$$W^2 = M_1 M_2$$

$$W = \sqrt{M_1 M_2}$$

Hence, in order to get the true weight of a body when the balance does not have equal arms, weigh the body first in one scale pan and then in the other. The square root of the product of the weights required to balance the body when placed first in one pan and then in the other is the true weight of the body.

Example.—An iron sphere when placed in the right-hand pan of a scale balance required 80.0 gm. to balance it. When placed in the left-hand pan it required 80.5 gm. to balance it. Find the true weight of the sphere.

$$\text{True weight} = W = \sqrt{M_1 M_2}$$
$$= \sqrt{80.0 \times 80.5} = 80.249 \text{ g.}$$

86. Wheel and Axle.—A wheel and axle (Fig. 58) consists of a large and small wheel fastened together to the same axle. It is in reality a lever of the first class with the axle as fulcrum. To a rope wound around the axle or around the smaller wheel is fastened the weight which is to be lifted. The force is applied to the rim of the wheel and will be called F. If r is the radius of the axle and R the radius of the wheel, then the moment of force tending to produce rotation counterclockwise is FR and that tending to produce rotation clockwise is Wr.

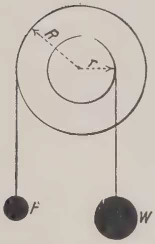


FIG. 58.—Wheel and axle.

$$W \times r = F \times R$$
$$W/F = R/r = \text{mechanical advantage.}$$

From the above equation it is seen that the mechanical advantage of a wheel and axle, which is equal to the force exerted by the wheel and axle divided by the force applied to it, is equal to the radius of the wheel divided by the radius of the axle.

Example.—Let the radius of the wheel be 30 cm. and the radius of the axle be 7 cm. When a force of 10 kg. is applied to the wheel, find the force exerted on the rope wound around the axle.

Applying the law of moments,

$$W \times r = F \times R,$$
$$W \times 7 = 10 \times 30,$$
$$W = \frac{300}{7} = 42.8 \text{ kg.}$$

$$\text{The mechanical advantage} = \frac{W}{F} = \frac{R}{r}$$
$$= \frac{30}{7} = 4.28.$$

87. The Windlass.—A windlass used for drawing water out of a well is one form of wheel and axle. The weight is attached to a rope wound on the drum, and the force is applied to the handle of the crank. The radius of the drum is the weight arm, and the length of the handle of the crank is the force arm. This older form of windlass has been replaced for the most part by a more modern pump which consists of a chain of buckets. This chain of buckets is lifted by means of a cogwheel. Since the chain is endless, the weight of the chain on one side balances that on the other side so that the load which is lifted consists only of the water in the buckets. The same relation exists between the force applied to the crank and the weight to be lifted as held in the former case.

A derrick used in lifting heavy weights consists of a double wheel and axle. Such a derrick is often used in lifting the front end of a wagon when unloading it into an elevator. As in the case of the windlass, the force is applied to the cranks attached to one of the axles. The mechanical advantage is found in this case as in the case of the simple wheel and axle.

There are many other applications of the wheel and axle. In the clothes wringer the weight arm is the radius of the roller. In the coffee mill and in the breadmixer the weight arm varies in length according to the position on the axis. In the breadmixer it may be taken as approximately one-half the radius of the mixer

Problems

1. The crank on a grindstone is 8 in. long and the diameter of the stone is 36 in. If a force of 30 lb. is applied to the crank, what force will be exerted on the rim of the wheel?

2. In a pump consisting of a chain of buckets the radius of the crank is 2.5 ft. and the radius of the cogwheel on which the buckets move is 8 in. How many pounds of water can a man lift by exerting a force of 35 lb. on the crank?

3. In a clothes wringer the radius of the roll is 2.5 cm. and the length of the crank is 35 cm. A force of 25 kilograms is exerted on the crank. What is the force tending to drag the clothes through the rolls?

4. The drum of an ordinary capstan for house moving is 16 inches in diameter, and the lever to which is hitched a horse pulling 200 lb. is 15 ft. long. Find the pull on the rope, assuming that friction can be neglected.

5. The drum of a stump puller (Fig. 59) is 18 in. in diameter. The lever to which the horse is hitched is 12 ft. long. If the force required to uproot a stump is 500 lb., with what force must the horse pull?

88. Pulleys.—A pulley consists of a wheel with a grooved rim called a sheave free to move about an axle which is mounted in a frame called a block. A flexible rope or cord passes over the groove in the rim of the wheel. To the ends of this rope is applied the weight and the force which overcomes this weight.

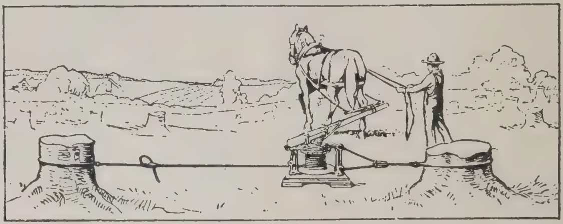


Fig. 59.—The stump puller.

In the case of a simple fixed pulley as in Fig. 60 equal forces or weights applied to the ends of the rope just balance each other. Neglecting friction, the tension in the rope is everywhere the same, and the mechanical advantage of the pulley is unity. There is therefore no advantage in such a pulley except that it is sometimes more convenient to pull down on the rope than it is to lift the weight directly.

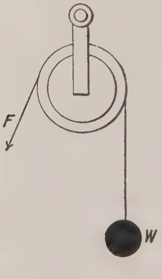


Fig. 60.—Single pulley.

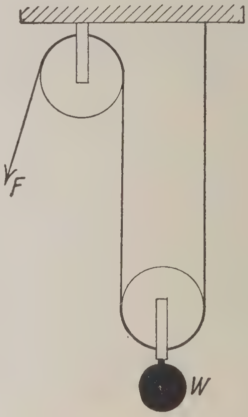


Fig. 61.—Fixed and movable pulleys.

When the pulley is movable as in Fig. 61 and the weight attached to it, it is evident that the weight is supported by two parts of the cord. It is therefore necessary for each of the cords to exert a pull equal to only one-half of the weight. If the weight is lifted, it moves only one-half as far as the free end of the cord to which F is applied. Applying the principle of work to this simple machine, it is seen that if the weight W is lifted a ft. and the force F moves b ft. then,

$$W \times a = F \times b.$$
$$W/F = b/a = 2 = \text{mechanical advantage}$$

89. Combination of Pulleys.—Fixed and movable pulleys are combined in a variety of ways according to the needs. It is quite common to use a fixed block with two or more sheaves and a movable block with two or more sheaves in it as shown in Fig. 62. The weight is fastened to the movable block, one end of the rope is attached to the movable block, and the force applied to the other end of the rope. If the weight is supported by five ropes, the pull on each rope is 1/5 of the weight. Hence, the force required to move the weight is only 1/5 as great as the force required to lift it without the system of pulleys. Fig. 63 shows another possible way in which pulleys can be combined.

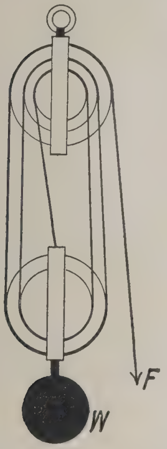


Fig. 62.—Block and tackle.

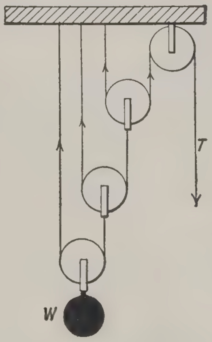


Fig. 63.—Combination pulley.

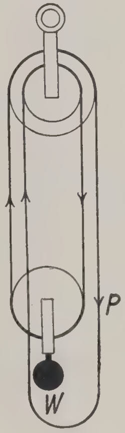


Fig. 64.—Differential of pulleys.

90. The Differential Pulley.—Where heavy weights are to be lifted, use is often made of a differential pulley (Fig. 64). This pulley consists of two sheaves of different diameters in the fixed block. These sheaves are rigidly fastened together. In the movable block there is one sheave. An endless chain passes over each of the sheaves. Teeth-like projections extend from the rim of the sheaves. Over these teeth the links of the chain fit so that the chain will not slip.

Let the force P move down far enough to cause the fixed pulley to turn around once. If R is the radius of the larger sheave and r the radius of the smaller sheave, the work done by P will be $2\pi R \times P$. Since r is the radius of the smaller sheave in the fixed pulley the length of chain unwound in one revolution is $2\pi r$. The weight W will be lifted $\frac{1}{2}(2\pi R - 2\pi r)$, and the work done on W is therefore $W\pi(R - r)$. Applying the principle of work to the pulley, we have, since **input** = **output,**

$$W \times 2\pi \frac{(R - r)}{2} = P \times 2\pi R$$

$$W/P = \frac{2R}{\mathrm{R} - r} = \text{mechanical advantage.}$$

The mechanical advantage will be determined by the radius of the larger sheave in the fixed pulley and by the difference between the radii of the sheaves in the fixed pulley. When the difference between the radii of these two sheaves is small, the mechanical advantage of the differential pulley is large.

Example.—A man wishes to lift a mass of 1 ton by means of a differential pulley. The diameter of the larger pulley is 8 in. and that of the smaller 7 in. Find the force the man must exert.

$$\frac{\text{Weight lifted}}{\text{Force exerted}} = \frac{2R}{R - r}$$

$$\frac{W}{F} = \frac{2R}{R - r} = \frac{2 \times 8}{8 - 7} = \frac{16}{1}$$

$$W = 2{,}000 \text{ lb.}$$

$$\frac{2{,}000}{F} = \frac{16}{1},$$

and

$$F = \frac{2{,}000}{16} = 125 \text{ lb.}$$

Problems

1. In a system of one fixed and one movable pulley what load can be pulled by a horse which exerts a force of 450 lb.? If the horse walks at the rate of 2 miles per hour, how much work is done in 10 min.? At what rate is the horse working?

2. Find the mechanical advantage of a differential pulley in which the radius of the larger pulley is 6 in. and the radius of the smaller pulley is 5 in.

3. What is the mechanical advantage of a block and tackle, if each block contains four pulleys, the fixed end of the rope being fastened to the fixed block?

4. What force is required to raise a load of 2 tons hung from a movable pulley and a fixed block with two pulleys, the rope being fastened to the movable pulley?

91. Inclined Plane.—Suppose a man must raise a block of ice from the ground into a wagon. Evidently the simplest way is to lift it vertically. But if the weight is too heavy for the man to lift he can get a plank and put one end of the plank on the ground and the other on the rear of the wagon. Although the ice is too heavy to lift the man can push it up the plank which acts as a simple machine known as an **inclined plane.** When a body rests on such an inclined plane (Fig. 65) the weight of the body acts vertically downward. The plane pushes on the body in a direction perpendicular to the surface of the plane. If there is no friction between the plane and the body it is necessary to apply a third force to the body in order to keep it from slipping or to make it move up the plane. Every street or road which is not level is such an inclined plane. Experience teaches that the steeper the plane the greater the force required to haul the load up the incline. The force necessary to make the body move up the incline may be applied (1) parallel to the face of the plane or (2) parallel to the base of the plane.

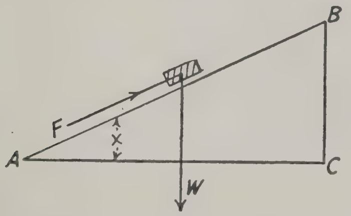


FIG. 65.—Mechanical advantage of inclined plane; force parallel to the plane.

92. Force Parallel to the Face of the Plane.—When the force is applied parallel to the face of the plane (Fig. 65) it is easy by an application of the principle of work to find the relation between the force F necessary to drag the body up the plane and the force W necessary to lift the body directly. Suppose the weight W is pushed from the bottom to the top of the plane a distance, l. The weight W has been lifted a height h, and the work done is

$$Wh \text{ ft.-lb.}$$

If F is the force parallel to the face of the plane, the work done by this force in moving the body the entire length of the plane is

$$Fl \text{ ft.-lb.}$$

Neglecting friction these two works must be the same.

$$F \times l = W \times h$$

$$\frac{W}{F} = \frac{l}{h} = \text{mechanical advantage.}$$

Since the mechanical advantage is defined to be the ratio of the force overcome to the force applied, W/F is the mechanical advantage of the plane, and this is equal to the ratio of the length of the plane to its height. In order, therefore, to have the mechanical advantage as large as possible, the inclination of the plane should be as little as possible.

Example.—A mass rests on an inclined plane whose length is 8 ft. and whose height is 3 ft. Find the mechanical advantage.

$$\text{Mechanical advantage} = \frac{\text{force to lift mass}}{\text{force to push it up plane}} = \frac{\text{length}}{\text{height}} = \frac{l}{h} = \frac{8}{3}$$

93. Grade.—The ratio of the height of an inclined plane to its length is of much importance in the construction of roads. This ratio is expressed by engineers as so many feet rise per hundred feet along the plane. Suppose a road rises 3 ft. per hundred feet along the incline, then the road has a a 3 per cent. grade. On such a grade the force required to move the load up the incline is only 3 per cent. of the force required to lift it directly. Consequently a team of horses can pull a much heavier load up such a grade than can be lifted directly. In the construction of roads it becomes important to have the slope as gentle as possible.

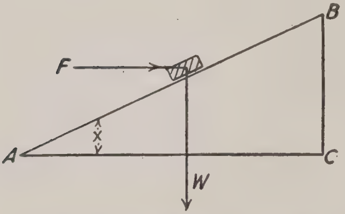


FIG. 66.—Mechanical advantage of an inclined plane—force parallel to the base.

94. Force Applied Parallel to the Base.—The force F to push the body up the inclined plane may be applied parallel to the base of the plane (Fig. 66). In this case the work done by the force F in moving the body up the plane from A to B is

$$F \times b,$$

where b is the length of the base of the plane. In this and in all cases the displacement must be measured in the direction in which the force acts. Since the force acts parallel to the base of the plane, the displacement must be measured parallel to the base of the plane. The work done in lifting the body is,

$$W \times h,$$

the same as in the preceding case. Applying as before the principle of work,

$$F \times b = W \times h$$
$$W/F = b/h = \text{mechanical advantage.}$$

To get the mechanical advantage when the force is applied parallel to the base of the plane, divide the length of the base by the height of the plane. Since the length of the plane is not as great as the length of the incline which is the hypotenuse of the triangle, the mechanical advantage in this case is not as great as in the preceding case.

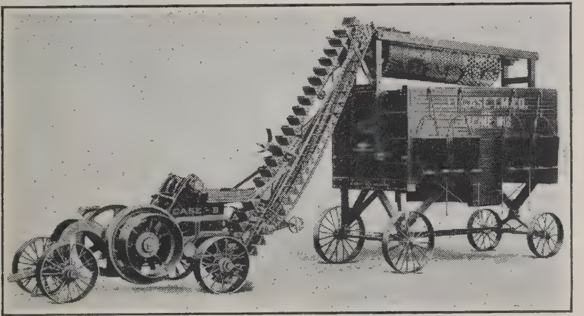


FIG. 67.—Rock crusher—an inclined plane.

Example.—A mass rests on an inclined plane whose length is 8 ft. and whose height is 3 ft. If it pushed up the plane by a force acting parallel to the base of the plane, what is the mechanical advantage?

$$\text{Mechanical advantage} = \frac{\text{weight}}{\text{force parallel to base}}$$
$$= \frac{\text{length of base}}{\text{height}}$$
$$= \frac{b}{h} = \frac{7.4}{3} = 2.4$$

95. Rock Crusher.—In a rock crusher (Fig. 67) there is an application of the principle of the inclined plane. By means of an endless chain of buckets the crushed stones are carried up an inclined plane to the desired height from which they are allowed to fall to the ground. When the buckets are all filled the work required to move the buckets up the incline is the same as that required to lift them directly. However, the force required to push the buckets up the incline is less than the force required to lift them directly. The smaller force acts through the larger distance and the larger force through the smaller distance.

96. Dump Cart.—A dump cart by means of which heavy loads can be removed from the wagon or cart depends on the principle of the inclined plane. Gravity pulls on the load in the wagon and leaves an unbalanced force which acts down the plane, causes whatever masses may be in the wagon to move downward and roll out.

97. Haystacker.—A haystacker (Fig. 68) is a form of inclined plane. The force applied to the load of hay causes it to move up the incline. This force is less than the force required to lift the load directly. The force exerted by the horses to raise the hay is therefore less than they would need to exert without this stacker. In the end the work done is the same. In one case the work is done more rapidly and in the other case it is done more slowly.

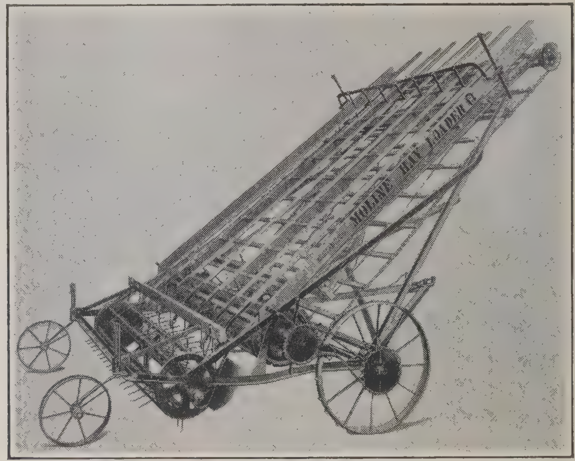


FIG. 68.—A haystacker—an inclined plane.

98. Wedge.—If instead of pulling the load up the inclined plane we push the inclined plane under the load, the inclined plane is known as a wedge (Fig. 69). Wedges are usually double inclined planes with the force acting parallel to the base of the plane. The force is often applied by a blow from a hammer. To make the mechanical advantage of the wedge large, the angle of the wedge should be small. It then requires less force to drive the wedge. It is impossible to give a simple expression for the mechanical advantage in this case because friction can not be neglected, and the resistance to be overcome is not constant all over the face of the plane. The principle of the inclined plane is used in many cutting tools such as the ax, the chisel, and the plane.

99. The Screw.—The screw, which consists of a cylindrical rod of metal around which has been cut a groove or thread rising uniformly, is really another form of the inclined plane. A similar thread cut in the iron base or nut fits the thread of the screw. When the screw is turned around once it moves upward a distance equal to the distance between the successive threads. This distance between the successive turns of the thread is called the **pitch of the screw.** If the head of the screw is supporting a weight of 1 ton, and the screw makes one complete turn, the weight is lifted the distance between two successive threads; i.e., through a distance equal to the pitch of the screw. If the pitch of the screw is 0.01 ft., the work done by the screw when turned once around is $2000 \times 0.01 = 20$ ft.-lb. The effort is exerted through a distance equal to the circumference of the circle traced out by one end of the bar or handle used to turn the screw.

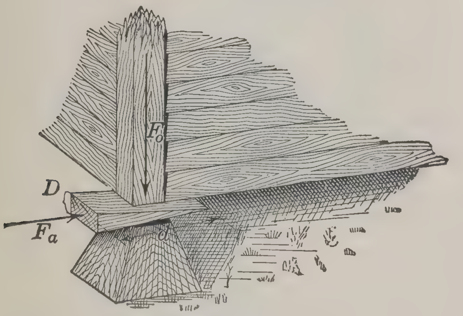


Fig. 69.—The wedge—two inclined planes.

100. The Jack Screw.—When large forces must be exerted a type of screw known as the jack screw is often used (Fig. 70). Let p be the pitch of the screw, W the weight to be lifted, F the force applied to the lever arm and r the radius of the lever arm. In one complete turn of the screw the output is the weight lifted times the distance through which it is lifted.

$$W \times p;$$

The input is the force applied times the distance through which it acts.

$$F \times 2\pi r.$$

By the principle of work,

$$W \times p = 2\pi rF$$
$$W/F = 2\pi r/p = \text{mechanical advantage.}$$

Hence the mechanical advantage is equal to the circumference traced out by the end of the lever in one complete revolution divided by the pitch of the screw. This mechanical advantage may be made large by making the pitch of the screw small or by making the lever arm long.

Example.—Suppose the length of the lever arm used with a jack screw is 5 ft., the pitch of the screw is 1 in. and the weight to be lifted is 10 tons.

$$\text{Output} = \text{input}$$
$$20{,}000 \times 1/12 \text{ ft.-lb.} = F \times 5 \times 2\,\pi \text{ ft.-lb.}$$
$$F = \frac{20{,}000}{120\pi} = 53 \text{ lb.}$$

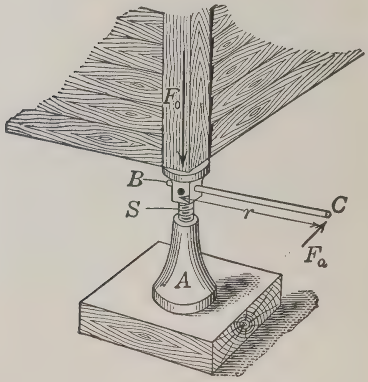


FIG. 70.—The jackscrew.

The friction in the screw will make it necessary to apply a force somewhat greater than the theoretical value.

101. Applications of the Screw.—The letter press, the screw propeller of a ship, the vise (Fig. 71), the cider press, etc., are familiar examples of this simple machine.

102. Combinations of Simple Machines.—Machinery is a more or less complicated combination of levers, pulleys, wheels and axles and screws.

For example, the builders' crane is made up of a movable pulley to which the weight to be lifted is attached. This pulley has a mechanical advantage of 2. The force is applied through three wheels and axles. Each of these wheels and axles has a mechanical advantage equal to the radius of the wheel divided by the radius of the axle. The total mechanical advantage of the machine is found by multiplying together the mechanical advantage of the separate parts.

Problems

1. The length of a gravity conveyor is 45 ft. and its height is 5 ft. Find how much force is necessary to push up this conveyor a box weighing 800 lb. If the box is pushed up in 2.5 min., find the work done and the horsepower.

2. If a road crusher is inclined 30° to the horizontal and is 25 ft. long, neglecting the weight of the buckets, how many pounds of crushed stone can be lifted per hour by an engine which develops 15 horsepower?

3. The average walking draft of a horse weighing 1,600 lb. is 250 lb. If it requires a traction of 60 lb. to pull 1 ton on good level road, find the steepest grade up which four horses can pull a load of 9 tons.

4. A hammer drives a wedge which is 2 inches thick and 1 ft. in length a distance of $\frac{1}{2}$ in. at each stroke. If the wedge supports a weight of 1.5 tons what is the average force exerted by the hammer? Neglect friction.

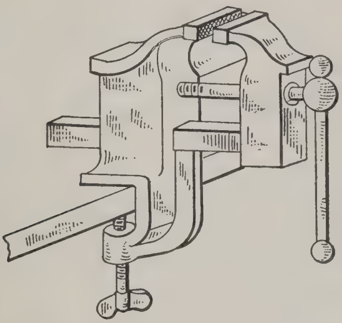


FIG. 71.—The machinist's vise.

5. Find the theoretical and actual mechanical advantage of a jackscrew of 30 per cent efficiency whose screw has 10 threads to the inch and is turned by a rod giving a lever arm of 3 ft.

6. In a cider press the lever arm is 6 ft. long. The screw has 8 threads to the inch and the diameter of the piston in the press is 1 ft. Find the total force and the force per square inch exerted on the apples by the press, when a force of 125 lb. is applied to the end of the lever.

7. A load of 6 tons is being lifted by a jackscrew whose pitch is 1/4 in. What force is necessary at the end of a lever arm which is $2\frac{1}{2}$ ft. long, assuming that friction reduces the efficiency of the machine 35 per cent?

8. A farmer wishes to make a bench-vise with a lever which is 9 in. long and a screw of such pitch that when he applies a force of 75 lb. to the lever it will cause a pressure of 2,500 lb. on a block in the vise. Find the number of threads to the inch on the screw.

CHAPTER IX

ELASTICITY AND STRENGTH OF MATERIALS

103. Stress.—When a cable supports an elevator or a belt transmits power from one pulley to another, there is a pull applied to each end which tends to pull one part away from the neighboring parts. In such a case the belt or cable is sustaining a stress, and if this stress is sufficiently large, the belt or cable breaks. When one end of a post is placed on a foundation and a load is placed on its top, the post resists a push at each end which tends to shorten it and may crush it. The former is a tensile stress and the latter a compressive or crushing stress.

There is another kind of stress like that which is shown when a screw is broken by trying to drive it into hard wood with a screw driver. In this case the molecules have been pushed aside with such force that they slide over each other. When a beam is bent or a rod twisted, one layer of molecules is sheared over a neighboring layer and made to occupy a new position. Stresses of this kind are called shearing stresses. When equal force is applied on all sides of a body its volume is changed but its shape remains the same. The force which produced the change in volume is called a stress.

Stress is measured by the force per unit area. It is found by dividing the total force by the area over which the force acts. **The stress is, therefore, defined to be the force per unit area.**

104. Strain.—The term strain is applied to any change occurring in the dimensions or shape of a body when forces are applied to it. A wire from which a weight is hanging becomes longer as the load is increased. This elongation is called a **longitudinal strain.** This longitudinal strain is calculated by dividing the change in length by the original length. It is therefore the change in length per unit length.

Let L = the original length of the wire,
l = the change in length due to the added load,
l/L = the longitudinal strain = change in length per unit length.

Example.—A piece of brass wire 27.1 cm. long is found to be stretched 0.133 cm. by a load of 0.5 kilogram. What is the longitudinal strain?

$$\text{Longitudinal strain} = \frac{\text{increase in length}}{\text{original length}} = \frac{0.133}{27.1} = .0049 \text{ cm. per cm. of length.}$$

A body which is subjected to uniform normal stress all over its surface changes its volume. For example when pressure is applied to an enclosed volume of water, the volume of the water is decreased. This change of volume with increase of pressure is more noticeable in the case of gases than in the case of liquids. This kind of strain is known as **volumetric strain.**

Let V = the original volume of the liquid,

v = the change in volume produced by the application of pressure,

Then v/V = volumetric strain or change in volume per unit volume.

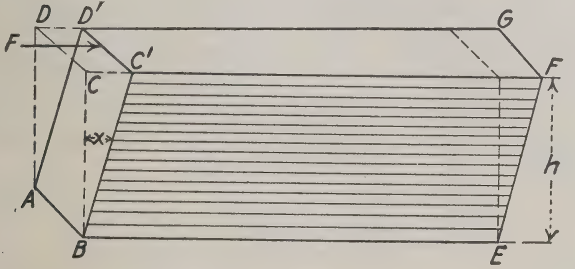


FIG. 72.—Shearing strain.

Example.—A steel cylinder having a volume of 900 cu. in. is subjected to a hydrostatic pressure and its volume decreases 0.57 cu. in. Find the volumetric strain.

$$\text{Volumetric strain} = \frac{\text{change in volume}}{\text{original volume}} = \frac{0.57}{900} = 0.000633 \text{ per cu. in.}$$

Shearing strain occurs when a body is subjected to a shearing stress. If one cover of a thick book is held firmly on the table and a force parallel to the top of the table is applied to the other cover, the shape of the book is changed. Its thickness remains the same. If the book is replaced by a rectangular block the shearing strain produces the result shown in Fig. 72. In cases of this sort the shearing strain is measured by determining the angle through which a vertical line in the body has been rotated by the shearing force. This angle is denoted by x in Fig. 72 and it is measured in radians.

Since $\tan x = \frac{CC'}{BC}$ and since for small angles $\tan x$ is approximately equal to x,

$$\text{Shearing strain} = x = \frac{CC'}{BC} \text{ approximately.}$$

Example.—A rectangular block of metal 10 cm. high rests on a horizontal table. A force which is parallel to the surface of the table is applied to the upper surface of the block and produces a displacement of 1 mm. in the face $C'G$. (Fig. 72.) What is the angle of shear?

$$\text{Shearing strain} = x = \frac{0.1}{10} = 0.01 \text{ radian.}$$

105. Elasticity and Coefficient of Elasticity.—Elasticity is that property of a body by virtue of which it tends to return to its original shape and dimensions when the disturbing forces have been removed. In many kinds of material this return is very perfect if the material has not been loaded beyond a certain limit called the **elastic limit.** If the body is loaded beyond this limit it does not completely recover its original shape and dimensions. In some cases the return of the body to its original condition is delayed for some time especially after long repeated distortions.

The coefficient of elasticity is defined as,

$$\text{Coefficient of elasticity} = \frac{\text{stress}}{\text{strain}}$$

The stress as already defined is taken to mean the force per unit area producing the deformation and the strain means the percentage or fractional deformation produced by the stress.

106. Hooke's Law.—If observations are made on the change in the length of a wire produced by hanging various loads on it, it is found that the change in length is proportional to the load which produced it. When one end of a rod is held firmly and the other end twisted, the angle through which the end is twisted with respect to the fixed end is proportional to the moment of force which produced the twist. In like manner experiments show that beams are bent and springs are stretched amounts which are proportional to the loads which were applied. These observations may be stated in the form of a single law which was discovered by Hooke and bears his name. Hooke's law states that, **strains are proportional to the stresses producing them.**

$$\text{Stress} = \text{constant} \times \text{strain.}$$

This law is obeyed by many substances if they are not strained beyond their elastic limit.

107. Young's Modulus of Elasticity.—Where a rod or wire (Fig. 73) is pulled or pushed so that there is a longitudinal stress and a longitudinal strain the modulus of elasticity is called Young's modulus of elasticity. It is defined as follows:

Let M = the pull or weight applied to the wire,
a = the cross section of the wire,
L = the original length of the wire,
l = the change in length produced by weight M.

Then the longitudinal stress $= M/a =$ force per unit area on cross section of the wire.

$$\text{Longitudinal strain} = \frac{\text{change in length}}{\text{original length}} = l/L$$

$$\text{Young's modulus} = \frac{\text{stress}}{\text{strain}}$$

$$= \left(\frac{M}{a}\right) \div \left(\frac{l}{L}\right) = \frac{M}{a} \times \frac{L}{l}$$

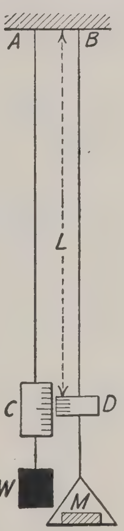


FIG. 73.—Young's modulus of elasticity.

Example.—A wire 120 in. long with a cross section of 0.125 sq. in. hangs vertically. When a load of 450 lb. is applied to the wire, it stretches 0.015 in. Find Young's modulus of elasticity.

$$\text{Young's modulus} = \frac{\text{stress}}{\text{strain}} = \frac{M/a}{l/L} = \frac{450}{120} = 2.88 \times 10^7 \text{ (lb. sq. in.)}$$

108. Volume Elasticity.—When a body like a piece of iron is immersed in a liquid so that a uniform pressure is applied to its surface, there is a change in volume and its volumetric elasticity must be considered.

Let p = the increase of pressure applied,
v = the change in volume produced by this pressure,
V = the original volume.

$$\begin{array}{l}\text{Coefficient of}\\ \text{volumetric elasticity}\end{array} = \frac{\text{volumetric stress}}{\text{volumetric strain}}$$

$$= \frac{\text{force per unit area}}{\text{change in volume per unit vol.}}$$

$$E = \frac{p}{v/V}$$

Example.—The coefficient of volume elasticity of copper is 6,600,000 lb. per sq. in. A sphere of copper having a volume of 100 cu. in. is subjected to a pressure of 1,000 lb. per sq. in. Find the change in volume which takes place.

$$E = \frac{\text{change in pressure per unit area}}{\text{change in volume per unit volume}} = \frac{p}{v/V}$$

$$E = 6{,}600{,}000 \text{ lb. per sq. in.}$$

$$p = 1{,}000 \text{ lb.}$$

$$6{,}600{,}000 = \frac{1{,}000}{\dfrac{v}{100}}$$

$$v = 1/66 = 0.016 \text{ cu. in.}$$

109. Stiffness and Strength of Beams.—It is important to be able to calculate the bending (Fig. 74) which will be produced in a beam by a given load. Experiments on beams of different size and shape have shown that the bending for a given load depends on the length of the beam, on its breadth and on its depth. The relation between these dimensions and the stiffness of a beam of rectangular cross section can be expressed as follows:

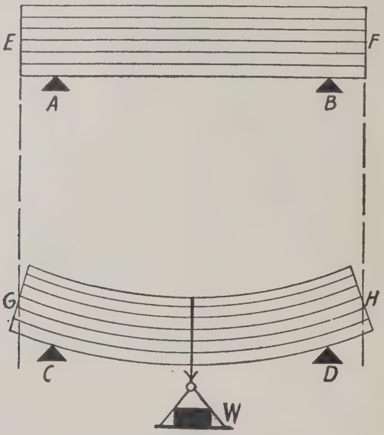


Fig. 74.—Bending of beams.

$$\text{Stiffness factor} = \frac{\text{breadth} \times (\text{depth})^3}{(\text{length})^3}$$

Example.—Find the stiffness factor of a rectangular beam which is 20 ft. long, 4 in. broad and 6 in. depth.

$$\text{Stiffness factor} = \frac{4 \times (6)^3}{(12 \times 20)^3} = \frac{864}{13{,}824{,}000} = 0.000063$$

The stiffer of two beams is not necessarily the stronger. This means that the bending of a beam does not give at once the true measure of its strength. The strength does, however, depend on the same dimensions on which the stiffness depends but it depends on these dimensions in a different way from the way in which the stiffness depends on them.

$$\text{Strength factor} = \frac{\text{breadth} \times (\text{depth})^2}{\text{length}}$$

Example.—Find the ratio of the strengths of the following beams:

Beam A: length 20 ft., breadth 3 in., depth 6 in.
Beam B: length 30 ft., breadth 6 in , depth 4 in.

$$\frac{\text{Strength of } A}{\text{Strength of } B} = \frac{3 \times 6^2 \div 20}{6 \times 4^2 \div 30} = \frac{27}{16}$$

110. Cross Sections of Beams.—Where rectangular wooden beams are used they are placed so that the depth is greater than the breadth. Both the strength and the stiffness are proportional to the breadth, but the stiffness increases as the cube of the depth and the strength increases as the square of the depth. In order to place the beam so that the stiffness and the strength will be as large as possible, the depth must be made as great as possible. In steel beams, in order to get the greatest stiffness and strength for a given weight of beam, account is taken of the fact that in bending, the top layer of a beam is shortened and must resist compression and the lower layer is lengthened and must resist a tension. The central layer, however, remains the same length. To make the beam as strong or as stiff as possible for a given amount of material, as much material as possible should be put in the upper and lower layers and as little as possible near the middle. For this reason iron beams are designed as indicated in Fig. 75. They are called I-beams and give the greatest strength and stiffness for a given amount of iron.

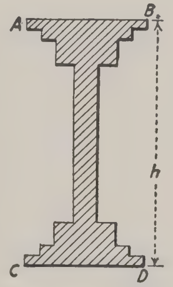


Fig. 75.—An I-beam.

111. Inelastic and Elastic Bodies.—The motion of bodies after collision depends greatly on their degree of elasticity. If the bodies are without any elasticity, they do not recover their original form or dimensions after collision. For example, when two spheres of putty collide, they are permanently deformed, consequently do not exert any force on each other after collision and do not tend to separate again, but move on as one body. On the other hand, when elastic or partly elastic bodies collide, they are only slightly deformed and tend to regain their original shape immediately after collision. While such bodies are regaining their original shape, they exert a force on each other and tend to separate. It is found by experiment that the relative velocity of the bodies after collision bears a definite ratio to their relative velocity before collision. This ratio of the relative velocity after collision to the relative velocity before collision is called the **coefficient of restitution.**

112. Impact of Inelastic Bodies.—In Fig 76 two inelastic bodies of masses m_1 and m_2 and velocities v_1 and v_2 respectively are represented just before and after collision. After collision they move on as a single body of mass $(m_1 + m_2)$ and velocity v. A law known as the **Law of Conservation of Momentum** follows directly from Newton's second Law of Motion and is easily verified by experiment. **It states that the total momentum before impact = total momentum after impact always.**

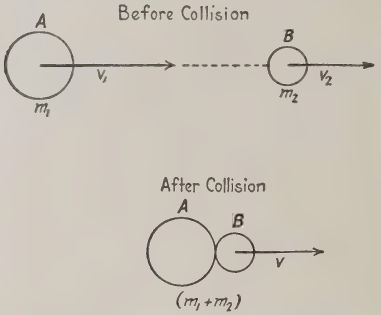


FIG. 76.—Collision of inelastic bodies.

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v$$

$$v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}$$

If the bodies are moving in opposite directions before collision,

$$m_1v_1 - m_2v_2 = (m_1 + m_2)v$$

$$v = \frac{m_1v_1 - m_2v_2}{m_1 + m_2}$$

Example.—Two inelastic bodies of mass 400 g. and 800 g. are moving in the same direction with velocities of 50 cm. per sec. and 20 cm. per sec. respectively. Find their common velocity after collision.

Momentum before collision = momentum after collision

$$m_1v_1 + m_2v_2 = (m_1 + m_2)v$$

$$400 \times 50 + 800 \times 20 = (400 + 800)v$$

$$v = \frac{20{,}000 + 16{,}000}{1{,}200} = \frac{360}{12} = 30 \text{ cm. per sec.}$$

113. Impact of Elastic Bodies.—The direct collision of two perfectly elastic spheres A and B, moving in opposite directions is represented in Fig. 77. At collision the spheres are deformed and during the recovery from this deformation they exert forces on each other which cause the spheres to separate and one or both of them to move in a direction opposite to the direction of motion before collision. Let m_1 and m_2 be the masses of the spheres, v_1 and v_2 their respective velocities before collision and u_1 and u_2 the corresponding velocities after collision.

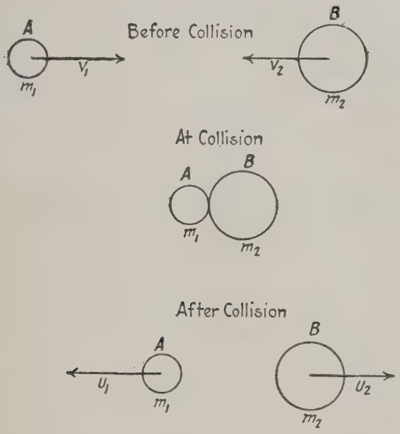


Fig. 77.—Collision of elastic bodies.

Momentum before collision = momentum after collision.

$m_1v_1 - m_2v_2 = -m_1u_1 + m_2u_2$, when the direction of motion of both bodies is reversed.

$m_1v_1 - m_2v_2 = m_1u_1 - m_2u_2$, when the direction of motion of B alone is reversed.

If the bodies are moving in the same direction before collision, and if their momenta are such that they move in the same direction after collision,

$$m_1v_1 + m_2v_2 = m_1u_1 + m_2u_2$$

Example.—Two elastic spheres of masses 40 lb. and 60 lb. moving in opposite directions with velocities of 8 ft. per sec. and 2 ft. per sec. respectively, collide. If the velocity of the larger sphere after collision is 3 ft. per sec., what is the velocity of the smaller sphere?

$$\text{momentum before collision} = \text{momentum after collision}$$

$$m_1v_1 - m_2v_2 = -m_1u_1 + m_2u_2$$

$$40 \times 8 - 60 \times 2 = +40 \times 3 + 69u_2$$

$$u_2 = \frac{40 \times 8 - 60 \times 2 - 40 \times 3}{60} = \frac{80}{60} = 1.33 \text{ ft. per sec.}$$

Problems

1. Taking Young's modulus for iron to be 2×10^9 grams per sq. cm. Find the increase in length of an iron wire 4 m. long when it is stretched by a force of 5 kg., the radius of the wire being 0.4 mm.

2. A vertical rod 2 m. long and 0.3 sq. cm. cross section has suspended from it a weight of 150 kg. It increases in length 0.06 cm. Find Young's modulus of elasticity.

3. If the volume of a gas is 1,500 cu. cm. under a pressure of 12,500 g. per sq. cm. and if an additional pressure of 500 g. per sq. cm. to the gas reduces its volume to 900 cu. cm., find the coefficient of volume elasticity of the gas.

4. The coefficient of volume elasticity of water is 20×10^6 grams per sq. cm. Find the change in the volume of the water in a tube which is 1 m. long and 6 sq. cm. in cross section, when a force of 30 kg. per sq. cm. is applied to the water.

5. An inelastic ball having a mass of 200 g. and a velocity of 15 cm. per sec. strikes a second inelastic ball of mass 150 g. moving in the same direction as the first ball with a velocity of 30 cm. per sec. Find the velocity of the balls after collision.

6. A perfectly inelastic sphere of mass 40 lb. strikes normally an inelastic sphere of 200 lb. which is at rest. After collision the two spheres move on as a single body with a velocity of 4 ft. per sec. Find the original velocity of the 40 lb. mass.

CHAPTER X

MOLECULAR FORCES AND MOTIONS

114. Molecular Theory of Matter.—Any form of matter is made up of very small particles called molecules. These molecules are packed together in the substance and held together more or less securely depending on the nature of the substance. In between these molecules are spaces which are unoccupied. Even in solids these spaces are so large that the molecules do not permanently touch each other. In liquids these intervening spaces are still larger and in gases under ordinary conditions they are very large in comparison with the size of the molecules. It is customary to assume that the molecules are spherical in shape but this is not exactly true. They are in rapid motion and in the case of gases strike against each other and rebound much as steel balls when shaken vigorously in a closed vessel.

115. States of Matter.—There are three states in which matter may occur. It may be a **solid** as ice, iron or wax; a **liquid** like water, mercury or oil; or a **gas** like steam, air or hydrogen. Under suitable conditions it is possible for a body to change from the solid to the liquid state, from the liquid to the gaseous state and from the solid to the gaseous state. Thus water is found in the form of ice when its temperature is sufficiently low or as water at higher temperatures or as steam at still higher temperatures. A solid like camphor will pass directly from the solid to the gaseous state without first passing through the liquid state. This is evident from the odor which is received from it directly as a solid. It has been found possible to change ordinary atmospheric air into a liquid by lowering its temperature and increasing sufficiently the pressure on it. This liquid air may also be solidified. Thus substances which occur normally in nature in one state may under suitable conditions be a gas, a liquid or a solid.

A solid is that state of matter in which the molecules strongly cling together and tend to keep the same relative positions.

Since the molecules of a solid are rather rigidly held together, solids resist any tendency to change their shape or size and therefore preserve their original form unless acted upon by some outside force.

In a liquid the molecules tend to cling together but are free to move with respect to each other. **Liquids do not resist forces tending to change their shape,** but resist forces tending to change their volume. Because the molecules of a liquid can be easily displaced with respect to each other the layers of a liquid flow freely over each other, and a liquid assumes the form of the vessel in which it is placed.

In a gas the molecules are free to move about without restriant and thus tend to separate indefinitely. Gases do not resist forces tending to change their shape and only to a small degree do they resist forces tending to change their volume. Since the molecules of a gas can move freely and tend to separate indefinitely the gas fills up entirely the space of the containing vessel and exerts a pressure on it.

116. Diffusion of Gases.—One of the best evidences for believing that a gas is made up of a large number of molecules is found in the rapidity with which an odor penetrates to all parts of a room. Suppose that a bottle of ammonia water or some other substance with a pronounced odor is opened in a room. In a very short time the odor has penetrated to all parts of the room. Here the gas giving the odor whether it comes from a solid or a liquid consists of molecules which hit their neighbors and in turn are struck by other molecules. By this buffeting back and forth the individual molecules are widely scattered and soon are present in all parts of the room. The pleasant odor of flowers or trees is a case of this scattering of the molecules of one gas among the molecules of the air. This process of scattering the molecules of one gas among the molecules of another gas is called **diffusion of gases.**

It is diffusion of this kind which keeps the air which is composed of oxygen and nitrogen in such a state of uniform mixture.

117. Absorption of Gases.—Since the molecules of a gas have a tendency to separate from each other as far as possible a gas which is free will distribute itself as widely as possible. If its molecules which are in rapid motion strike the surface of a liquid, some of them will be retained and ultimately distributed though the liquid. This gives rise to absorption of the gas by the liquid. For example, ammonia water is a mixture of ammonia gas and water. This tendency to absorb gases is greater in some liquids than in others.

If milk is left exposed to gases which arise from decaying organic matter or to other gases, these gases may be absorbed by the milk. The warmer the air surrounding the milk, the greater is the tendency of the molecules of the gas to plunge into the milk. This tendency to absorb the molecules of a gas does not go on indefinitely. Some of the molecules which have been caught by the milk or other liquid escape again. When the number of molecules which escape per second from the surface is equal to the number which are caught, the liquid will not absorb more gas and is saturated.

The foliage of plants is freely permeable to gases. All leaves are actively engaged in absorbing and exhaling certain gaseous ingredients of the atmosphere, during the whole of their healthy existence. As rapidly as these gases are removed from the atmosphere surrounding the plant, they are resupplied from without by diffusion.

118. Diffusion of Liquids.—In liquids as well as in gases the molecules are free to move about with greater or less freedom according to the nature of the liquid. If a little vinegar is placed in a pail of water all of the water soon becomes sour. When a lump of sugar is placed in a cup of coffee the contents of the whole cup is sweetened by the distribution of the molecules of the sugar throughout the coffee. When a piece of fuchsin is dropped quickly to the bottom of a test tube containing water which is placed where it can not be jarred, a diffusion of fuchsin from the bottom to the top of the tube is observed. This diffusion is made evident by the changes of color which take place in the tube.

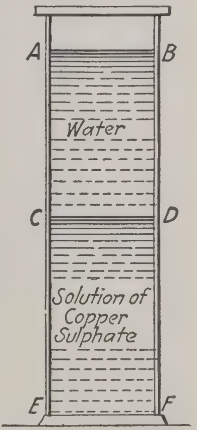


Fig. 78.—Diffusion of copper sulphate.

The diffusion of liquids may also be demonstrated by taking a tall glass jar (Fig. 78) filled with water and by means of a thistle tube extending to the bottom of the jar pouring into the jar a solution of copper sulphate until the bottom of the jar is filled to a height of one or two centimeters. If the jar is allowed to stand for some days without being disturbed, an upward diffusion of the copper sulphate can be observed. The force of gravity tends to keep the copper sulphate on the bottom but diffusion overcomes this force and the copper sulphate rises in the jar. The action which takes place in this case is very similar to that which takes place in the diffusion of gases except that it takes place more slowly in liquids than in gases. The motion of the molecules in diffusion is such that they seldom travel far without encountering neighboring molecules. Between collisions their paths are a series of short straight lines. In liquids the distances between collisions is less than in gases. Hence, the molecules diffuse more slowly in liquids than in gases.

119. Solutions.—If a crystal of some substance like salt or sugar is placed in a vessel of water it soon distributes itself to every portion of the water. The solid is said to go in solution in the liquid. This process of solution arises from the fact that the molecules of liquid attract the molecules of the solid and pull them away from their neighbors. This will not go on indefinitely for after a while no more of the molecules of the solid will distribute themselves among the molecules of the liquid. The solution is then **saturated.** The amount of the solid which will go into solution depends on the nature of the solid, on the nature of the liquid and on the temperature of the solution. Usually the amount of the substance which will go into solution increases as the temperature of the solution is raised. This is doubtless due to the fact that at the higher temperatures the molecules of the solid move with greater velocity and are more easily detached from their neighbors.

Liquids like water will dissolve more or less freely all gases. When such a solution occurs the gases are uniformly distributed throughout the liquid. The dissolved gas may be thought of as distributed between the molecules of the liquid. Thus ammonia gas or hydrochloric acid gas is readily dissolved in water, and soda water is a solution of carbon dioxide in water.

120. Illustrations of Solutions.—The process of solution is very important in nature. Plants receive their food from the soil in the form of liquids, solids or gases which are in solution in the water in the soil. The fertility is determined by the rate at which this plant food may be dissolved and carried to the plant. In humid regions where there is an excessive amount of water in the soil the concentration of the salts in solution in the soil may become too small and the plants cease to grow. On the other hand in arid regions where the amount of water in the soil is small, the concentration of the salts becomes too large and the plants cease to grow for this reason. Many of the fluids which have a part in the vital processes of the body are solutions of one sort or another. A considerable part of cooking, preserving and canning is concerned with solutions and the changes in the concentrations of solutions. In the manufacture of maple syrup, a dilute solution of sugar is changed into a more concentrated solution by evaporating some of the water in which the sugar is dissolved. When salt is used to preserve food, it forms a solution in the water or juices of food and then enters the food as a solution.

121. Osmosis.—When two miscible liquids or solutions are separated by a porous membrane, there is a tendency for one of the liquids to diffuse more rapidly through the membrane than the other liquid does. If water is separated from a solution of sugar by a suitable membrane, the water molecules on one side of the membrane diffuse through the membrane more rapidly than the water molecules on the other side do. This will cause the volume of the solution to increase and the volume of the pure water to decrease. If the membrane is fixed, a change in the level of the solution and the water takes place. This process of diffusion through a semi-permeable membrane is called **osmosis.**

The following experiment illustrates osmosis. Take a carrot (Fig. 79) and cut out its interior, filling the space with a thick syrup. Insert in the mouth of the cavity a rubber stopper through which projects a long glass tube and immerse the carrot in a wide mouthed jar filled with water. The water from the jar begins to flow through the pores of the carrot into the space filled with syrup. There is also a tendency for the syrup to flow out through the carrot into the jar. The tendency of the water to flow inward is greater than the tendency of the syrup to flow outward. Hence, the volume of the syrup in the carrot increases and the solution rises in the tube.

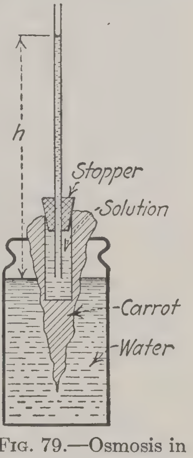


Fig. 79.—Osmosis in a carrot.

122. Illustrations of Osmosis.—When dried beans are placed in water the vegetable sack surrounding the bean acts as a membrane through which the water diffuses from the outside to the inside of the bean. The bean swells because of this flow of water into its interior. For this reason dried fruits such as prunes and raisins swell when they are cooked, and may even burst if the pressure becomes sufficiently large. If marine animals like oysters are transferred from salt water to fresh water, more water flows into the animal through the membrane which serves as its covering. A dilation of the animal results since the conditions for osmosis are present. Another illustration of this kind is found in the boiling of sausages. The animal membrane which acts as a covering for the sausage is freely permeable to the water, but does not so readily allow the passage of dissolved salts. Hence the water flows in through the membrane causing it to distend and usually burst during cooking. If the membrane is punctured before this rupture has occurred a quantity of liquid squirts out.

If fresh fruits are placed in strong sugar solution they begin to shrivel and decrease in size. This is also due to osmosis. In this case the stronger solution is on the outside and the weaker on the inside. Hence the flow of water is from the inside to the outside causing the fruit to shrink. Such a shrinkage is observed in the canning of peaches. After the peaches have been placed in the cans and covered with a strong sugar solution, they shrink on being heated to the required temperature for canning.

When celery is placed in water it becomes crisp due to the absorption of water by osmosis. Salts which are packed with fresh meats soon become brine by extracting juices from the meats. The meats shrink on account of the losses of these juices.

123. Crystalloids and Colloids.—Those substances which, like cane sugar, glucose, etc., pass through animal membranes most readily have been called **crystalloids.** Substances like gums, starches and albumens which do not diffuse through membranes are called **colloids.** The crystalloids in solution are in the molecular state; that is, the substance has been divided up into molecules. The colloids are less finely divided. Each colloidal particle contains a large number of molecules. The size of the colloidal particles varies greatly from one colloid to another. These particles are really in suspension whereas the crystalloids are in solution. Milk is a good illustration of a colloid. Ruby glass owes its color to the presence of gold particles in the colloidal state. When crystalloids are dissolved they produce marked changes in the properties of the solvent. If they are dissolved in water they diminish its vapor pressure, lower its freezing point and raise its boiling point. When colloids are added to water they produce scarcely any effect.

124. Osmotic Pressure.—It has been seen that if two solutions differing in concentration are separated by a semi-permeable membrane, the solvent will migrate from the solution where the concentration is small to the solution where the concentration is larger. The force which is responsible for the flow of the solvent in osmosis is called **osmotic pressure.**

In order to get a measure of this pressure, the following experiment is satisfactory. Take a thistle tube (Fig. 80) over the mouth of which is fastened a piece of bladder or parchment. Fill the thistle tube up to the beginning of the glass stem with a strong solution of sugar. Now immerse the thistle tube in a beaker filled with pure water so that the level of the water in the vessel and the level of the solution in the tube are the same. A diffusion of the water takes place through the membrane into the solution and the solution rises in the tube. This rise continues until the pressure of the solution in the tube is sufficient to prevent any more water from flowing into the tube. This pressure necessary to prevent the flow of the water into the solution is equal to the **osmotic pressure.**

125. Laws of Osmotic Pressure.—Two important laws have been established concerning osmotic pressure. They are as follows:

1. In fairly dilute solutions in which the molecules are not dissociated, the osmotic pressure is proportional to the concentration of the dissolved substance.

2. In a solution of given concentration the osmotic pressure increases as the temperature is increased, and the rate of increase is the same for all solutions.

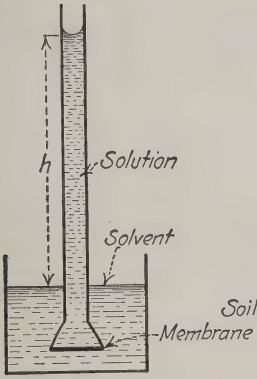


Fig. 80.—Osmotic pressure.

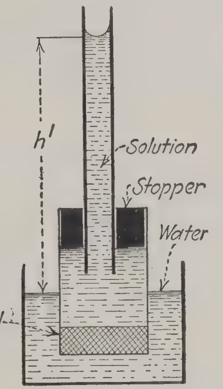


Fig. 81.—Osmotic pressure in soils.

126. Osmotic Pressure in Soils.—It has been found that a soil may sometimes act as a semi-permeable membrane and that water is sometimes moved through it by osmotic pressure. Fig. 81 shows an osmotic pressure tube in which a clay soil is made to act as a semi-permeable membrane. The lower end of the tube is covered with a cotton cloth supported by a wire gauze. On this is a column of clay soil which acts as a semi-permeable membrane. The tube is filled with a solution and closed with a rubber stopper through which passes a glass tube. When the larger tube is immersed in a vessel of water, the water rises in the narrow tube. This experiment shows that water will tend to flow through the soil toward the more concentrated solution. By increasing the thickness of the clay subsoil which is used for the membrane the efficiency of the membrane can be increased and the height to which the liquid rises will be correspondingly increased.

127. Cohesion and Adhesion.—In liquids the molecules move freely with respect to each other, but are held together by the attractive forces of the molecules on each other. This tendency of the molecules to cling together is most noticeable in the case of the more viscous liquids like heavy syrups and least noticeable in the more mobile liquids like alcohol and water. Not only do the molecules of a liquid cling to each other, but they also cling to the molecules of a solid as shown when a piece of glass is dipped into a vessel of water. The molecules of water adhere to the glass and form a thin film over its surface. The attraction of like molecules for each other is called **cohesion.** The attraction of unlike molecules for each other is called **adhesion.** The nature of the forces acting between the molecules is the same whether the molecules are alike or unlike.

It is cohesion which causes a drop of water to form. It is also cohesion which holds together so firmly the molecules of iron, copper and other solid substances. When glue or cement is used to fasten two pieces of wood or crockery together the adhesion of the glue or cement for the wood or the crockery causes a firm joint to be formed. Sometimes the adhesion is so large that the joint thus formed is stronger than the material out of which it is formed. This force of attraction of molecules for each other is very large but acts only through small distances.

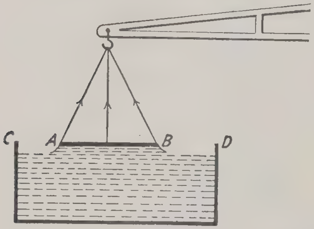


FIG. 82.—Cohesive forces in liquids.

To get a measure of this force suspend from one arm of a balance a circular glass plate. (Fig. 82.) Place weights enough on the opposite pan of the balance to just balance this circular plate. Having adjusted the plate so that it is horizontal, elevate a vessel of water so that the surface of the water just touches the under surface of the glass plate. Now find what weight must be added to the weights in the pan in order to raise the glass plate away from the surface of the water. This weight is a measure of the attractive forces of the molecules for each other. It is found that when the plate comes away from the water its lower surface is wet. Thus the molecules of water have pulled away from each other and not the molecules of water away from the molecules of glass. Hence the force of cohesion between the molecules of water is less than the force of adhesion between the molecules of glass and water.

128. Cohesion in Soils.—A good example of the action of cohesion is found in the behavior of soils. When the particles of soil are large, the cohesion between the particles is small and the soils are easily blown about by the winds. The ease or difficulty of plowing or cultivating soil depends largely on the cohesive forces between the particles of the soil. When the cohesion is large the soil is difficult to cultivate and easily forms clods. When the cohesion is small the soil pulverizes easily and is easily cultivated. The cohesive forces in the soil may be measured either in the wet or in the dry condition, and its value will depend on the conditions under which it is measured.

129. Surface Tension.—If the surface of a liquid is examined it has the appearance of being covered with a thin elastic film. It requires a force to break this film and the amount of this force depends on the nature of the liquid. For this reason if a needle is greased and gently placed on the surface of water it will float although the density of the needle is greater than that of the water. For this same reason some insects can walk on the surface of liquids like water. This apparent surface film is due to the molecular attraction in the liquid.

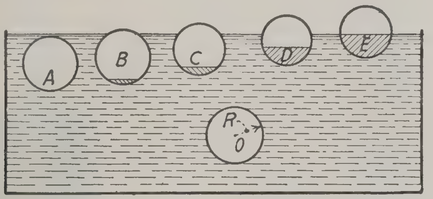


Fig. 83.—Forces on a molecule inside of a liquid.

A molecule in the mass of the liquid is in equilibrium as it is attracted equally in all directions by the molecules on all sides. But molecules in the surface of the liquid are not attracted by molecules on the side away from the liquid. On account of this unbalanced force the molecules at the surface are pulled toward the interior of the liquid with a force due to the attraction of the molecules lying below the molecules at the surface. The difference between the behavior of the molecules inside the liquid and those at the surface is seen by considering a sphere drawn around a molecule on the surface and around one in the interior of the liquid (Fig. 83). Let all the molecules which act on the molecule 0 at the center be included in this sphere. Now the forces on this molecule 0 are balanced and there is no tendency for the molecule to move. But the forces on the molecule at the center of the sphere *E* are not balanced since there are no molecules in the upper half of the sphere. This causes the molecule at the center to be pulled downward by the unbalanced attractions. This unbalanced force makes the liquid take the shape which has the least possible area under the conditions. Hence, a drop of mercury or water becomes spherical. If a drop of oil is placed in a mixture of water and alcohol of the same density, the drop of oil does not rise or fall, and under the action of the molecular forces it assumes a spherical form.

If the molecules of a liquid have less attraction for each other than for the molecules of the solid with which they are in contact, the liquid adheres to the solid and wets it. If, however, the molecules of the liquid cling to each other with a greater force than that with which they adhere to the solid, the solid is not wet by the liquid. Such is the case when mercury is in contact with glass, but when a drop of distilled water falls on a clean surface of glass it spreads over the glass in a thin layer. This is because the surface tension causes the water which adheres to the glass to extend as far as possible. If a hot body is brought near such a film of water the surface tension is lessened by the increase in temperature and the film of water pulls away leaving the glass dry. The surface tension of oil is small compared to that of water so that when a drop of oil is placed on a surface of water the oil spreads in a thin film over the surface of the water. For this reason it is difficult to keep a surface of water or mercury free from oil.

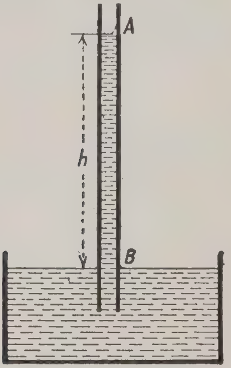


FIG. 84.—Rise of water in a capillary tube.

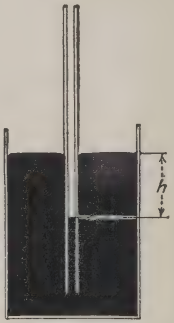


FIG. 85.—Depression of mercury in capillary tube.

130. Capillarity.—If a piece of glass tube of very small bore has been thoroughly cleaned and is then dipped into water (Fig. 84) the water will wet the inside of the tube and rise in it. If the liquid does not wet the tube (Fig. 85) as mercury in a glass tube, the liquid is depressed in the tube. In general, liquids rise in tubes which they wet and are depressed in tubes which they do not wet. The smaller the bore of the tube the greater the height to which the liquid rises or the greater the amount which it is depressed. This rise or depression of liquids in tubes of small bore is known as **capillarity.** It is caused by the molecular forces which are responsible for surface tension. The molecules of the liquid have an attraction for each other and also for the molecules on the surface of the wall of the tube, the former being a case of cohesion and the latter a case of adhesion. If the cohesive forces between the molecules of the liquid are greater than the adhesive forces between the molecules of the liquid and the molecules on the wall of the tube, the liquid is pulled away from the tube and depressed. Mercury in a glass tube is a case of this kind. If, however, the adhesive forces are the greater, the liquid wets the side of the capillary tube and rises in the tube. Water or alcohol in a glass tube rises in this manner.

131. Cause of the Variation in the Height of Rise.—The reason for the fact that liquids like water will rise higher in small tubes than in large ones is found in the relation between the volume of the tube and its internal circumference. When a glass tube is thrust in water, the molecules in the surface of the wall just above the water pull up on the row of molecules lying nearest them and raise them above the level of the water in the vessel. This carries up the whole column of water until the weight of the column is equal to the force of attraction between the molecules of glass and the molecules of water. As each molecule of glass exerts a fixed attraction on each molecule of water near it, a tube of larger circumference will be able to lift more than a tube of small circumference. Now the circumference of the tube increases as the diameter so that a tube which is 0.1 mm. in diameter will lift 1/10 as much as a tube which is 1 mm. in diameter. The volume and hence the weight of the water lifted increases as the square of the diameter. The larger tube will therefore lift its water only 1/10 as high as the smaller tube.

Let a tube of radius r be inserted in a liquid of density d (Fig. 84). Let the mean elevation of the liquid be h. Assume that the angle which the surface of the liquid makes with the wall of the tube is very small so that all the force of attraction is directed upward. The force due to the surface tension T must be balanced by the weight of the column of liquid of height h.

The length of the film around the tube is $2\pi r$; the force upward is

$$2\pi r T.$$

The weight of the liquid supported by this force is

$$\pi r^2 h d.$$

Hence for equilibrium we have

$$\pi r^2 d h = 2\pi r T$$

$$h = \frac{2\pi r T}{\pi r^2 d} = \frac{2T}{rd}$$

From this equation it is seen that the smaller the bore of the tube the greater the height to which the liquid rises. Fig. 86 shows the difference of level of liquids in tubes of unequal diameters.

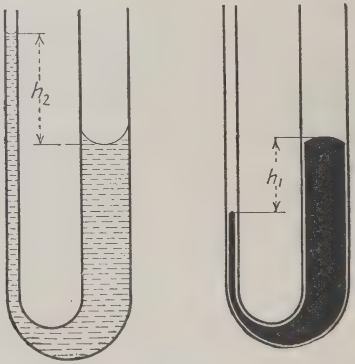


FIG. 86.—Unequal heights of liquids in tubes of unequal areas.

132. Illustrations of Capillarity.—There are many familiar illustrations of capillary action in nature. The oil in a lamp rises in the wick by capillary action. Ink spreads in a blotter or water in a lump of sugar by this same action. If one end of a towel dips in a bucket of water and the other end hangs over the bucket, the towel soon becomes wet throughout and drains the water out of the bucket. If the towel is covered with a preparation which prevents the water from adhering to its fibers the towel does not become wet and the action does not take place. In this case a surface film spreads over the cloth but does not enter it. By this means the cloth has been rendered waterproof. Cravenette cloth has been treated in this way.

The method of making cloth waterproof may be understood from the following experiment. Take a sieve made out of fine copper gauze which has been dipped into melted paraffin. By this means each wire has been covered with paraffin and water will not adhere to it. Lay a piece of paper carefully on the bottom of the sieve and fill the sieve with water. When the paper is carefully removed the surface film of water will prevent the water from flowing through the sieve.

Example.—The liquid in a capillary tube rose to the height of 7 cm. The diameter of the tube was 0.1 mm. and the density of the liquid 0.8. If the angle between the liquid and the surface of the tube is zero, find the surface tension of the liquid.

$$\text{Surface tension} = \frac{\text{height} \times \text{density} \times \text{radius of tube}}{2}$$

$$T = \frac{hdr}{2} = \frac{7 \times 0.8 \times 0.01}{2} = 0.028 \text{ gm. per cm.}$$

Example.—In two capillary tubes of different diameters the liquid rose in one case to a height of 10 cm. and in the other case to a height of 20 cm. Find the ratio of the diameters of the tubes.

$$T = \frac{hdr}{2}$$

$$T = \frac{h'dr'}{2}$$

$$hr = h'r'$$

$$h/h' = r'/r$$

$$\frac{r'}{r} = \frac{10}{20} = \frac{1}{2}$$

Hence, the tube in which the liquid rises the highest is one-half the radius of the other tube.

133. Capillary Action in Soils.—The distribution of water in soils depends largely on capillary action. The spaces left between the soil grains form imperfect triangular capillary tubes. The cross section of such a tube formed by four spherical grains is seen in Fig. 87. These tubes extend in all directions through the soil and their effective diameters are roughly proportional to the diameter of the grains so that the effective diameter of these tubes is small in finely divided soils and large in coarse soils. Through these spaces the water rises as in capillary tubes thus supplying water from the lower soil which is damp to the upper soil which is drier. If the soil is made too loose, the effective diameter of these tubes become too large and the water is prevented from rising any considerable height from the lower soil toward the upper soil. Hence constant cultivation keeps the soil loose and therefore lessens evaporation by preventing the conduction of water from the lower layers to the surface.

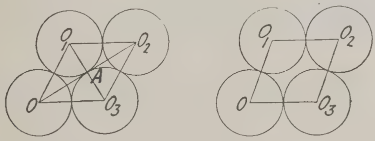


Fig. 87.—Capillary tubes between grains of soils.

To get a measure of the power of this capillary action to lift the water in the soil, take a glass funnel (Fig. 88) and join it by means of a piece of rubber tubing to a thick walled capillary tube which is about 90 cm. in length. The lower end of the tube is now placed in a vessel filled with mercury. The capillary tube is filled with water which has been previously boiled to expel the air which it contains under ordinary conditions. Over the top of the funnel is placed a piece of cotton cloth and the funnel is then filled with finely divided clay soil. On account of capillary action the water will rise through the soil to replace that which evaporates from the surface. As the water is lifted up through the soil by capillary action and allowed to evaporate, it will be replaced by mercury in the capillary tube. The height to which the mercury rises in the tube gives a measure of the lifting power of the capillary action in the soil.

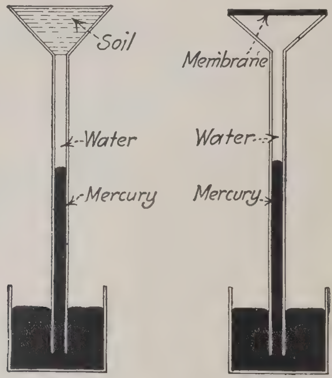


FIG. 88.—Rise of water by evaporation from a soil.

CHAPTER XI

PRESSURE OF LIQUIDS

134. Characteristics of Liquids.—The molecules of a liquid at rest are displaced by the slightest force and for this reason a liquid has no shape of its own but takes the shape of the containing vessel. Hence liquids yield to a continued application of force tending to deform them or to change their shape in any way. They, however, manifest wide differences in their readiness to yield to distorting forces. Water, alcohol and ether are very mobile liquids which yield readily to forces tending to change their shape. Glycerine is less mobile and tar is still less so. There is no sharp line of separation between liquids and solids. In warm weather paraffin candles yield under their own weight and bend double. Although shoemaker's wax will break readily when cold, it behaves like a very viscous liquid at higher temperatures. All liquids offer large resistance to forces tending to change their volume. For example it requires a pressure of 1,500 lb. per sq. in. to cause the volume of water to change one-half of one per cent.

135. Pressure in a Liquid under Action of Gravity.—If an empty can is pushed down into the water, the force of the liquid against the can tends to push it upward. When a block of wood is forced under the water, a pressure resists the action of the force tending to submerge the body and the body when released returns to the surface. The fact that large ships float on the surface of the water shows that there is a force on their lower surface sufficient to overcome their weight.

Consider a cylindrical vessel partly filled with liquid of density d. Let R be the radius of the vessel and h be the depth of the liquid in the vessel. The volume of the vessel is,

$$v = \pi R^2 \times h$$

and the weight of the liquid is, $W = vd = \pi^2 Rhd$.

This weight is sustained by the bottom of the vessel and will exert on each unit area of the bottom a force p such that,

$$p = \frac{\text{weight}}{\text{area of base}} = \frac{\pi R^2 hd}{\pi R^2} = hd$$

This force per unit area is called the **pressure.** It is proportional to the depth of the liquid and to its density.

Example.—Find the pressure per unit area on the bottom of a tank which is 10 ft. in diameter and 25 ft. deep, if the tank is filled with water which weighs 62.5 lb. per cu. ft.

Total pressure = height × area × weight per cu. ft.

$$= 25 \times \pi \times 5^2 \times 62.5 = 122{,}750 \text{ lb.}$$

$$\text{Pressure per unit area} = \frac{\text{total pressure}}{\text{area}} = \frac{122{,}750}{\pi \times 5^2} = 1{,}562 \text{ lbs. per sq. ft.}$$

136. Pressure in Vessels of Different Shape.—Where a vessel has vertical sides, the pressure on the bottom is equal to the height of the liquid times its density. If instead of considering vessels with vertical sides we consider vessels in which the sides flare out (Fig. 89), one might expect the pressure on each square centimeter of the bottom in case B to be greater than in case A because there is more water in B. The pressure on each square centimeter of the bottom is the same since each square centimeter of the bottom holds up only the column of water above it. The extra water above the slanting sides is held up by these sides and does not press on the bottom. If the area of the base is the same in case A and case B, the total downward force on the base in the two cases is the same.

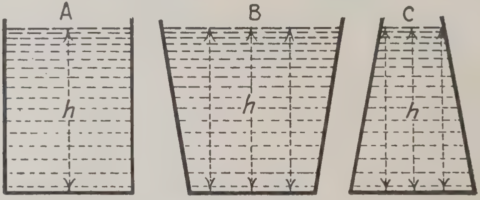


FIG. 89.—Pressure independent of shape of vessel.

When the vessel is conical as in case C the total force on the base is the same as in the preceding cases. It is easy to see that in this case the pressure on the base directly under the top is the same as in the other cases. The slanting walls press down on the liquid standing on the remainder of the base with a force which, when added to the weight of the liquid on that part of the base, makes the pressure on each square centimeter of the base equal to the pressure on each square centimeter directly under the top.

137. Liquid in Communicating Vessels.—It is a matter of common experience that liquids seek their own level in communicating vessels. For this reason water stands at the same level in the spout of a tea kettle and in the tea kettle itself. If tubes of various sizes are connected, liquid poured into one of these tubes (Fig. 90) will come to the same level in all the tubes. This result is to be expected from the fact that the pressure in a liquid depends on the depth below the free surface. If points in the interior of the liquid are at the same level, the pressure at these points must be the same or the liquid would flow from one point to the other until the pressure was equalized.

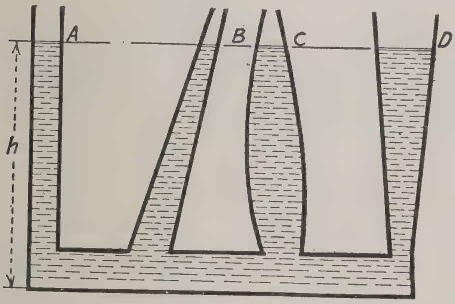


FIG. 90.—Level of liquids in communicating vessels.

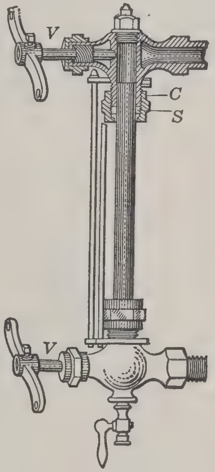


FIG. 91.—The water gage.

The water gauge, in a steam boiler (Fig. 91), is an illustration of this principle. This gauge which consists of a heavy walled glass tube is connected at the top with the steam space and at the bottom with the space filled with water in the boiler. The water in this glass tube stands at the same height as the water in the boiler and thus serves to indicate the height of the water in the boiler.

Artesian or flowing wells illustrate on a large scale the tendency of water to seek its own level, whatever the form or shape of the connecting vessel. The earth's surface in many places is made up of different strata. Some of these are porous to water and others are not porous. If a porous stratum happens to lie between two strata which are not porous, and the strata are then folded, the conditions are favorable for the formation of an artesian well. Rain sinks by gravity into the porous stratum. Since this stratum has an impervious stratum above and below it, the water fills the porous stratum until some opportunity of escape is provided. When a well is sunk into the porous stratum, the water from the well rises until it reaches the level of the water in the stratum. Wherever the well is drilled, the water will rise to the same level.

138. Pressure Transmitted by Liquid.—Since liquids can not resist forces tending to shear one layer of molecules over another, they have the power of transmitting pressure from one part of the liquid to all other parts. This power of a liquid to transmit pressure is illustrated in Fig. 92. A bottle is filled with water and closed with a rubber stopper to which a force is applied. This force is transmitted in all directions and breaks the bottle.

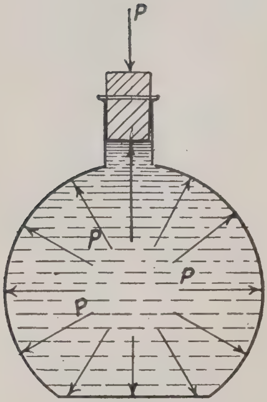


FIG. 92.—Pressure transmitted equally in all directions.

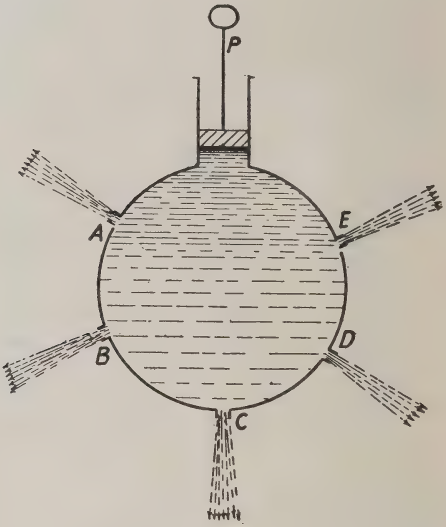


FIG. 93.—Pressure uniform at all points on the surface of a sphere.

Liquids press against surfaces which are vertical or oblique as well as against those which are horizontal. The force is always found to be perpendicular to the surface against which it acts. Otherwise, owing to the mobility of the fluid, the component of the pressure along the surface would produce a flow of the liquid tangent to the surface. Liquids will be forced through a hole in the side of a pail near the bottom as well as through a hole in the bottom. The application of pressure to the piston (Fig. 93) forces water from all the holes equally far in all directions. A comparison of the forces in different directions at any point in a liquid shows that when a liquid is at rest, the pressure at a point is the same in all directions. Where a liquid is at rest and is acted on only by gravity, the pressure is the same at all points on the same level.

139. Pascal's Principle.—Suppose there is a vessel filled with water (Fig. 94) and fitted with two equal pistons *A* and *B*, and suppose that the pistons carry equal weight so that there is no tendency for either of them to move. Now let a weight of one pound be added to *A*. This will cause a pressure to be transmitted throughout the liquid, and it will be found that a weight of 1 lb. must be added to *B* in order to keep it from moving. This is a special case of a general principle discovered by Pascal and known as Pascal's Principle. **This principle states that if pressure is applied to an enclosed gas or liquid it is transmitted without diminution in all directions.**

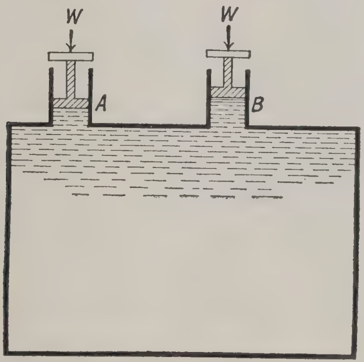


FIG. 94.—Pressure transmitted in all directions without diminution.

The expansion of a rubber balloon when it is being filled with air illustrates this principle. As the air is being forced into the balloon an equal pressure is exerted in all directions so that the balloon expands equally in all directions and becomes a sphere.

140. Multiplication of Force by Transmission of Pressure.—Consider two cylinders which are connected together and filled with water (Fig. 95). Assume that each cylinder is fitted with a piston which moves without friction. Let *A* be the area of the larger piston and *a* the area of the smaller piston. When a weight of *w* pounds is placed on the smaller piston, it produces a pressure of w/a pounds to the square inch on the water under this piston. This pressure is transmitted undiminished throughout the water and acts on the larger piston. The upward pressure on the larger piston is then w/a pounds per square inch. Since the area of the larger piston is A the total force exerted on this piston by the water is $\frac{w}{a}A$. Hence by the application of a force of w pounds it has been possible to produce a force A/a times as large.

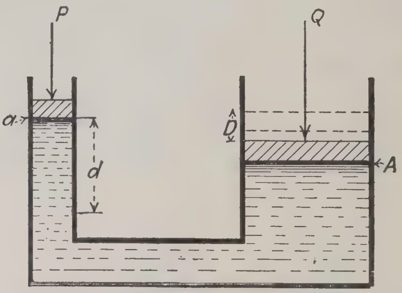


FIG. 95.—Multiplication of force by transmitted pressure.

Let the smaller piston be displaced a distance d and let D be the corresponding displacement of the larger piston.

$A \times D =$ volume swept out by larger piston,
$a \times d =$ volume swept out by smaller piston.

Since the liquid can be assumed incompressible,

$$A \times D = a \times d$$

The work done on the smaller piston is,

$$\text{Work} = \text{force} \times \text{distance} = w/a \times a \times d = w \times d$$

Work done by the larger piston is,

$$\text{Work} = w/a \times A \times D = w \times d.$$

Hence work done on smaller piston = work done by larger piston.

Example.—The larger piston in Fig. 95 has an area of 100 sq. in. and the smaller an area of 5 sq. in. A force of 250 lb. is applied to the smaller piston. Find the force exerted by the larger piston.

Pressure per sq. in. on smaller piston = 250/5 = 50 lb. per sq. in. and the pressure per sq. in. on the larger piston is also 50 lb.

Total force on the larger piston = pressure per sq. in. × area = 50 × 100 = 5,000 lb.

141. The Hydrostatic Bellows.—This bellows consists of two disks of wood connected by water proof canvas in such a way as to form a collapsible drum (Fig. 96). A small pipe passing through the lower disk opens into the drum and is then turned so that it is vertical when the disks are horizontal. If now the drum is filled with water and a man stands on the upper disk water is forced into the vertical tube. The striking thing is that the height of the water is the same whatever the cross section of the tube. Hence a small weight of water balances the weight on the disk.

The explanation of this fact is found in Pascal's principle. If the cross section of the tube is one square inch and the area of the upper disk is 500 sq. in. then the weight on the disk will be 500 times as large as the weight of the water in the tube.

142. Hydraulic Elevator.—Pascal's principle also explains the action of hydraulic elevators, which make use of hydraulic pressures for lifting heavy loads. At the bottom of the elevator (Fig. 97) is a well or pit. In this well is set a long hollow cylinder in which there moves a long plunger or piston which is firmly fastened at the top to the elevator cage. The admission of water under pressure acts against the lower end of the piston and lifts it and with it the elevator and its load. The weight of the elevator is partly counterbalanced by hanging a heavy weight over the pulley. The checking of the flow of water into the cylinder stops the elevator and by allowing the water to flow out of the cylinder the elevator descends by its own weight. In order to make the elevator move more rapidly than the plunger and thus get more speed the motion of the plunger is communicated to the elevator through a set of pulleys so that the cage of the elevator may be made to move several times as fast as the plunger.

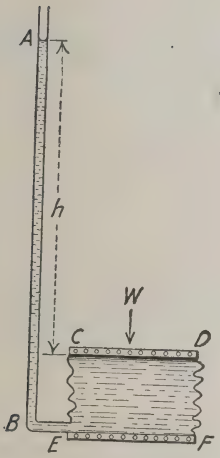


Fig. 96.—Hydrostatic bellows.

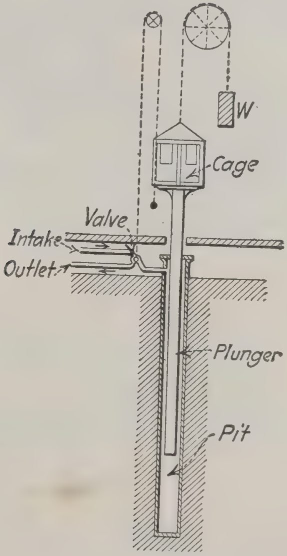


Fig. 97.—Hydraulic elevator.

Example.—The pressure in a water main is 50 lb. per sq. in. and the diameter of the plunger of a hydraulic elevator is 10 in. How heavy a load can the elevator lift?

Total force on the elevator = pressure × area of plunger

$$= 50 \times \pi \times 5^2 = 50 \times 78.6 = 3{,}930 \text{ lb.}$$

143. Water Motors.—By employing rotary water motors or water wheels, the pressure of water may be used to run various kinds of machinery. Common types of water motors are shown in Fig. 98. The stream of water issues with great velocity in the form of a jet and strikes against the blades or buckets on the rim of the wheel. After impact with these blades or buckets, the water which has lost most of its velocity, runs away to the drain or sewer. The impact of the water against the blades or buckets causes the wheel to rotate. By connecting the shaft of the water motor to the machinery, the desired power can be developed. Fig. 99 shows a Pelton water wheel.

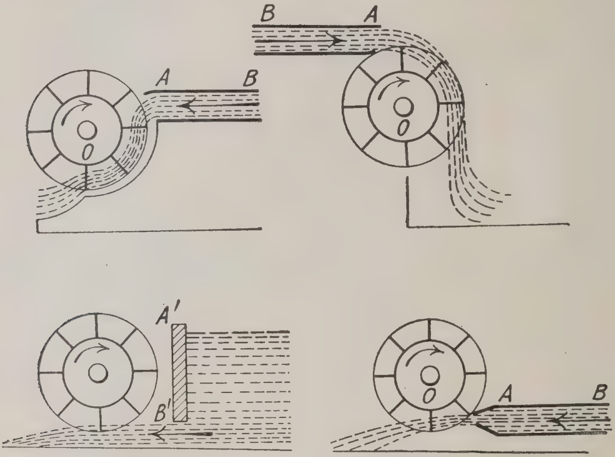


FIG. 98.—Types of water wheels.

144. Hydraulic Ram.—When there is present an abundant supply of water which has a fall of only a few feet, it is possible by means of a hydraulic ram to elevate some of this water to a much greater height than that of the source from which the water is obtained. The hydraulic ram operates on the principle that by allowing a large amount of water to fall through a small distance it is possible to raise a small amount of water through a much greater distance. The essential parts of a hydraulic ram are shown in Fig. 100. Through the pipe *DE* the water flows from the spring into the reservoir and then out at the waste valve *B* which is normally open. The water flowing through the pipe *DE* and out at the waste valve *B* soon attains considerable velocity. On account of the momentum which it thus possesses it pushes up with sufficient force to close the waste valve *B*.

When the escape of the water is thus checked at *B*, the force of the water lifts the check valve *A* and enters the chamber *C*. As the water continues to flow into the chamber *C* the air in this chamber is compressed until the pressure of the air and the weight of the water are sufficient to stop a further flow of water into this chamber and force the water from *C* through the pipe *FG*. At this time the pressure below the waste valve *B* is sufficiently reduced to allow the valve to open and the flow of water through the waste valve to begin again. The water continues to flow with increasing velocity until it exerts sufficient force to close the waste valve again. The action is then repeated as before. The operation is automatic with an intermittent flow at the delivery pipe.

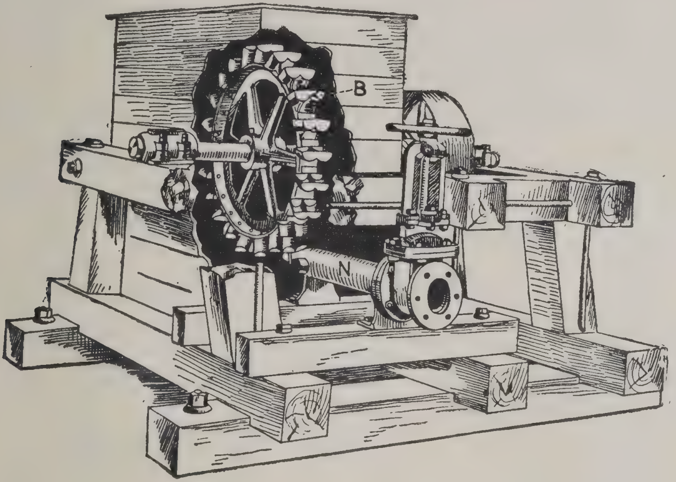


Fig. 99.—Pelton water wheel.

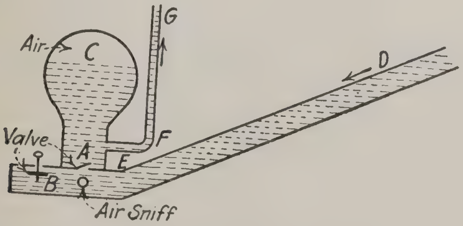


Fig. 100.—Hydraulic ram.

145. Hydraulic Crane.—An hydraulic crane (Fig. 101) consists essentially of a ram and cylinder each carrying a set of pulleys. A rope passing continuously over these pulleys as in an ordinary block and tackle is fastened at one end to the load to be lifted. The other end is fastened to one of the sets of pulleys. When water is forced into the cylinder, the pressure on the ram forces the two sets of pulleys apart and lifts the load at the free end of the rope. The mechanical advantage of the crane is determined by the number of pulleys and the diameter of the ram.

146. Hydraulic Press.—The hydraulic press consists of a strong cylinder (Fig. 102) in which works a cylindrical piston *C*. By means of a small pump *D* oil is forced into the large cylinder through a check valve *K* which prevents its return. In consequence of Pascal's principle, whatever pressure is communicated to the liquid by the pump is transmitted undiminished to the walls of the containing cylinder and the piston *C*. If the large piston *C* has 100 times the area of the small piston *D*, the force exerted on *C* will be 100 times that applied to *D* and on the downward stroke of the small piston the large piston *C* will be moved only 1/100 the distance through which the small piston moved. If the oil is incompressible, the work done on the large piston is just equal to that done on the small piston. That is, the input of the machine is just equal to the output. In order to still further increase the pressure exerted on the piston *C*, the small piston is ordinarily forced down by means of a lever.

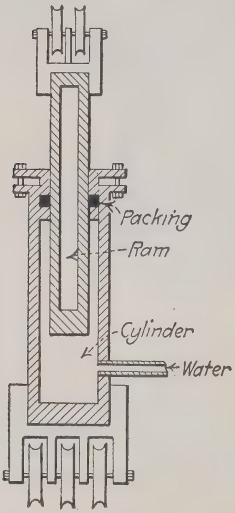


Fig. 101.—Hydraulic crane.

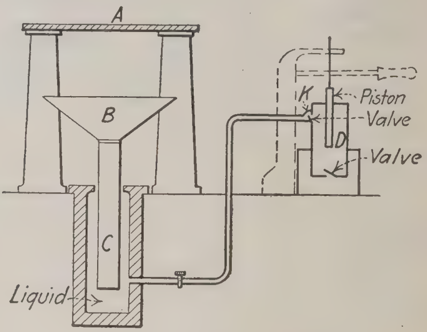


Fig. 102.—Hydraulic press.

Hydraulic presses are used in baling paper, cotton, etc., in punching holes through steel plates, and in extracting oil from seeds. By means of them a small force operating through a large distance produces a large force operating through a small distance.

Problems

1. A hole in the vertical side of a tank containing water is 2 ft. in diameter and is closed by a flap. The center of the hole is 10 ft. below the surface of the water. Find the pressure necessary to hold the flap in position.

2. A diver goes down into sea water (density 1.03 g. per cubic centimeter) to a depth of 15 m. What is the pressure which he experiences in excess of atmospheric pressure? Give the result in kilograms per square centimeter.

3. A dam, Fig. 103, is 50 ft. long and 2 ft. high. If the water just reaches the top, what is the total pressure against the dam?

4. A rectangular tank 4 ft. long, 2 ft. broad and 2 ft. deep, is full of water. If water weighs 62.5 lb. to the cubic foot, find the total force on the bottom, on one side and on one end.

5. In a simple hydraulic elevator the ram is 4 in. in diameter and has a stroke of 15 ft. The water is supplied from the mains at a pressure of 600 lb. per square inch. Neglecting friction, what total load can be raised and how much work is done?

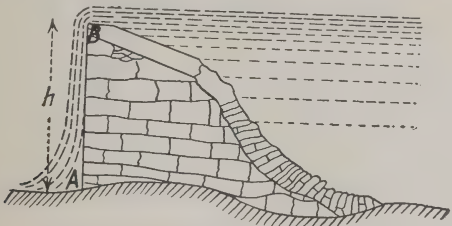


Fig. 103.—Pressure against a dam.

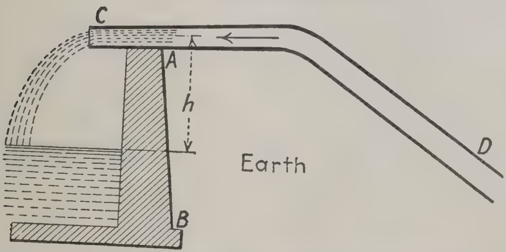


Fig. 104.—Loss of power by elevating delivery pipe.

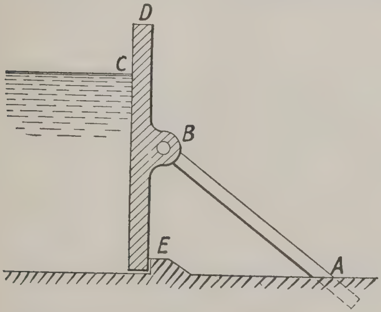


Fig. 105.—Automatic flood gate.

6. Find the power lost in pumping water (Fig. 104) over an embankment which is 8 ft. high, if the pump delivers 200 cu. ft. per minute.

7. A lock gate is 20 ft. wide and the depth of water on the two sides is 24 ft. and 10 ft. respectively. Find the total pressure on the gate and the point of its application.

8. An hydraulic crane has a ram 10 in. in diameter and the speed of its lift is 15 times the speed of the ram. If the crane has an efficiency of 60 per cent., what load can be lifted by a water pressure of 1,800 lb. per sq. in.?

CHAPTER XII

ARCHIMEDES' PRINCIPLE

147. Buoyancy of Liquids.—It is a matter of common experience that bodies are lighter in water than they are in air. A fresh egg will sink in pure water but will float in water to which has been added a considerable quantity of salt. This fact offers a convenient method of testing the concentration of a salt solution. Sufficient salt is added to just float the egg. By adding more salt the lifting effect of the solution is increased. A piece of iron sinks in water but floats in mercury. This is because the density of the mercury is greater than that of the iron. When a diver lifts a stone under water and then brings it to the surface, he finds that the stone is much heavier when it is brought above the surface. The diver also observes when he is in the water up to his neck that he is lifted up by the water so that it is difficult for him to retain his footing. In case of lighter bodies like wood or cork this lifting effect may be sufficient to keep parts of the body above water. This resultant upward pressure of a liquid on a wholly or partly immersed body is called **buoyancy.** It is a force acting vertically upward and counterbalancing in whole or in part the weight of the body.

That point through which the force of buoyancy acts is the **center of buoyancy.** This point lies at the center of gravity of the displaced liquid. The buoyant forces of all the displaced liquid might be replaced by a single force acting through the center of buoyancy without altering the behavior of the body.

148. Archimedes' Principle.—Suspend from one arm of a balance (Fig. 106) a hollow cylindrical cup and a piece of brass which has been nicely turned in the form of a cylinder so that it will just fit the cavity inside the cup. Now counterbalance the weight of the cup and cylinder by adding the necessary weights to the other pan of the balance. When a vessel of water is brought up in such a way that the cylinder *C* is completely immersed, it is observed that the side of the balance carrying the cylinder rises, showing that the water is pushing up on the cylinder. If water is now poured into the cup until it is just filled, the equilibrium of the balance is restored. Since the weight of a volume of water equal to that displaced by the cylinder is sufficient to compensate for the lifting effect of the water on the cylinder, it is evident that the cylinder is lifted up by a force equal to the weight of the displaced water. If the experiment is repeated using kerosene or some other liquids instead of water, the same result is always found to be true. The loss in the weight of the immersed body is equal to the weight of the volume of liquid displaced by it. This principle which is known as Archimedes' principle may be stated as follows: **The loss of weight of a body immersed in a liquid is equal to the weight of the displaced liquid, or a body immersed in a liquid is buoyed up by a force equal to the weight of the liquid displaced by it.**

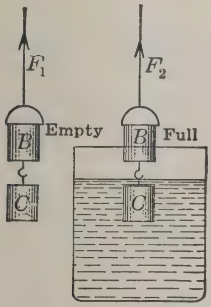


FIG. 106.—Test of Archimedes' principle.

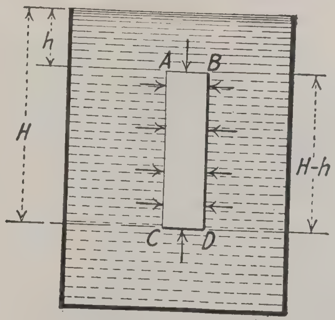


FIG. 107.—Proof of Archimedes' principle.

149. Explanation of Archimedes' Principle.—If a rectangular block (Fig. 107) is immersed in a vessel of liquid the pressures on the vertical sides are equal and in opposite directions. These forces will not therefore tend to move the block in the liquid. Upon the upper face of the block there is a downward pressure equal to the weight of the column of liquid having this face as a base and having a height h. On the lower face there is an upward pressure which is equal to the weight of a column of liquid which has an area equal to the area of the lower base and a height H equal to the depth of this face below the surface of the liquid. The upward pressure exceeds the downward pressure by the weight of a column of liquid having a base equal to the area of the cross section of the block and a height equal to the height of the block. Now the volume of this column is just equal to the volume of the liquid displaced by the immersed block, and the weight of this column is equal to the weight of the displaced liquid. The same sort of reasoning will hold for a body of any shape in any liquid. Hence a body immersed in a liquid is lighter by the weight of the volume of liquid which it has displaced.

Example.—A piece of iron weighs 76 grams in air and 68 grams in water. What volume of water is displaced? What is the density of the iron?

The mass of water displaced = loss of weight of iron in water.
= 78 − 68 = 10 grams.

Volume of water displaced = volume of iron = 10 cu. cm.

$$\text{Density of iron} = \frac{\text{weight of iron}}{\text{volume}} = \frac{78}{10} = 7.8 \text{ g. per c.c.}$$

150. Archimedes' Principle Applied to Fishes.—Fishes are capable of moving toward the surface or into deep water by regulating the quantity of water which they displace and therefore the force which urges them upward. By a distension of the air bags in their bodies they can change their volumes and thus change the buoyancy of the water on them. This gives an upward force which causes them to rise. By a contraction of these air sacks the volume of the body is diminished and the fishes sink. When a fish in a vessel of water is placed under the bell jar of an air pump and the air removed from the bell jar, these air sacks are ruptured and the fish sinks to the bottom no longer able to float.

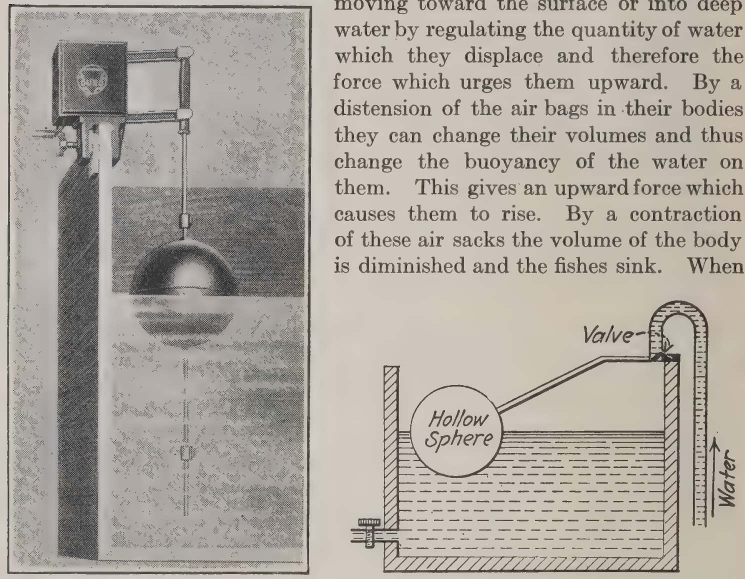


Fig. 108.—Float valve and automatic control for water supply.

151. The Float Valve.—In many cases it is desirable to have an automatic cut off for the water which flows into a tank in order to regulate the supply of water. The flow of water from a pump operated by a windmill is often controlled in this way. In such cases (Fig. 108) a float is provided to operate the cut-off valve when the tank is sufficiently full. This float is a light hollow brass or copper sphere which has less weight than an equal volume of water. This sphere is buoyed up by the water in the tank according to the principle of Archimedes. As the water rises in the tank the float is buoyed with greater and greater force until the valve to which it is attached by means of a lever is closed. In the case of the windmill the buoyant force on the float is also made to stop the windmill.

152. Floating Ship or Boat.—Since according to the principle of Archimedes every floating body displaces a weight of liquid equal to its own weight, it is possible to calculate how much water will be displaced by a ship or boat when it is afloat. It is evident that the ship will sink until the lifting force of the water is sufficient to just balance the weight of the ship. For this reason the ship will sink deeper when loaded than when empty. Since salt water is somewhat heavier than fresh water the boat will sink further in fresh water than in salt water. A modern submarine can sink below the surface of the water by letting more water into certain tanks and thus making the boat heavier than it was when riding on the surface. It rises from below the surface by again pumping this water out of the tanks, thus making the boat lighter than the water which it displaces.

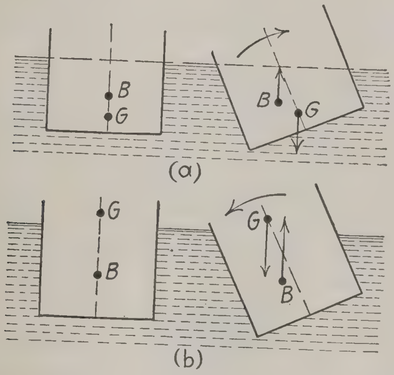


FIG. 109.—Center of buoyancy and stability of floating bodies.

Example.—A barge is 30 ft. long and 16 ft. wide and has vertical sides. When a team of horses is driven on board it sinks 2 in. further into the water. How much do the horses weigh?

Volume of displaced water = 30 × 16 × 1/6 = 80 cu. ft.

Weight of water displaced = 80 × 62.5 = 5,000 lb.

Weight of horses = weight of displaced water = 5,000 lb.

153. Stability of Floating Bodies.—The stability of a floating body depends on the position of its center of gravity with respect to its center of buoyancy. When the center of gravity lies below the center of buoyancy (case *a*, Fig. 109) and the body is slightly displaced from its position of equilibrium, there are two forces acting on the body, and these forces act in such a direction that they tend to restore the body to its original position. If the center of gravity lies above the center of buoyancy and the body is slightly displaced, the two forces acting on the body are in such a direction that they cause the body to move further away from its original position. Where the center of gravity and the center of buoyancy coincide, a displacement of the body does not call forth any forces tending to move the body toward or away from its original position. The body therefore remains in any position in which it happens to be. When the hold of a ship is loaded, the center of gravity of the ship is lowered and its stability increased. If, however, the load is on the upper deck, the center of gravity is raised and the tendency of the ship to upset is increased.

154. Density and Specific Gravity.—In order to determine the density of a substance it is necessary to determine its weight and its volume. The density is then found by dividing the weight by the volume. The weight of the substance is easily determined by weighing, but it is sometimes difficult to find the volume, especially when the body has an irregular shape. In such cases the volume may be determined by an application of Archimedes' principle. Since the body displaces a volume of water equal to its own volume and since each cubic centimeter of water weighs 1 gram, the loss of weight in water is numerically equal to the volume of the immersed body.

The measure of the density of a body depends on the units in which the mass and the volume are measured. In the C.G.S. system the density is the number of grams per cu. cm. In the British system it is the number of pounds per cu. ft.

The specific gravity of a body is the ratio of its density to the density of water at 4°C. Since in the C.G.S. system a gram is defined to be the weight of a cu. cm. of water at 4°C, the numerical value of the density and the specific gravity in this system is the same. In the British system, however, they are very different.

155. Density of Solids Heavier than Water.—When a body is heavier than an equal volume of water and is insoluble in water, its volume can be determined by finding its loss in weight when weighed in water. This loss of weight is equal to the weight of the water displaced, and if this loss of weight is expressed in grams, it is numerically equal to the volume of the body in cubic centimeters. By dividing the mass of the body by this volume, the density or specific gravity is obtained.

156. Density of Solids Lighter than Water.—If the body is lighter than water but insoluble, its volume may still be determined by this method by fastening to the body a sinker large enough to force it below the surface of the water. In this case (Fig. 110) the combined weight of the body and the sinker is first determined when the sinker is immersed in water and the body above the surface of the water. The body is then also submerged and the combined weight redetermined. The change in weight is due to the buoyant force of the water on the body and equal to the weight of the water displaced by the body. It therefore gives the volume of the body in cu. cm. The density is then determined as in the preceding case.

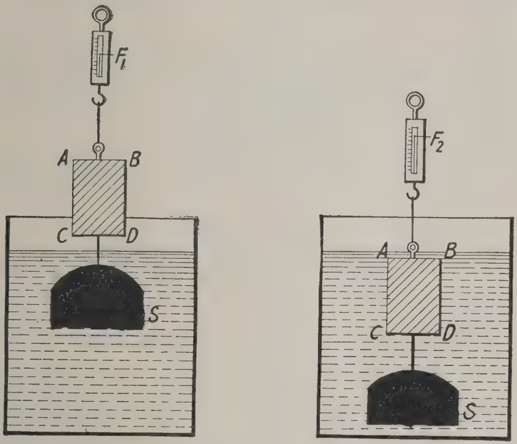


FIG. 110.—Density of floating bodies.

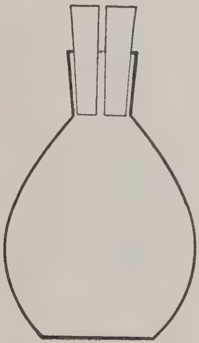


FIG. 111.—Specific gravity bottle.

Example.—A piece of cork weighs 50 g. in air. When it is fastened to a sinker and the sinker alone immersed in water, the combined weight of sinker and cork is 200 g. When both sinker and cork are immersed they weigh 75 g. What is the density of the cork?

Loss of weight due to submerging the body = 200 − 75 = 125 g.

Volume of water displaced by body = 125 cu. cm.

$$\text{Specific gravity} = \frac{\text{weight of body}}{\text{volume}} = \frac{50}{125} = 0.4$$

157. Specific Gravity Bottle.—A specific gravity bottle (Fig. 111) is often used to determine the density of liquids. It consists of a glass bottle of known volume. This bottle is first weighed empty, then full of water and then full of the liquid the density of which is desired. The difference between the weight of the bottle empty and its weight filled with water gives the weight of the water in the bottle. Likewise, the difference between the weight of the bottle empty and its weight filled with the liquid gives the weight of the liquid in the bottle. The specific gravity is then found by dividing the weight of the liquid to fill the bottle by the weight of water to fill it.

$$\text{Specific gravity} = \frac{\text{weight of liquid}}{\text{weight of equal vol. of water.}}$$

Example.—A specific gravity bottle when empty weighs 240.30 g. When filled with water it weighs 390.30 g.; and when filled with alcohol it weighs 360.30 g. Find the specific gravity of alcohol.

Weight of water = 390.30 – 240.30 = 150.00 g.

Weight of equal vol. of alcohol = 360.30 – 240.30 = 120.00 g.

$$\text{Specific gravity} = \frac{\text{weight of alcohol}}{\text{weight of equal vol. of water}}$$
$$= \frac{120.00}{150.00} = .8 \text{ g per cu. cm.}$$

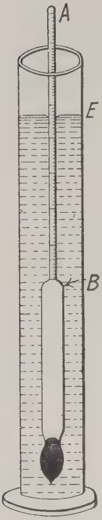


Fig. 112.—Hydrometer.

158. Hydrometer.—Another application of the principle of Archimedes is found in the hydrometer, which is extensively used for determining the density of liquids. It is usually made of a cylindrical glass tube (Fig. 112) which is provided with a narrow graduated glass stem. At the lower end of the hydrometer is placed a sufficient amount of mercury or shot to make the hydrometer sink to the desired level in the liquid. The depth to which the hydrometer sinks is determined by the density of the liquid, for the hydrometer sinks into the liquid until the buoyancy of the liquid is sufficient to overcome the pull of gravity on the hydrometer. The hydrometer then floats freely in the liquid, and the point at which the surface of the liquid touches the stem can be read and the density found directly from the graduated stem. Since the weight of the hydrometer remains the same, it will sink further in light liquids like alcohol or kerosene than in heavier liquids like brine. For this reason the larger numbers are near the bottom of the scale and the smaller numbers near the top. Such hydrometers are now commonly used in finding the density of the acid in storage batteries (Fig. 113).

159. Lactometer.—A form of hydrometer is much used by dairymen in testing milk. For this reason it has taken the name of lactometer. Except for the way in which the scale is graduated it does not differ from the hydrometer which has been just described. The specific gravity of cow's milk usually lies between 1.027 and 1.035. In order to determine the density of the milk it is then only necessary to know these last two figures. It is sufficient to make the scale of the lactometer run from 20 to 40 which really means from 1.020 to 1.040. It is seen that the specific gravity of milk is greater than that of water. This is because the milk contains such substances as albumen, sugar and various salts besides water. It, however, contains other substances like butter fat which are lighter than water. The specific gravity alone does not give very definite information concerning the composition of the milk.

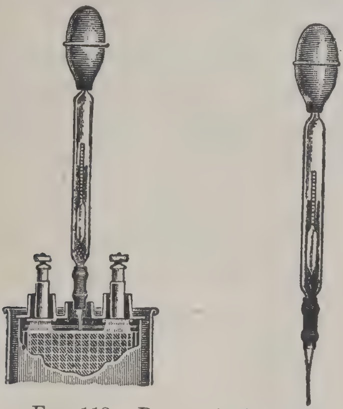


Fig. 113.—Battery hydrometer.

160. Liquids in Communicating Tubes.—Let two liquids which do not react chemically be placed in a bent tube (Fig. 114). When the liquids are at rest, the less dense liquid stands at a height h_1, above the junction of the two liquids. The pressure exerted by this column of lignter liquid is just balanced by the weight of the column of heavier liquid which stands above the junction of the liquids. Let d_1 be the density of the lighter liquid; d_2 the density of the heavier liquid; h_1 the height of the lighter liquid above the junction and h_2 the height of the heavier liquid. Then

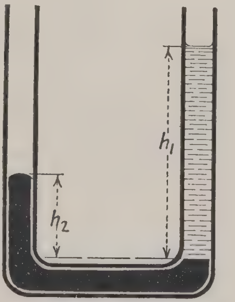


Fig. 114.—Density of non-miscible liquids by balanced columns.

$$h_1 \times d_1 = h_2 \times d_2$$
$$h_1/h_2 = d_2/d_1$$

Hence, the heights of the two liquids above their surface of separation are inversely proportional to the densities of the liquids.

In case the liquids react chemically, the bent tube is inverted and the ends placed in cups containing the liquids (Fig. 115) whose densities will be denoted by d_1 and d_2. The air from the upper part of the bent tube is partially removed and the stop cock closed. The pressure above both liquids inside the tube is the same, and the atmospheric pressure on the liquids in the open vessels is the same. The difference between the pressure inside the tube and the atmospheric pressure is in each case balanced by the rise of the liquid in the tube. These two differences in pressure is the same and we have,

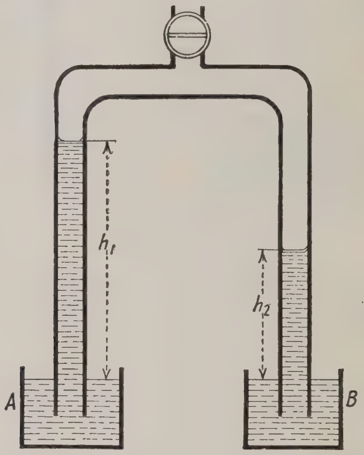


FIG. 115.—Density of miscible liquids by balanced columns.

$$h_1 \times d_1 = h_2 \times d_2$$
$$h_1/h_2 = d_2/d_1$$

Example.—If one of the beakers in Fig. 115 contains sulphuric acid and the other contains water, and if the height of the column of water is 40 cm. when the height of the column of acid is 30 cm., find the density of the sulphuric acid.

$$\frac{\text{Density of acid}}{\text{Density of water}} = \frac{\text{height of water}}{\text{height of acid}}$$
$$d_2/d_1 = h_1/h_2 = 40/30$$
$$d_2 = 1.33 \text{ g. per cu. cm. density of acid.}$$

Problems

1. What is the density of a body whose mass is 783 g., if it weighs 258 g. when immersed in a fluid whose density is 1.45 g. per cubic centimeter?

2. A body which has a density of 2.58 g. per cubic centimeter weighs 524 g. when immersed in a liquid whose density is 0.725 g. per cubic centimeter. What is the mass of the body?

3. A piece of wood weighing 46.5 g. is immersed in water by the aid of a sinker weighing 75.5 g. in water. The combined weight of the wood and sinker in water is 32.5 g. What is the density of the wood?

4. Ten tons increase in the cargo of a certain ship made the ship sink 1.5 in. further into the water. What is the area of a horizontal section of the boat at the water line?

5. A piece of zinc weighs 42 g. in air and 37.8 g. when immersed in oil having a specific gravity of 0.7. Find the specific gravity of the zinc.

6. A piece of iron weighing 4 lb. is immersed in oil weighing 50 lb. per cu. ft. It is supported by means of a cord to which it is tied. Find the pull on the cord if the iron weighs 0.25 lb. per cu. in.

7. A block of tin weighs 347.5 g. in air. What is the density of a liquid in which it weighs 29.8 g.?

8. A diver with his suit weighs 325 lb. It requires 1/25 of a cu. ft. of lead to cause him to sink. Find the number of cu. ft. of water which he displaces.

9. Two liters of a liquid of specific gravity 1.25 are mixed with 5 liters of a liquid of specific gravity 2.4. The mixture shrinks to 4.6 liters. Find the specific gravity of the mixture.

CHAPTER XIII

PROPERTIES OF GASES

161. Composition of the Air.—Just as water is the most widely distributed and most important of liquids, so the air is the most important and intimate of gases. It consists for the most part of two elements which are mixed together but never chemically combined. These elements are oxygen and nitrogen. In spite of the fact that there is no chemical union between them, the composition of the air is extraordinarily constant. Up to a height of seven miles it always contains about twenty-one parts of oxygen to seventy-nine parts of nitrogen. Besides, the oxygen and nitrogen the air contains small parts of other gases, the most important of which are water vapor and carbon dioxide.

162. Weight of Air.—To an ordinary observer the air seems to have no weight and to offer little resistance to bodies moving through it. Yet smoke rises through the air and small balloons ascend out of sight. This is because the air is denser than the smoke or the gas with which the balloon is filled. The heavier air crowds the lighter gas upward as a piece of wood is forced to the surface of the water because it is lighter than water.

If a hollow glass sphere provided with a stopcock is weighed when the stopcock is open and then connected to an air pump by which as much as possible of the air is removed from the sphere, and if now the stopcock be closed and the sphere weighed a second time, it is found that the second weight is less than the first. The difference between these two weights is the weight of the air removed from the sphere. If the volume of the sphere is known and if it is almost completely exhausted a fair approximation to the density of the air can be obtained by this method. A liter of air at the temperature of melting ice and under standard conditions weighs 1.293 grams.

Example.—A hollow glass flask weighs 25.556 g. when it is empty. When it is filled with dry air at atmospheric pressure it weighs 26.849 g. The volume of the flask is 1 liter. What is the weight of one cubic centimeter of air?

Weight of air = 26.849 − 25.556 = 1.293 g.

$$\text{Density of air} = \frac{\text{weight of air}}{\text{volume of flask}} = \frac{1.293}{1{,}000} = .001293 \text{ g. per c.c.}$$

163. Buoyant Effect of the Air.—Since air has weight, it produces a certain buoyant effect on bodies immersed in it just as liquids do on bodies immersed in them. As the density of the air is so much less than that of liquids its buoyant effect is also much less than that produced by liquids. That air exerts a lifting effect on bodies immersed in it may be shown by suspending a lead ball from one side of a small balance and a large hollow brass sphere from the other side. The hollow sphere is just large enough to balance the lead ball when both are in air. If the balance together with the suspended spheres is placed under a bell jar and nearly all the air removed by means of a pump, the lead ball will no longer balance the hollow sphere. It is observed that the hollow sphere is now too heavy for the lead ball. This is because the lifting effect of the air on the hollow brass sphere is greater than its lifting effect on the lead ball for the reason that the hollow sphere displaces more air than the lead ball and when this lifting effect has been removed the lead ball is not large enough to balance the hollow sphere.

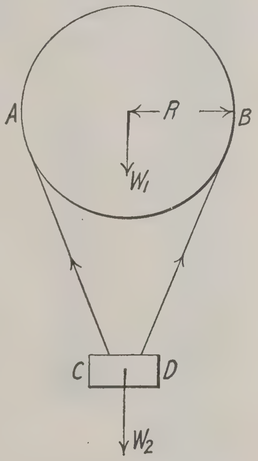


FIG. 116.—Buoyancy of air on a balloon

The lifting capacity of a balloon (Fig. 116) is equal to the weight of the air which it displaces. The density of the air becomes less at higher elevations and the lifting effect of the balloon decreases as it ascends. For this reason ballast is carried in the car or basket and this ballast is thrown over when it is desired to make the balloon rise higher. By allowing some of the gas in the balloon to escape, the balloon can be made to descend.

Example.—A balloon has a volume equal to that of a sphere 15 ft. in radius. What is the total weight which it will lift when the density of the air is 2 lb. per cu. yd.?

Volume of balloon = $4/3\pi(15)^3$ = 14,132 cu. ft.

Weight of air displaced by balloon = volume × density
= 14,132 × 2 = 28,264 lb.

164. Pressure of the Air.—Since a gas like air has weight, a column of it will exert a pressure just like a column of liquid exerts a pressure. Though a cubic foot of air weighs little, the height of the atmosphere is large, and the weight of all this air pressing on the earth is large. That air exerts a pressure is shown by taking a thistle tube and covering the mouth with rubber dam so that it is air tight. If the air in the thistle tube is partially removed by means of a pump, the pressure of the air on the outside of the rubber dam is greater than the pressure on the inside so that the rubber dam is stretched and forced inward.

If a fairly large syrup can containing a small amount of water is placed on the fire and the water boiled for a while, the escaping steam carries most of the air out of the can. If the can is now removed from the fire and at the same instant closed by inserting a rubber stopper, the water vapor which filled the can soon condenses and leaves the can nearly free from air. This reduces the pressure on the inside of the can and the atmospheric pressure on the outside causes the can to collapse.

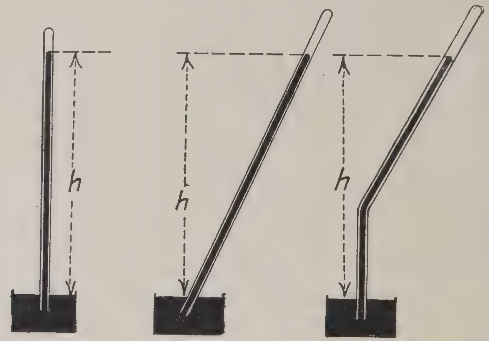


Fig. 117.—Torricelli's experiment.

165. Torricelli's Experiment.—An Italian named Torricelli first proved that the atmosphere exerts a pressure. He was able to measure this pressure in the following manner. A glass tube 100 cm. long was closed at one end and completely filled with mercury. Care was taken that no air was held between the mercury and the sides of the tube. The finger was then placed over the open end of the tube and the tube inverted in a basin of mercury. When the finger was removed from the open end of the tube under the mercury, the mercury in the tube sank until the level of the mercury in the tube was about 76 cm.

(Fig. 117) higher than its level in the basin. The pressure of the atmosphere on the mercury in the basin supported the mercury in the tube so that it did not sink further. Since each cu. cm. of mercury weighs 13.56 grams, the weight of this column is 76 × 13.56 = 1,033 grams for each square centimeter of cross section. This means that the atmosphere presses with sufficient force on each square centimeter to support a weight of 1,033 grams. Since mercury is 13.56 times as heavy as water, the atmospheric pressure would be able to support a column of water which is 13.56 times as high as this column of mercury or 1,033 cm. which is equal to a height of about 33 ft.

If this tube is carried up a mountain the level of the mercury in the tube will drop because there is less air above the tube in the new position than there was at the base of the mountain. The greater the height to which the tube is carried the greater is this drop. On the top of a mountain one mile in height the column of mercury stands 4.5 in. lower than at the base of the mountain.

Example.—Assuming that the average density of the air at distances not greater than one mile above the surface of the earth is 0.001273 g. per cu. cm. find the change in the pressure on a balloon which has risen to a height of 1 mile.

Change in pressure = weight of column of air between balloon and earth.

Change in pressure = 5,280 × 30.48 × 0.001273 = 20.6 g. per sq. cm.

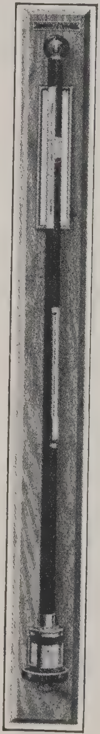


FIG. 118. Cistern barometer.

166.—The Mercury Barometer.—From this experiment of Torricelli has been developed the barometer, an instrument of the highest importance in modern life. There are two forms of mercury barometers. One of these forms (Fig. 118) is known as a cistern barometer and consists of a Torricellian tube. The pressure of the air on the mercury in the cistern supports the mercury in the tube as in the experiment of Torricelli. As the pressure of the atmosphere varies from hour to hour or from day to day, the height of this column varies. These variations in the height of the barometer are made a basis for weather predictions.

The other type of mercury barometer consists of a U-tube partly filled with mercury as indicated in Fig. 119. If both ends of the tube were open, the mercury would stand at the same height in both parts of the tube. One end of the tube is, however, closed, and there is a vacuum above the mercury in this part of the tube. The weight of the column of mercury which stands between the level of the mercury in the closed tube and the level of the mercury in the open tube is supported by the atmospheric pressure on the mercury in the open tube. The length of this column then gives a measure of the atmospheric pressure. At sea level the length of this column is 76 cm. of mercury. Hence it is customary to say that the atmosphere exerts a pressure which is equal to the weight of a column of mercury 76 cm. in height.

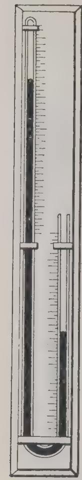


Fig. 119.—Siphon barometer.

167. The Aneroid Barometer.—The aneroid barometer consists of an air-tight box (Fig. 120) having a corrugated top. From this box most of the air has been removed so that the pressure inside the box is much less than atmospheric pressure. The top of the box is a flexible diaphragm which moves inward and outward as the pressure on it is changed. As the pressure decreases the diaphragm moves outward and as it is increased the diaphragm moves inward. These movements of the diaphragm are slight but they are magnified by a system of levers, which are connected to a pointer which moves over a dial and thus indicates the change in pressure. The position of the pointer is marked when the atmospheric pressure is 76 cm. of mercury. By marking the points above and below this point the variation of the pressure above and below normal can be determined.

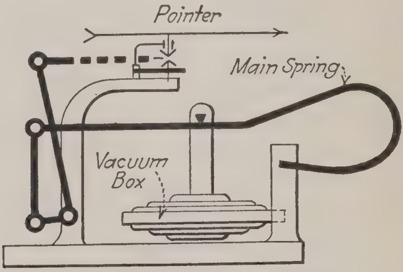


Fig. 120.—Aneroid barometer.

Example.—An aneroid barometer reads 75.35 cm. of mercury at sea level and 68.15 cm. on a nearby hill. How high is the hill?

$$\text{Change in pressure} = 75.35 - 68.15 = 7.20 \text{ cm. of mercury.}$$
$$= 7.20 \times 13.56 = 97.92 \text{ g. per sq. cm.}$$
$$\text{Height in cm.} = \frac{7.20 \times 13.56}{.001293} = 76{,}000 \text{ cm}$$

168. Depth of the Atmosphere.—Judging from the rate at which the pressure of the air decreases as a barometer is carried upward it is thought that the depth of the atmosphere is about 30 miles. Since the atmosphere becomes less and less dense as we ascend, at the height of 30 miles the air will be very rarefied but it cannot be said to have entirely disappeared.

169. Applications of Atmospheric Pressure.—The breathing of animals is an application of atmospheric pressure. A reduction of pressure is made by a movement of the lungs and the diaphragm and then the greater pressure of the outside air causes a fresh supply to fill them. Sucking and drinking animals take advantage of atmospheric pressure to aid them in this operation. By reducing the pressure in the mouth they really allow water to be forced into the mouth by the outside atmospheric pressure. Even in eating, the atmospheric pressure helps to force the food from between the teeth after it has been masticated.

170. The Siphon.—The siphon is an instrument for transferring liquid from a place of higher level to a place of lower level. It consists of a bent tube (Fig. 121) with the short end immersed in the liquid at the higher level and the other end immersed in the vessel into which the liquid is to be transferred. If the flow is once started the liquid will continue to flow until it is at the same level in the two vessels.

Evidently, the water must flow through the tube if the force pushing it in one direction is greater than the force pushing it in the opposite direction. The pressure on the surface of the liquid in A is atmospheric pressure. The force on the water in the top of the tube pushing it to the right is atmospheric pressure less the weight of the column of liquid of height h_2. The pressure on the liquid in the top of the tube pushing it to the left is atmospheric pressure less the weight of the column of liquid of height h_1. Hence the pressure toward the right exceeds the pressure toward the left by the difference in the weight of these two columns of liquid. The pressure toward the right $= P - dh_2$ and that toward the left $= P - dh_1$.

$$\text{The net pressure} = d(h_1 - h_2).$$

If the water in the two vessels has the same height, the pressure disappears and the flow ceases. The greater the difference in level, the greater the force and the more rapid the flow. Since the atmospheric pressure forces the water past the bend in the tube, the water will not flow, when the bend is higher than about 30 ft. above the level of the water in the vessel.

Example.—In the siphon shown in Fig. 121 the height of C above A is 1.8 in. and the height of C above B is 36 in. Find the force driving the water through the siphon.

Pressure driving water through the siphon

$$= (h_1 - h_2)p = \frac{(36 - 1.5) \times 62.5}{12} = 180 \text{ lb. per sq. ft.}$$

$$= 1.2 \text{ lb. per sq. in.}$$

171. The Intermittent Spring.—A siphon of the form indicated in Fig. 122 will give an intermittent flow. When the water fills the vessel so that the siphon is completely covered the operation of the siphon begins and the flow will continue until the vessel is empty. The vessel will then again fill and the process will be repeated.

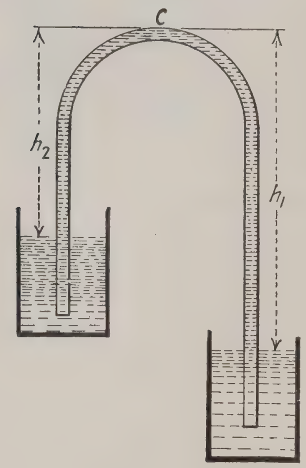


FIG. 121.—Siphon.

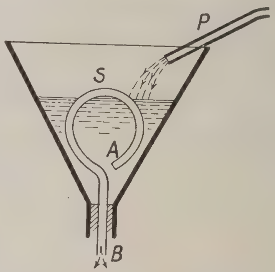


FIG. 122.—A form of siphon.

An intermittent spring (Fig. 123) operates on the principle of such a siphon. The flow of water from the spring is periodic. After the spring has been once emptied sufficient time must elapse for it to fill to the height at which the water will start to flow through the siphon.

172. Compressibility of Gases.—If we undertake to decrease the volume of a liquid by the application of pressure, it is found that it is necessary to apply enormous pressure in order to get appreciable changes in volume. The behavior of gases in this respect is quite different. It is easy to compress a body of air so that it occupies only one-third or one-tenth of its original volume. As soon as this pressure is removed the air or other gas springs back to its original volume. The tires of automobiles are ordinarily filled with air. As more and more air is forced into the tire the volume of the tire increases very little but the air taken from the outside is forced to occupy much less volume than it originally occupied. As the air is forced into the tire the pressure it exerts is more and more increased.

The tendency of air to expand is seen when a puncture occurs in an automobile tire and the air suddenly escapes. This tendency of a gas to expand also manifests itself when a toy balloon is placed under a bell jar and the air is removed from the bell jar by means of a pump. The balloon expands and bursts as soon as the pump is set in action. Before the pump is set in operation, the pressure of the air on the inside and on the outside of the balloon is nearly the same. That on the inside is only slightly larger than that on the outside. As the pressure on the outside is removed the air on the inside expands as much as possible causing the rupture of the balloon.

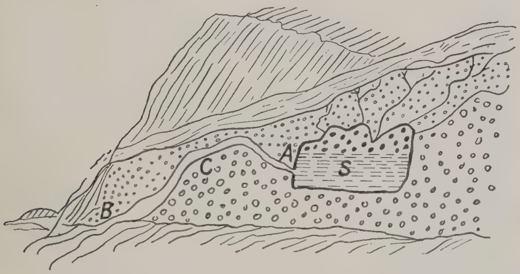


Fig. 123.—The intermittent spring.

173. Boyle's Law.—Relation between Pressure and Volume of a Gas. The relation between the volume of any mass of gas and the pressure exerted by the gas upon the walls of the containing vessel was investigated by Robert Boyle and is known as Boyle's Law. **This law states that at constant temperature the volume of a given mass of gas is inversely proportional to the pressure to which it is subjected.** Thus if v_1 and p_1 denote the original volume and pressure and v_2 and p_2 denote the final volume and pressure,

$$p_1v_1 = p_2v_2 = \text{Constant}$$

or for a constant temperature the product of the pressure and the volume is a constant. By pouring mercury into the open end of the tube (Fig. 124), the pressure on the air in AC is increased and its volume decreased. Since the density is inversely proportional to the volume, this law states that at constant temperature the density of a gas is proportional to the pressure.

$$\frac{p_1}{p_2} = \frac{d_1}{d_2}$$

For high pressures and low temperatures this law is only an approximation. Gases which can be liquefied by the application of pressure do not obey this law near the temperature and pressure at which they begin to liquefy. Fig. 125 shows the relation between the pressure and volume of a gas at different temperatures.

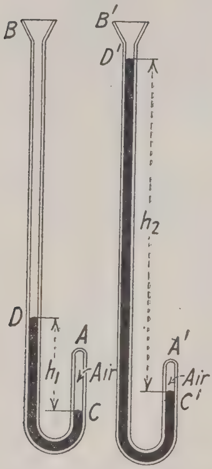


FIG. 124.—Boyle's law —pressure times volume is constant.

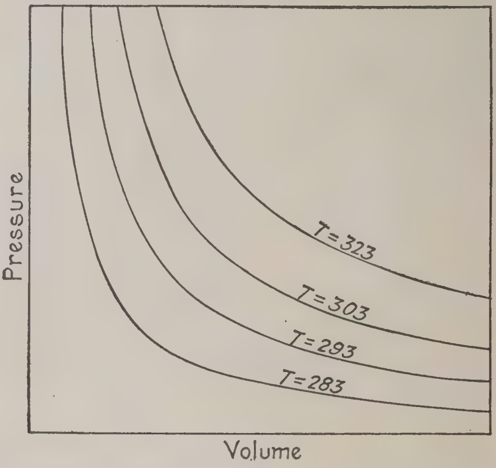


FIG. 125.—Relation of pressure to volume at different temperatures.

Example.—One hundred cubic inches of air under a pressure of 400 lb. per sq. in. is allowed to expand until the pressure is 20 lb. per sq. in. What is the new volume?

Original pressure × original volume = final pressure × final volume

$$P_1V_1 = P_2V_2$$
$$P_1 = 400 \text{ lb. per sq. in.}$$
$$V_1 = 100 \text{ cu. in.}$$
$$P_2 = 20 \text{ lb. per sq. in.}$$
$$400 \times 100 = V_2 \times 20$$
$$V_2 = 2{,}000 \text{ cu. in.}$$

174. Applications of Boyle's Law.—There are many important applications of this law. When the faucet of a kerosene can (Fig. 126) is turned on, only a small amount of kerosene will run out of the can, if the cap *A* is screwed down so that the can is air tight. In this case, as the kerosene runs out of the can, the volume of air above the kerosene increases and its pressure decreases according to Boyle's law. On the kerosene flowing out at the faucet, there is under normal conditions a pressure of one atmosphere. When the pressure of the air in the can together with the weight of the kerosene exerts a pressure of one atmosphere, the flow ceases. If the screw cap *A* is now opened so that atmospheric pressure acts undiminished on the upper surface of the kerosene, the flow begins again.

It is difficult to pour a liquid from a small necked bottle without considerable sputtering as the air goes inside the bottle to force the liquid out. This fact is illustrated in Fig. 127. Suppose there is air at atmospheric pressure above the liquid at *P*. Then the weight of the column of liquid will be sufficient to cause a flow. While the flow is in progress the air above the liquid expands and its pressure decreases. Before the flow ceases, the pressure will be decreased too far for equilibrium. This is because the liquid once in motion tends to keep moving until forced to stop. When the flow has ceased more air will rush in through the mouth of the bottle. Too much air for equilibrium will rush in and the pressure above the liquid becomes too large. The flow will then begin again. This action causes the familiar gurgling which is noticed when liquids are poured from bottles. To prevent this gurgling and get a steady flow, sometimes an auxiliary tube is inserted in the stopper. The air flows in through this tube continuously and a steady flow of liquid is obtained.

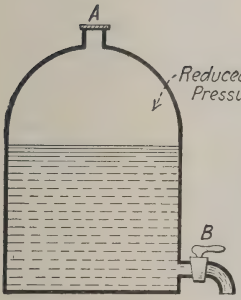


FIG. 126.—Flow of liquid from a can.

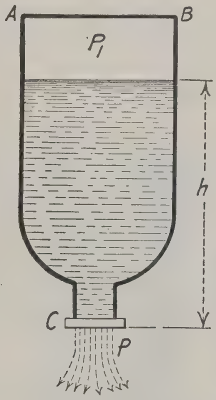


FIG. 127.—Flow of water from a bottle.

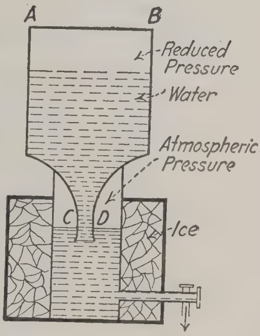


FIG. 128.—Water cooler.

The drinking fountain (Fig. 128) is also an illustration of the expansion of gases according to Boyle's law. The bottle containing some air is inverted. The air at atmospheric pressure in the upper part of the bottle exerts sufficient pressure to force some water out of the faucet. As the water flows out more air is sucked in through the mouth of the bottle as in the preceding case.

In the common firebellows (Fig. 129), as the two sides *B* and *C* are pushed apart, the volume of the enclosed air is increased and its pressure decreased in agreement with Boyle's law. More air rushes in from the outside through the valve *D*. As the sides *B* and *C* are pushed together the volume of the enclosed air is decreased and the pressure is increased. This additional pressure on the inside closes the valve and air is forced out through the nozzle. The nozzle *A* is made small so that little air will come through it, in comparison with that which comes through the valve *D* when the volume of the bellows is being increased.

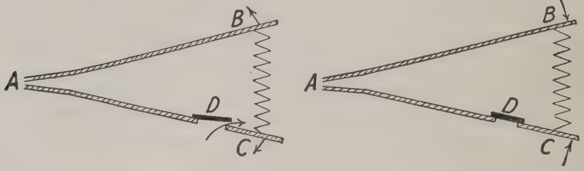


Fig. 129.—Fire bellows.

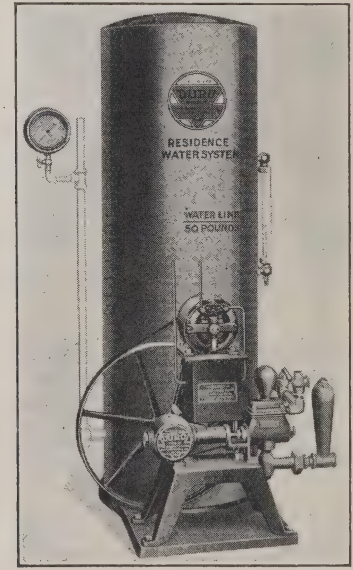


Fig. 130.—Air-pressure tank for water system.

175. The Air-pressure Tank.—A modern and highly practical method of storing pumped water for use on a farm is the pneumatic or compressed air water tank. The principle of this tank which is an application of Boyle's law is very simple. An air-tight steel tank (Fig. 130) similar to a steam boiler is placed in the basement of a suitable building where it will not freeze. This storage tank is connected to a force pump. When the outlet pipe is closed, water is pumped through the inlet pipe. At the beginning, the tank is completely filled with air. As more and more water is forced into the tank, the air above the water is more and more compressed according to Boyle's law. When the tank is half full of water, the air occupies one-half of its original volume and consequently exerts double its original pressure. As the pumping of the water into the tank continues, the pressure of the air above the water increases and it becomes increasingly difficult to force more water into the tank. In this way any desired pressure can be obtained in the tank. When a faucet is opened in any part of the house, water is forced from the tank through the faucet by means of the pressure of the air in the tank. As the water is allowed to flow, the volume of air above the water increases and the pressure of the air on the water decreases, so that when the tank is nearly empty, the pressure on the water is very small. When the tank is again filled with water, the pressure is again restored.

Example.—When the water in the pressure tank (Fig. 130) stands at the water line, the compressed air above the water exerts a pressure of 30 lb. to the sq. in. When enough water has run out of the tank to allow the air to become 1.5 times its original volume, to what height will the water rise in the house?

By Boyle's law, $P_1V_1 = P_2V_2$
since $V_2 = 3/2\ V_1$, $P_2 = 2/3\ P_1$
Hence the new pressure is 2/3 the original pressure = 20 lb. per sq. in. If the atmospheric pressure at the time = 15 lb. per sq. in., the effective pressure for lifting water in the house is,

$$20 - 15 \text{ lb. per sq. in.} = 5 \text{ lb. per sq. in.}$$
$$= 5 \times 144 = 720 \text{ lb. per sq. ft.}$$
$$\text{Pressure} = \text{density} \times \text{height}$$
$$720 = 62.5 \times h$$
$$h = 11.5 \text{ ft.}$$

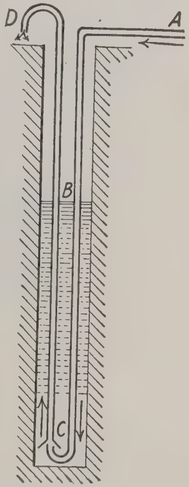


FIG. 131.—Compressed-air lift.

176. Compressed-Air Lifts.—The force exerted by air when compressed is often used for lifting water from deep wells. If an air compressor is available the process is quite simple. A water pipe *DC* (Fig. 131), open at both ends is submerged in a well, and a smaller pipe *AC* delivers compressed air into it at its lower end, which is considerably below the level of the water. The compressed air escaping from the smaller pipe rises through the pipe *CD*, lifting with it a column of water. This water is discharged into an open tank above the ground where the bubbles of air escape.

177. Suction Pump.—Water for household or farm purposes is usually lifted out of moderately deep wells by a suction pump. This pump (Fig. 132) consists of a cylinder which is connected to a pipe *S*. The lower end of the pipe *S* is immersed in the water in the well. At the bottom of the cylinder there is a valve *B* which opens upward. A plunger *P* containing a valve *A* opening upward is moved up and down in the cylinder by means of a pump handle. The valve *B* in the cylinder prevents any water above it from passing downward. As the handle is forced downward the plunger is raised with the valve *A* closed. The water above the plunger is thus raised and flows out of the spout. The upward stroke of the piston reduces the pressure in the space below the plunger. The reduction of the pressure in this space allows the pressure of the air on the water in the well to force more water up the pipe *S*, through the valve *B*, into the cylinder. When the piston makes its next downward stroke, the valve *B* closes and the water above the valve is entrapped in the cylinder. During the downward stroke the valve *A* in the piston opens and the water flows above the piston. The upward stroke is again repeated and the water flows out of the spout as before. In order for the pump to operate, the valve *B* must not be more than 30 ft. above the surface of the water in the well.

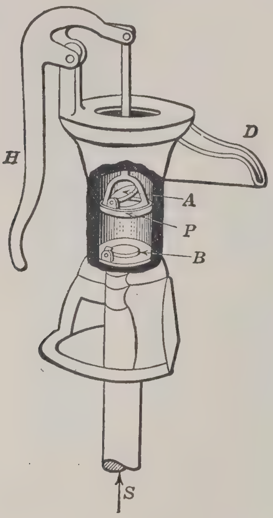


FIG. 132.—A suction pump.

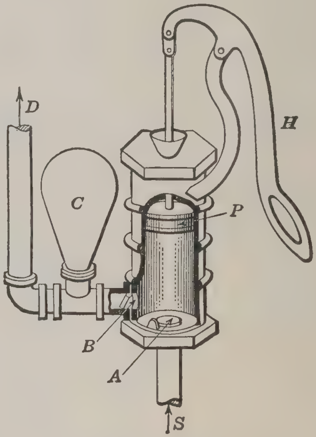


FIG. 133.—A force pump.

178. Force Pump.—In the force pump (Fig. 133) the suction pipe *S* with its valve *A* is just like this portion of the suction pump. An outlet pipe with a valve *B* is connected to the lower part of the cylinder. As the piston moves downward, the water in the cylinder is forced through the valve B into the delivery pipe D. Raising the piston allows the valve A to open and water to be forced through it by the atmospheric pressure on the water in the well. On the downward stroke of the piston, this valve closes and the valve in the delivery pipe opens.

In order to obtain a steady stream of water from the pump, an air cushion C is provided. On the downward stroke of the piston, the air in this chamber is compressed by the water flowing into it from the delivery pipe. While the piston is making its upward stroke, the compressed air in this chamber expands and forces water through the delivery pipe. There results in this way a more or less steady stream of water through the delivery pipe. The compressed air in this chamber tends to prevent the jars and shocks which would accompany the starting and stopping of the water if it flowed only on the downward stroke of the piston.

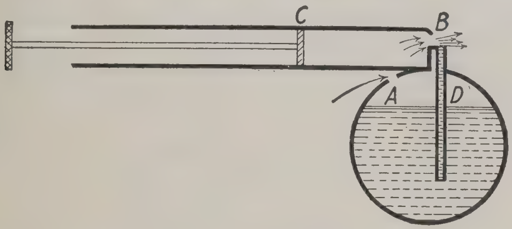


Fig. 134.—A sprayer.

179. Sprayer.—In the sprayer (Fig. 134) the forward stroke of the piston compresses the air in the cylinder, forcing a steam of air past the end of the tube D. The other end of this tube is immersed in the liquid which is to be sprayed. The stream of air flowing past the open end of the tube reduces the pressure on the liquid in the tube. Atmospheric pressure acting on the surface of the liquid in A forces more liquid up into the tube D from which it is carried away by the stream of air. There results from this mixture of air with the fine particles of liquid, the spray which is much used on the farm and in the garden.

180. Measuring Pumps.—Pumps are often used in measuring the volumes of liquids, especially in the sale of gasoline. The ordinary piston pump can be used for this purpose, if means are provided for defining the length of the stroke and insuring that each stroke of the piston will discharge the same volume of liquid. This requires that the valves be tight and the piston close fitting so as to prevent leakage or slippage of the liquid past the valves or the piston. Such pumps may discharge either on the up stroke or they may discharge either on the upward or downward strokes.

In the former case, they are said to be single acting pumps and in the latter case they are known as double acting pumps.

Problems

1. If the earth's atmosphere had a uniform density of 0.0012 grams per c. c., how high would the atmosphere be when the barometer reads 75.4 cm. of mercury?

2. The barometer reads 75 cm. of mercury at the surface of a pond. How deep must the pond be in order that an air bubble on reaching the surface may have double the volume it had at the bottom?

3. A barometer at the top of a tower reads 75.50 cm., and at the bottom it reads 76.00 cm. If the average density of the air is 0.0012 g. per c. c., how high is the tower?

4. What will be final pressure inside of a vessel of 600 c. c. capacity containing air at a pressure of 74 cm. of mercury when 2,500 c. c. of air, taken at a pressure of 95 cm., of mercury is forced into it?

5. A room is closed tightly when the barometer reads 75.5 cm. of mercury. The barometer falls so that it reads only 73.0 cm. of mercury. Find the total pressure which one of the walls of the room sustains if the wall is 8 ft. high and 15 ft. long.

6. An airship 550 ft. long has an average diameter of 50 ft. and is filled with hydrogen. If the total weight of the airship is 5,000 lb., find how heavy a load can it lift.

Specific gravity of air = 0.00125
Specific gravity of hydrogen = 0.000085

7. A cylinder having a cross section of 4 sq. cm. is closed by a frictionless piston and filled with air at a pressure of 75 cm. of mercury. If the cylinder is 20 cm. high and the piston weighs 5 kg., how far will the piston descend on account of its own weight.

8. A cylindrical jar is 80 cm. long and 20 cm. in diameter. It is sunk in water to a depth of 40 m. How much air must be pumped into it to keep the water from rising into it when the barometer reads 75 cm. of mercury?

PART II.—HEAT

CHAPTER XIV

TEMPERATURE AND ITS MEASUREMENT

181. Temperature.—The most familiar temperature is the temperature of the human body. Objects are said to be warm, hot, cool or cold compared with this temperature. This is not a very reliable or accurate standard. It is therefore necessary to look for some more accurate method of estimating temperatures. For this purpose it is customary at ordinary temperatures to use a thermometer, which depends for its operation on the fact that a liquid like mercury expands when its temperature is increased. Other substances besides mercury are used. Any substance which expands uniformly may be used between suitable temperatures.

182. Mercury Thermometer.—The most common type of thermometer for ordinary use is the mercury in glass thermometer. It consists of a small glass bulb to which is sealed a glass tube with a very small bore. The bulb and part of the tube are filled with mercury. The residual air above the mercury in the tube is then carefully removed so that the space above the mercury is empty. The glass tube is then sealed off. In order to use this bulb with its fine tube for a thermometer, it is necessary to have on the stem of this thermometer a scale divided into equal divisions called degrees.

Fixed points. The two fixed points which are ordinarily chosen for a thermometer are the melting point of ice and the boiling point of water under atmospheric pressure.

To determine the first of these fixed points the bulb of the thermometer is surrounded with finely divided ice or snow. This melting ice or snow keeps the same temperature while melting. After the mercury in the bulb has reached the same temperature as the ice or snow, the mercury in the stem of the thermometer does not further change. The point at which the mercury stands is now taken and used as one of the fixed points on the thermometer. On the Centigrade scale, this point is called 0°, but on the Fahrenheit scale it is arbitrarily called 32°. The bulb and as much as possible of the stem of the thermometer are now placed in steam rising from water boiling at atmospheric pressure. (Fig. 135.) The mercury expands and assumes a new position on the stem. This point does not change after the temperature of the thermometer has reached the temperature of the steam. This point is marked on the scale and used as a second fixed point for the thermometer. On the Centigrade scale this point is called 100°, and on the Fahrenheit scale it is called 212°.

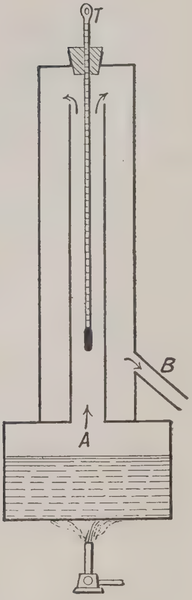


Fig. 135.—Testing fixed points of a thermometer.

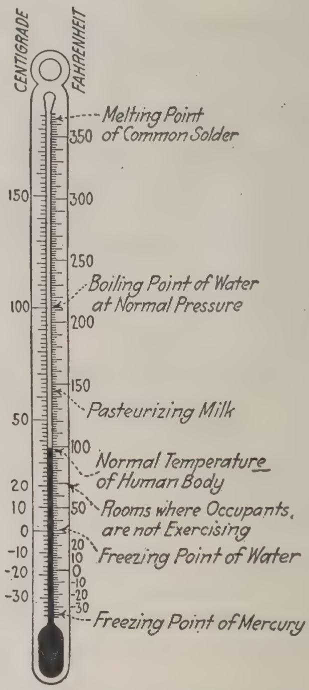

CENTIGRADE
FAHRENHEIT
Melting Point of Common Solder
350
150
300
250
Boiling Point of Water at Normal Pressure
100
200
150
Pasteurizing Milk
50
100
Normal Temperature of Human Body
20
10
50
Rooms where Occupants are not Exercising
0
-10
-20
-30
20
10
0
-10
-20
-30
Freezing Point of Water
Freezing Point of Mercury

Fig. 136.—Comparison of thermometer scales.

183. Comparison of Centigrade and Fahrenheit Thermometers.—Consider the thermometer shown in Fig. 136. On one side is a Centigrade thermometer and the other is a Fahrenheit thermometer. The freezing point on the Centigrade is 0° and that on the Fahrenheit is taken as 32°. The boiling point on the Centigrade is 100° and that on the Fahrenheit is 212°. Hence, one hundred divisions or degrees on the Centigrade scale correspond to one hundred and eighty on the Fahrenheit scale, and one degree on the Fahrenheit scale equals only 5/9 of one degree on the Centigrade.

To reduce from the Fahrenheit to the Centigrade scale first find how many degrees above or below the freezing point of water the temperature is on the Fahrenheit scale, then take five-ninths of this; the result will be the reading on the Centigrade scale. In other words, subtract 32° from the reading on the Fahrenheit scale and take 5/9 of the remainder.

Reading on Centigrade = 5/9 (Reading on Fahrenheit − 32°).
Reading on Fahrenheit = 9/5 (Reading on Centigrade) + 32°.

Example.—The reading of a thermometer for the temperature of a room is 77°F. What is the reading on the centigrade scale?

Reading on Centigrade = 5/9 (Reading on Fahrenheit − 32°)

$$= 5/9(77 - 32) = 5/9 \times 45 = 25°\text{C}.$$

Example.—The temperature of an oven is 125° Centigrade. Find its temperature on the Fahrenheit scale.

Reading on Fahrenheit = 9/5 (Reading on Centigrade) + 32°

$$= 9/5 \times 125 + 32$$
$$= 225 + 32 = 257°\text{F}.$$

Problems

1. The temperature on a summer day is 98°F. What is the temperature on the Centigrade scale?

2. The temperature of the earth is found to be 15°C. What is its temperature on the Fahrenheit scale?

3. The temperature of the blood is 98.4°F. What would a Centigrade thermometer indicate for this temperature?

4. On a winter morning the temperature is 25° below zero on the Centigrade thermometer. What is the temperature on the Fahrenheit thermometer?

5. Find the temperature at which the Centigrade thermometer and the Fahrenheit thermometer would show the same reading.

184. Maximum and Minimum Thermometers.—In making observations on the temperature of the atmosphere, the Weather Bureau usually finds it convenient to use a maximum and minimum thermometer, which registers the highest and lowest temperature reached since the last setting. There are several types of these thermometers is use. One of the most common forms is shown in Fig. 137. It consists of a bulb *A* filled with phenol or some other liquid having a large coefficient of expansion. A mercury column fills the lower part of the tube while the tube above C is also filled with phenol. When the temperature rises, the expansion of the liquid in the bulb A causes the mercury column to sink at B and rise at C, pushing upward a little index of iron in the tube above C, which in consequence of friction

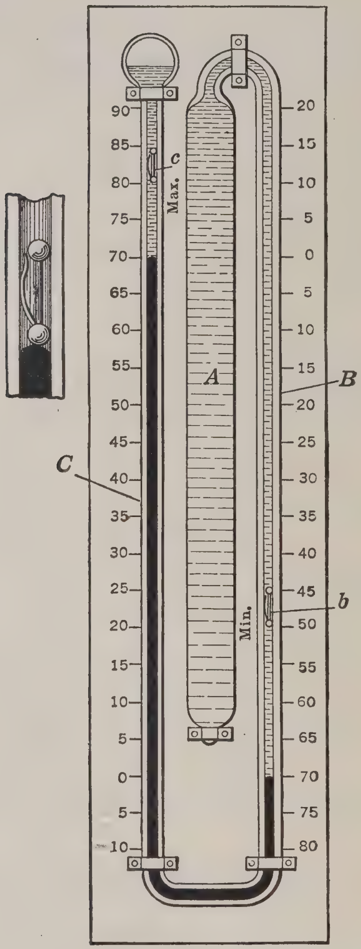

90
85
80
75
70
65
60
55
50
45
40
35
30
25
20
15
10
5
0
5
10
20
15
10
5
0
5
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
c
Max.
A
B
C
b
Min.

Fig. 137.—Maximum and minimum thermometer.

remains where pushed and marks the maximum temperature. On cooling, the contraction of the liquid in A causes the mercury to rise at B, pushing upward another little index of iron which marks the minimum temperature. To set the instrument, the indices are drawn downward against the mercury by means of a small magnet.

185. The Clinical Thermometer.—The thermometers commonly used by physicians are also maximum thermometers, having a short scale ranging from about 95° to 105° F. The tube is made very flat and narrow just above the bulb. The mercury passes readily through this constriction in rising; but as it contracts, capillarity causes the column to separate at that point, leaving the upper part of the mercury column to mark the highest temperature.

The usual type of this thermometer is made so that the front of the glass tube acts as a lens, magnifying the width of the mercury thread. To read such a thermometer, it should be held in the hand and turned until the mercury column suddenly appears magnified to a considerable width. This will occur when the clear corner of the triangular tube is directly in front. The mercury is then shaken back into the bulb by holding the thermometer firmly between the thumb and forefinger and giving a few brisk shakes from the wrist.

CHAPTER XV

MEASUREMENT OF HEAT

186. Unit of Heat.—Although heat is a form of energy and may be measured in the units in which energy is measured, it is convenient to use a unit which is based on the effect of heat in raising the temperature of a substance. In looking for a substance to use as a standard it is natural to choose water on account of the ease with which it is obtained. Since it always takes the same amount of energy to raise the temperature of 1 gm. of water from 4° to 5° Centigrade, it is possible to define an arbitrary unit in which to measure other quantities of heat. Here the choice of the unit is largely a matter of convenience. In this respect, however, it does not differ from the unit of length or the unit of mass which is also chosen arbitrarily.

187. Calorie.—The unit of heat in the C. G. S. system is called the **calorie.** **It is defined as the quantity of heat or energy which is necessary to raise the temperature of 1 gram of water from 4° to 5° on the Centigrade scale.** Because of the fact that the heat required to raise the temperature of 1 gram of water 1° Centigrade is not the same at all temperatures it is necessary to state the temperature at which the calorie is defined.

188. The British Thermal Unit.—In the English system of units, the unit of heat is known as the British Thermal Unit, (B.t.u.). **It is defined as the quantity of heat required to raise the temperature of 1 lb. of water from 60° to 61° on the Fahrenheit scale.** This is a much larger unit of heat than the calorie. Here, as in the case of the calorie, it is necessary to state the temperature, for the amount of heat required to raise one pound of water 1° Fahrenheit varies with the temperature.

189. Specific Heat.—So far we have been dealing with a single substance, namely water. It was convenient to choose this substance as a standard. The amount of heat required to raise another substance 1° may be compared with the amount of heat required to raise an equal mass of water 1°. In this way the specific heat of a substance may be defined. Thus, **the ratio**

between the amount of heat required to raise the temperature of any body one degree and the amount to raise the temperature of an equal mass of water one degree, is called the specific heat of the substance. Since the heat required to raise the temperature of 1 gram of water one degree is called a calorie, it is possible also to define the **specific heat of a substance as the number of calories required to raise the temperature of one gram of the substance one degree.** This is equivalent to the former definition except that in this case the mass chosen is the gram.

Let Q denote the quantity of heat added to a mass of M grams, let t and t' be the initial and final temperatures, and let C be the specific heat of the body. Then

$$Q = SM\ (t' - t)$$

Example.—Find the number of calories required to raise the temperature of 100 grams of aluminum from 25° to 75°C. Specific heat of aluminum is 0.088.

$$\begin{aligned} \text{Heat} &= \text{mass} \times \text{sp. heat} \times \text{change of temperature} \\ &= M \times S \times (t' - t) \\ &= 100 \times 0.088 \times (75 - 25) = 440 \text{ calories} \end{aligned}$$

190. Thermal Capacity.—Instead of using the specific heat of a substance which is the amount of heat necessary to raise the temperature of one gram of the substance one degree Centigrade, it is sometimes more convenient to use what is known as thermal capacity. **The thermal capacity is defined as the heat required to raise the temperature of the entire body one degree.** It differs from the specific heat only in the fact that the specific heat is the heat required to raise the temperature of unit mass of the substance one degree. To find the thermal capacity, it is only necessary to multiply the specific heat by the mass of the body. If, on the other hand, the thermal capacity of the body is given, the specific heat is found by dividing the thermal capacity of the body by its mass. That is,

$$\text{Thermal capacity} = MS$$

where M = mass of the substance and S = its specific heat.

To illustrate the difference between the specific heat and the thermal capacity of a body, place a number of metallic balls of equal mass but of different metals, side by side on a cake of wax. The balls, having been first heated to some convenient temperature, begin to melt their way into the wax. When the balls have come to the same temperature as the wax, it will be found that some have sunk to greater depths than others in the wax. The depth to which a ball sinks is a measure of the amount of heat it gives up in cooling to the temperature of the wax. The original temperature of the balls being the same, and the depths to which they sink being different, the quantity of heat which each contained was also different. The depth to which they sink also becomes a measure of their thermal capacities.

Example.—Find the thermal capacity of a clay soil which covers a field of one acre to a depth of 1 ft. Take 0.25 as the specific heat of the soil and 156 lb. per cu. ft. as its density.

$$\begin{aligned}\text{Thermal capacity} &= \text{mass} \times \text{specific heat}\\ &= \text{volume} \times \text{density} \times \text{specific heat}\\ &= 435{,}000 \times 156 \times 0.25\\ &= 17 \times 10^6 \text{ B.t.u.}\end{aligned}$$

If a piece of soapstone and a piece of iron of equal weight are heated to the same temperature and each of them dropped into an equal mass of water, it will be found that the temperature of the water into which the stone was dropped is increased more than the temperature of the water into which the iron was dropped. From this it is seen that a mass of soapstone stores up more heat than an equal mass of iron and consequently has a greater thermal capacity.

Problems

1. What is the thermal capacity of 1 cubic meter of aluminum which has a specific heat of 0.22 calories? Density of aluminum = 2.6.

2. How much heat is necessary to heat the air in a room, which is 6 × 5 × 3 meters from 0°C. to 25°C.? Assume the mass of the air remains constant. Take the specific heat of air at constant volume to be 0.24, and the weight of 1 liter of air = 1.29 grams.

3. A loosely packed loam soil weighs 70 lbs. per cu. ft., and a sandy soil weighs 106 lb. per cu. ft. The specific heat of the loam is 0.16 calories per gram, and that of the sandy soil is 0.11 cal. per gram. On a certain day 1,000 calories of heat are supplied to these soils by radiation from the sun. How much will the temperature of each be raised?

191. Measurement of Heat by Method of Mixtures.—One of the most familiar methods of measuring a quantity of heat is by imparting this heat to a known mass of water and observing the change which it produces in the temperature of the water. If the mass of the water is known and its rise in temperature is observed, the heat imparted to the water can be found by multiplying the mass of water by its rise in temperature. Since it requires one calorie to raise the temperature of one gram of water one degree Centigrade, to raise M grams from temperature t to temperature t' would require H calories where,

$$H = M(t' - t)$$

Let it be assumed that it is desired to determine the specific heat of iron. Let m grams of iron be heated to a temperature T. This can be conveniently done by placing the iron for a sufficient time in a steam bath. Meanwhile M grams of cold water are weighed out in a vessel of known mass M'. The temperature of the vessel and that of the water being the same, both can be denoted by t. The piece of iron is suddenly plunged into the vessel of water and after stirring the water, an equalization of temperature takes place. Such a vessel (Fig. 138) is known as a **calorimeter.** The iron falls in temperature, thus giving heat to the water and its containing vessel. The water and the containing vessel, on the other hand, gain heat and rise in temperature. After mixing the final temperature is noted and called t'. According to the law of conservation of energy, **the heat lost by the iron must be just equal to that gained by the water and its containing vessel.** The heat gained by the water is

$$M(t' - t),$$

its mass times its change in temperature. The heat gained by the vessel is

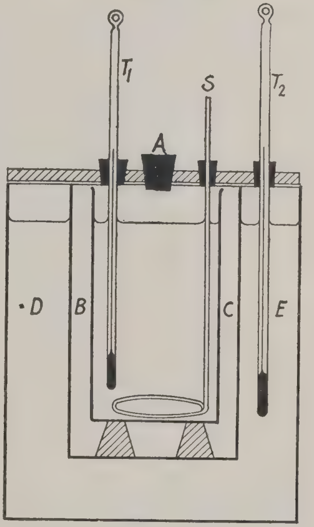


FIG. 138.—Calorimeter.

$$M'(t' - t)S',$$

its mass times its change in temperature times its specific heat. The heat lost by the iron is

$$m(T - t')S,$$

its mass times its change in temperature times its specific heat.

Heat gained = heat lost.

Hence, $M'(t' - t)S' + M(t' - t) = m(T - t')S$

$$S = \frac{M(t' - t) + M'(t' - t)S'}{M(T - t')} = \text{specific heat of iron.}$$

Example.—Suppose that 450 grams of lead are heated to 100°C. and then dropped into a calorimeter containing 100 grams of water at 10°C. After stirring, the temperature of the water rose to 21.5°C. Neglecting the heat given to the calorimeter, find the specific heat of lead.

Heat lost by lead = heat gained by water

Heat lost by lead = mass × specific heat × change in temp.

$$= 450 \times S \times (100 - 21.5)$$

Heat gained by water = mass × sp. heat × change in temp.

$$= 100 \times 1 \times (21.5 - 10)$$

Hence:

$$450 \times S \times (100 - 21.5) = 100 \times 1 \times (21.5 - 10)$$

$$S = \frac{100\,(21.5 - 10)}{450 \times (100 - 21.5)} = \frac{11.5 \times 100}{450 \times 78.5} = 0.033 \text{ calories sp. heat of lead.}$$

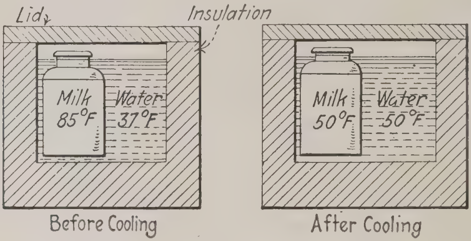


FIG. 139.—Cooling of milk.

Example.—Suppose that the atmosphere is warmer than the soil on a day in the spring when two inches of rain falls. Since the heat stored up in one pound of water is large, in this way there will be a considerable amount of heat conveyed to the soil. If this rain is all absorbed in one foot of the soil and if the temperature of the air is 80°F. and that of the soil is 50°F., find the resultant temperature after this rain has fallen. Specific heat of soil = 0.15; density of soil = 2.5.

Mass of water on 1 sq. ft. = 1/6 × 1 × 62.5 = 10.4 lb.

Mass of 1 cu. ft. of soil = 2.5 × 62.5 = 156 lb.

B.t.u. lost by water = mass of water × change in temp.

$$= 10.4\,(80 - t)$$

where t = resultant temperature

B.t.u. gained by soil = mass of soil × sp. heat × change of temp.

$$= 156 \times 0.15 \times (t - 50)$$

Hence:

$$156 \times 0.15\,(t - 50) = 10.4\,(80 - t)$$

whence $t = 59.2$°F

192. Method of Cooling Milk.—A common method of cooling milk depends on the transfer of heat as it takes place in the method of mixtures.

In Fig. 139 is shown a sketch of such a cooler. It consists of a vessel holding a large quantity of water into which is set the can of milk to be cooled. There is a transfer of heat from the milk to the colder water. The heat lost by the milk is gained by the water except for losses due to radiation.

Example.—Suppose that a can containing 10 gallons of milk at 85°F. is placed in 30 gallons of water at 37°F. Find the resultant temperature when both milk and water are at the same temperature.

Density of milk = 64.5 lb. per cu. ft.
Density of water = 62.5 lb. per cu. ft.
Mass of milk = 86.2
Mass of water = 250 lb.
T = final temperature of both milk and water
Heat gained by water = $250 \times 1 \times (T - 37)$
Heat lost by milk = $86.2 \times 1.1\ (85 - T)$

Hence:

$$250 \times 1 \times (T - 37) = 86.2 \times 1.1\ (85 - T)$$
$$T = 52°F.$$

193. Specific Heat of Soils.—When the same number of heat units is added to *equal* weights of different kinds of soils, their temperatures are not raised through the same number of degrees. This is because the thermal capacities of the soils are different. The following table shows how the specific heats of different soils differ.

Table of Specific Heats of Soils

Water	1.000	Loam	0.150
Moor earth	0.221	Pure clay	0.137
Humus	0.208	Sand	0.100
Sandy humus	0.141	Pure chalk	0.185
Clayey humus	0.158		

From this table it is evident that more heat is required to raise the temperature of one pound or one gram of water than to raise the temperature of an equal amount of dry soil one degree. For this reason dry soil will warm in the sunshine more rapidly than wet soils.

194. Specific Heat of Wet Soils.—When soils are wet, enough heat must be supplied to raise the temperature of the soil and at the same time raise the temperature of the water which they contain. This makes the thermal capacity greater, the wetter the soil. Sand has a small capacity for water and on this account its specific heat even when wet, is low. The weight of the sand per cubic foot or per cubic centimeter is great, so that if the heat required to raise the temperature of unit volume is desired attention must be paid to the greater density of the sand.

195. Measurement of Specific Heat of Soils.—The specific heat of a soil is frequently measured by the method of mixtures. Take a calorimeter (Fig. 138) containing a stirrer and a thermometer. The specimen of soil should be first heated to drive off all hygroscopic moisture and then weighed. It is then enclosed in a bag of oiled paper and suspended by a thread in a heater, near a thermometer. After the steam is turned into the jacket of the heater the soil is left until it has assumed the temperature of the heater. After the calorimeter has been weighed, it is filled with a known weight of water and its temperature is observed. Now bring the calorimeter under the heater. After reading the thermometer in the heater quickly lower the specimen of soil into the calorimeter. The calorimeter is then again covered and removed from under the heater. The water in the calorimeter is stirred until a constant temperature is reached. This temperature is noted and sufficient data are at hand for the determination of the specific heat of the soil.

Let t = the initial temperature of the soil.
m = the mass of the soil.
M = the mass of the water.
M' = the mass of the calorimeter.
S = the specific heat of the soil.
S' = the specific heat of the calorimeter.
T = the initial temperature of the water in the calorimeter.
T' = the final temperature of the water.
Heat lost by the soil = $Sm(t - T')$
Heat gained by the water = $M(T' - T)$
Heat gained by the calorimeter = $M'S'(T' - T)$
Total heat gained = total heat lost.

$$Sm(t - T') = M(T' - T) + M'S'(T' - T)$$
$$S = \frac{M(T' - T) + M'S'(T' - T)}{m(t - T')}$$

Example.—In a set of observation on the specific heat of a soil the following data were taken. Calculate the specific heat of the soil.

Mass of soil = 200 g.
Mass of water = 150 g.
Mass of calorimeter = 100 g.
Temperature of soil in heater = 100°C.
Temperature of water in calorimeter = 15°C.
Final temperature of the water = 35°C.
Specific heat of calorimeter = 0.09

$$S = \frac{150(35 - 15) + 100 \times 0.09(35 - 15)}{200(100 - 35)} = \frac{3{,}180}{13{,}000}$$
$$= 0.24 \text{ calorie} = \text{specific heat of soil.}$$

Problems

1. One hundred grams of water at 80°C. are thoroughly mixed with 500 g. of milk at 0°C. What is the temperature of the mixture? Specific heat of the milk 1.08.

2. If, for every pound of coal burned under a steam boiler, 15 pounds of gaseous products go up the flue at a temperature of 700°F., find the energy which is lost per pound of coal. The temperature of the boiler room is

75°C., and the specific heat of the gaseous products may be taken as 0.25 B.t.u.

3. In burning coke, steam is sometimes fed into the fire along with the air. How much additional energy is taken away from the fire by a pound of steam if it escapes up the flue at a temperature of 650°F.?

Take the specific heat of steam as = 0.48.

4. How many B.t.u. are required to raise the temperature of one ton of water from 40°F. to 280°F.?

5. An aluminum block weighing 150 g. at a temperature of 100°C. is dropped into a copper beaker weighing 250 g. The beaker contained 450 g. of water at 10°C. The resulting temperature is 16.5°C. If the specific heat of copper is 0.094, what is the specific heat of aluminum?

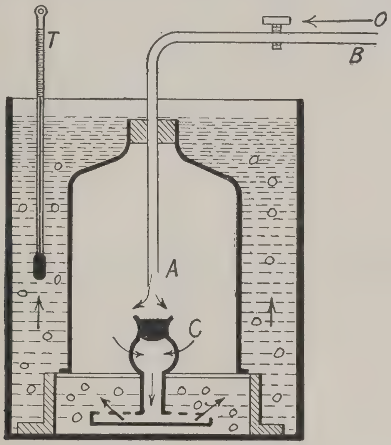


FIG. 140.—Calorimeter for heat of combustion of fuel.

6. The sun sends to the earth 2 calories per minute for each sq. cm. of the earth surface. Assuming that all this heat remains in a layer of soil 2 cm. deep and that the temperature of the soil is the same throughout the thickness, find the amount the temperature of a plot containing 100 sq. m. will be raised in 10 hours. Take the specific heat of the soil = 0.22 calories.

196. Heat Values of Fuels.—It is often important to know the quantity of heat given off when a known amount of a certain fuel is burned. A method of determining this heat is indicated in Fig. 140. A known mass of fuel is placed in the small crucible C, which is inside a bell jar. This jar is fastened down so that no air can get into it, and so the products of combustion can not escape except through the opening at the base of the bell jar. The bell jar is placed inside a vessel, the mass of which is known and this vessel then filled with a known weight of water. The temperature of the water is determined, and then a supply of oxygen is admitted through the opening at the top of the bell jar until all of the fuel has been burned. The products of combustion bubble up through the water. When the combustion is complete, the temperature of the water is again observed. From these data the heat of combustion of the fuel can be found.

$$\text{Heat of combustion} = \frac{\text{mass of water} \times \text{change in temp.}}{\text{mass of fuel}}$$

Example.—A sample of coal weighing 0.15 lb. was burned in the crucible in Fig. 140. The water weighed 100 lb., and its temperature at the beginning was 60°F.; the temperature at the end was 80°F. Find the heat of combustion of the coal. Neglect the heat given up to the vessel which forms the calorimeter.

$$\text{Heat of combustion per lb.} = \frac{\text{mass of water} \times \text{change in temp.}}{\text{mass of coal}}$$

$$= \frac{100 \times (80 - 60)}{0.15} = \frac{2{,}000}{0.15} = 13{,}300 \text{ B.t.u.}$$

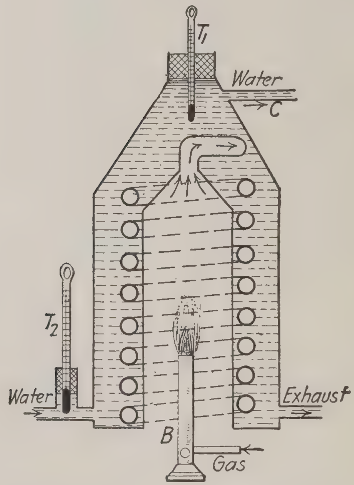


FIG. 141.—Calorimeter for heat of combustion of gases.

197. Continuous Flow Calorimeter.—Another method of measuring the heat of combustion of a fuel which can be burned as a gas is shown in Fig. 141. The gas to be studied is burned in a burner *B*, which is inside the calorimeter. The products of combustion pass out through a series of pipes. Around these pipes there is a continuous flow of water. This water enters the calorimeter at a constant temperature which is determined by the thermometer T_2. The water leaves by the outlet C. Its temperature is determined just before it leaves the calorimeter. The temperature of the water as it leaves is of course higher than it is when it enters. By weighing the water and measuring the quantity of gas which has been burned, it is possible to calculate the quantity of heat generated per cu. ft. or per liter of gas. If the rate of generation of heat is constant, the difference between the temperature of the ingoing and outgoing water will remain constant.

Let Q = number of heat units liberated.
V = the volume of the gas burned.
W = weight of water which flowed through the calorimeter.
T_2 = temperature of ingoing water.
T_1 = temperature of outgoing water.
S = specific heat of water.
$H = WS(T_1 - T_2)$.

$$\text{Heat per unit volume} = \frac{Q}{V} = \frac{WS(T_1 - T_2)}{V}$$

Example.—In determining the heat of combustion of natural gas it was found that 2,400 grams of water flowed through the calorimeter while 3 liters of gas were being burned. The temperature of the ingoing water was 20.0°C. and that of the outgoing water was 30.0°C. Find the number of calories generated by the combustion of 1 liter.

$$\text{Heat of combustion per unit volume} = \frac{\text{mass of water} \times \text{change of temp.}}{\text{volume of gas}}$$

$$\text{H} = \frac{2{,}400\ (30.0 - 20.0)}{3} = 8{,}000 \text{ calories per liter}$$

Problems

1. A sample of coal weighing 0.1 lb. and having a heat of combustion of 15,000 B.t.u. was burned in the crucible of the calorimeter of Fig. 140. The water in the vessel had a mass of 15 lb., and its temperature at the beginning was 60°F. The weight of the glass vessels amounted to 5.7, and the specific heat of the glass was 0.19. To what temperature was the water raised?

2. A sample of benzine weighing 9 g. was burned in a fuel calorimeter containing 18.5 lb. of water. The water equivalent of the calorimeter was 0.8 lb. The initial temperature of the water was 72°F. and the final temperature 92.0°F. Find the number of B.t.u. per pound of benzine.

3. A piece of coal weighing 1 gram was burned in a fuel calorimeter with a water equivalent of 0.5 lb. The mass of water in the calorimeter was 2.5 lb. The initial temperature of the water was 68.0°F. and the final temperature was 82.0°F. Find the heat of combustion of the coal in B.t.u. per pound.

4. A specimen of coal weighing 5 grams was burned in a copper calorimeter having a mass of 900 g. The calorimeter contained 5,500 grams of water at 22.0°C. The final temperature of the water was 30.0°C. Find the heat of combustion in calories per gram.

5. A quantity of gas, which, when measured at atmospheric pressure and at 20°C., had a volume of 1.25 cu. ft., was burned in a continuous flow calorimeter. During the test, 8 lb. of water flowed through the calorimeter. The temperature of the ingoing water was 62.0°F. and that of the outgoing water was 92.0°F. The products of combustion carried away 4.5 B.t.u. Find the heat of combustion per cu. ft.

6. In testing a specimen of gasoline, it was found that when 9 grams were burned in a fuel calorimeter containing 15 pounds of water, the temperature of the water in the calorimeter rose from 65°F. to 85°F. If the water equivalent of the calorimeter is 0.71 lb., how many B.t.u. are obtained from each pound of gasoline.?

7. One-tenth of a pound of coal was burned in a fuel calorimeter containing 80 lb. of water. The original temperature of the water was 66.5°F and the final temperature was 82.5°F. If the water equivalent of the calorimeter was 1.25 lb., how many B.t.u. are developed by each pound of coal?

8. When 1.5 cu. ft. of gas measured under standard conditions was burned in a fuel calorimeter containing 9.5 lb. of water, the temperature of the water rose from 65.5°F. to 90.0°. If the calorimeter had a water equivalent of 0.75 lb., and if the products of combustion and radiation carried off 3.5 B.t.u., how many B.t.u. were generated by the combustion of one cu. ft.?

CHAPTER XVI

EXPANSION BY HEAT

198. Expansion of Solids.—It is a familiar fact to any one who has seen a blacksmith put a tire on a wagon wheel, that the tires are made smaller than the wheel they are to fit. They are then heated and become large enough to be driven on. On cooling they shrink and grip the wheel. When a railroad track is laid, it is customary to leave a gap between the rails to allow for the expansion of the rails in the summer. The expansion of solids may be demonstrated in the following manner. A brass ball is made so that it will just pass through a ring when both ring and ball are cold. When the ball is heated, it will not pass through the ring, but when it is again allowed to cool, it passes through as before. From this experiment it is seen that solids expand when heated and contract when cooled.

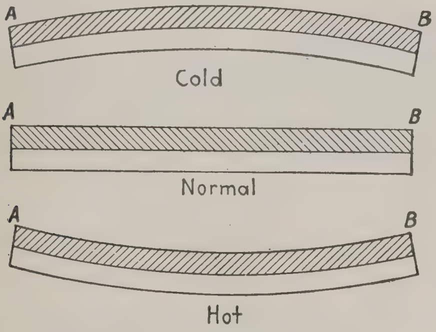


FIG. 142.—Unequal expansion of metals.

199. Unequal Expansion of Solids.—Careful experiments show that different substances expand at different rates. The unequal expansion of two metals such as iron and brass is shown in the following way. Rivet two strips of these metals together (Fig. 142) so as to form a composite bar. If now the bar is heated, it will be found to bend because of the unequal expansion of the metals. The brass expands more rapidly than the steel, so that the brass will be on the outside of the curve which the bar makes. The iron, expanding less rapidly than the brass will be on the inside. The bending of a bar which arises out of this equal expansion is of importance to the regulation of temperatures. In an incubator it is desired to keep the temperature as nearly constant as possible. When the temperature of the incubator changes, one of these compound bars will bend and may (Fig. 143), on account of the motion thus resulting, be made to control the supply of heat.

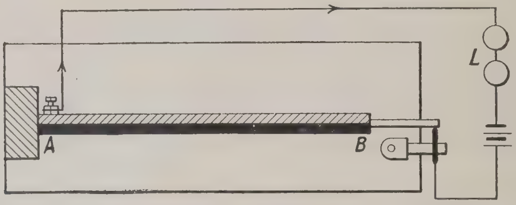


FIG. 143.—Temperature regulator of a thermostat.

200. Measurement of Expansion.—For many purposes it is necessary to know how much a substance expands when the temperature is raised one degree.

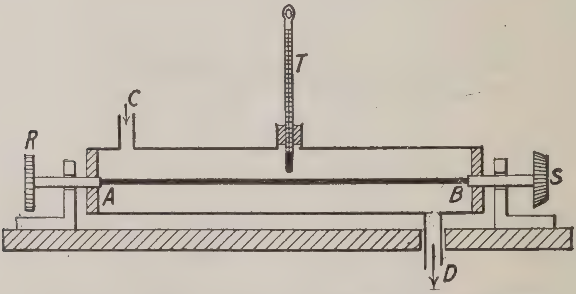


FIG. 144.—Measurement of coefficient of linear expansion.

To determine the expansion of a rod AB, insert the rod (Fig. 144) in a jacket which can be filled with steam or water as desired. The rod is fixed between two screws. One of these screws, R, is an adjusting screw; the other S is a micrometer screw. Adjust the micrometer screw, S, to read zero and then tighten up the screw R when water is flowing through the jacket. Read the temperature indicated by the thermometer T and replace the water in the jacket by steam from a boiler. Turn the micrometer screw until it is again tight. The amount the micrometer screw has been turned gives the increase in the length of the rod. Note the temperature of the water and measure the original length of the rod. Divide the increase in length by the original length. This gives the increase in length per unit length. Now divide the increase in length per unit length by the change in temperature. This gives the increase in length per unit length per degree change in temperature.

$$e = kl(t_1 - t_2).$$

where e = change in length.
k = the coefficient of expansion.
l = original length.
t_1 = highest temperature.
t_2 = lowest temperature.

or $$k = \frac{e}{l(t_1 - t_2)}$$

The factor k is the coefficient of expansion. **By it is meant the increase in unit length of the bar for one degree rise in temperature.** This quantity is a small fraction which varies from substance to substance. It is independent of the units in which the length is measured, but depends on the units in which the temperature is measured. For the Centigrade scale, it will have one value, and for the Fahrenheit scale, the value will be only 5/9 as large.

Example.—A telephone wire is one mile long when the temperature is 0°C. How much would its length increase by raising the temperature to 35°C.? Assume the coefficient of linear expansion of iron to be 0.000012 per degree Centigrade.

Change in length = original length × change in temperature × coefficient of expansion

Change in length in feet = 5,280 × 35 × 0.000012 = 2.22 ft.

Problems

1. What will be the increase in length in a 250 ft. span of a steel bridge when the temperature changes from 110°F. in summer to −10°F. in winter?

2. A sheet of aluminum has an area of 15 sq. ft. at 25°C. What is its area when the temperature is 40°C.? Coefficient of expansion of aluminum = 0.000023.

3. A tape used for measuring is made of steel and is correct at 15°C. If a farmer uses this tape to measure the area of a field and finds that the field contains 5 acres when measured on a day when the temperature was 5°C. what is the true area of the field?

4. A ball of cast iron has a volume of 250 cu. cm. at 10°C. Find its volume when heated to 350°C.

5. A certain rod is 40 in. long at 0°C. and 40.055 in. long at 55°C. What is the coefficient of expansion of the rod?

6. The driving wheel of a tractor is 6 ft. in diameter at 0°C. How much farther will it go per revolution at 25°C. than at 0°C.?

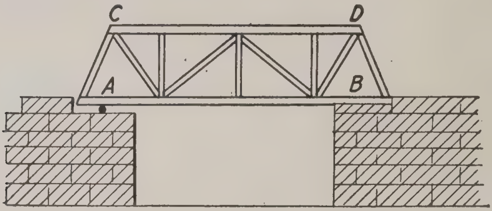


Fig. 145.—Expansion of a bridge.

7. A platinum rod and a brass rod each measure 1 meter at 0°C. They are heated until the brass rod is 2 mm. longer than the platinum rod. What is the temperature?

Coefficient of expansion of brass = 0.0000189

Coefficient of expansion of platinum = 0.0000089

8. How much longer must an iron rod be than one made of copper in order that each may expand the same amount when the temperature is raised 1°C.?

Coefficient of expansion of iron = 0.000012

Coefficient of expansion of copper = 0.0000167

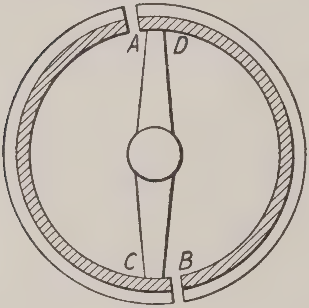


Fig. 146.—Balance wheel of a watch.

Illustrations of Expansion.—In the construction of an iron bridge (Fig. 145) provision is made for the expansion of the iron. To make it possible for the bridge to expand, one end of the bridge is mounted on a roller. As the length of the bridge increases or decreases, the end of the bridge moves forward or back without injuring the piers.

A wire fence on a farm is made of wires which are not straight but bent back and forth in kinks so that as the temperature of the fence increases, the wire is not stretched, but these bends in the wire take care of the increase or decrease in length.

In a watch, the balance wheel (Fig. 146) is made of two metals which are fastened together. As the temperature of the wheel increases, the end *B* and the end *A* are carried in, making the effective radius of the wheel less. The expansion of the radius tends to carry the rim of the wheel farther away from the center. At higher temperatures, the elasticity of the hair-spring is less, and this makes the wheel move more slowly. As the free ends of the wheel move in, the wheel vibrates more rapidly. These two tendencies, one making the wheel go more slowly and the other making it go more rapidly, are made to compensate each other so that the period of vibration remains the same at all temperatures.

201. Expansion of Liquids.—To show that liquids expand when heated fill a flask with kerosene or with water. Insert in it a stopper through which passes a glass tube. Allow the liquid to rise a small distance in the tube. Now insert the flask in a vessel filled with hot water. At once the liquid begins to expand, and the surface of the liquid in the tube rises. If two such flasks are put in the vessel side by side, and if one of these flasks is filled with water and the other filled with alcohol, it will be found that the alcohol expands much more than the water for the same rise in temperature. Kerosene and most oils expand more rapidly than water. In a hot water heating system, an expansion tank (Fig. 147) is provided to take care of the increase in the volume of the water when the furnace is heated.

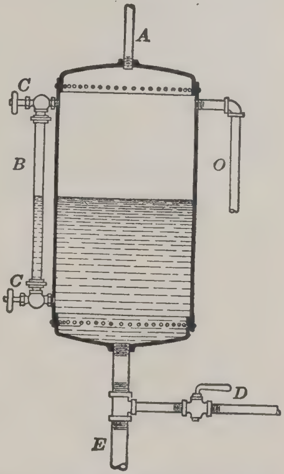


Fig. 147.—Expansion tank for hot-water system.

In the case of solids, we are usually concerned with the change in length for a given rise in temperature. In liquids and gases, on the other hand, we are concerned with the change in volume per unit change in temperature. **The coefficient of cubical or volume expansion may be defined as the change in volume per unit volume per degree change in temperature.** It may also be expressed as follows:

Let v = change in volume.
V = original volume.
t_1 = initial temperature.
t_2 = final temperature.
a = the coefficient of volume expansion.

then, $a = \dfrac{v}{V(t_2 - t_1)}$

Example.—A barrel which holds 30 gallons is completely filled with kerosene when the temperature is 0°C. How much kerosene will overflow when the temperature is raised to 75°C.?

Coefficient of expansion of kerosene = 0.00085.

Change in volume = original vol. × change in temp. × coefficient of expansion

= 30 gallons × 75 × 0.00085

= 1.9 gallons

Problems

1. A liter flask filled with mercury at 0°C. is heated to 100°C. The flask is made of glass with a coefficient of expansion small enough to be neglected. How much mercury runs out? Coefficient of cubical expansion of mercury = 0.000181.

2. A fruit jar having a capacity of 1 liter was filled with water at 100°C. How large an empty space will be left when the water cools to 20°C.? Coefficient of expansion of water = 0.000485.

3. A quart bottle filled with milk is pasteurized at 65°. How much is the empty space in the bottle when the milk has cooled to 20°? Coefficient of cubical expansion of milk = 0.00038.

4. A silver dish has a capacity of 1.0025 liters at 150°C. At what temperature will its capacity be 1 liter? Coefficient of cubical expansion of silver = 0.0000564.

202. Maximum Density of Water.—It has been seen that liquids like alcohol and kerosene expand whenever heated. The expansion of water at temperatures in the neighborhood of 4° is very peculiar. Fill a large tube with water, as in the preceding experiment, and then place it in a vessel filled with cracked ice. When the temperature has become constant, note the position of the liquid in the glass tube. On removing the flask from the vessel of ice water and allowing it to warm gradually, it will be noticed that the liquid in the glass tube first descends and then rises on further heating. This means that the water on being heated above 0°C., at first contracts and then expands. Careful experiments show that water has its least volume at 4°C. When it is warmed above this temperature or cooled below this temperature, it increases in volume. As the volume of the liquid increases, its density decreases; and as the volume of the liquid decreases, its density increases. Hence, where the volume of the liquid has its least value, the density has its greatest value. The facts with respect to water may therefore be expressed by saying that the volume of the liquid is a minimum at 4°C (Fig. 148) and the density is a maximum.

203. Unequal Expansion of Substances.—The unequal expansion of various substances when heated is a matter of much importance in the formation of soils. If the sun shines on rocks made up of minerals containing different kinds of crystals which do not expand at the same rate, the unequal expansion or contraction of these crystals tends to loosen them. In this way the rock breaks up into small fragments which by and by form soil.

204. Expansion of Gases.—Gases as well as liquids and solids expand when heated and contract when cooled. If a spherical bulb (Fig. 149) has a small glass tube with its end immersed in a beaker of water it will be found that on heating the bulb, air bubbles out of the beaker and as the bulb is allowed to cool the liquid rises in the tube. From this experiment it is seen that the volume of the gas increases as the temperature rises and decreases as the temperature falls. If the pressure is kept constant, the whole volume of the air occupies more space than it did when it was cold. Consequently, hot air weighs less per cubic inch or per cubic centimeter than cold air. That is, the density of air or any other gas decreases as the temperature rises.

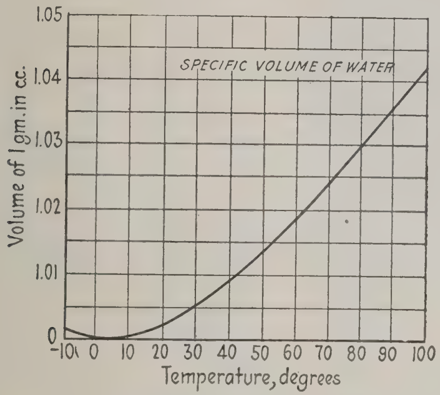


FIG. 148.—Specific volume of water.

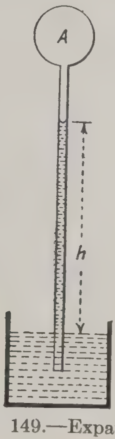


FIG. 149.—Expansion of gases.

205. Rise of Smoke in a Chimney.—When the fire is lighted in a fireplace, the air in the neighborhood of the fire is immediately warmed. (Fig. 150.) This causes its density to decrease due to the expansion of the air. Since the density of the warm air is less than that of the cold air nearer the top of the chimmey, the heated air rises and the cold air descends to take its place. The heated air in this case behaves like a light gas in a balloon. It weighs less than the cold air which it displaces. The hot air is consequently pushed up by the cold air and an upward circulation of the air takes place. After a short time the entire chimney is filled with hot air, and this entire volume of hot air is pushed up just as a body immersed in a liquid is lifted by the liquid which it displaces. Since a tall chimney has a larger volume of hot air than a short chimney has, the lifting effect in the tall chimney is greater than in the short chimney. Hence tall chimneys will draw better than low ones.

206. Expansion of Gases in Cooking.—In making bread, the yeast gives off carbon dioxide. This gas is liberated all through the dough, and there is formed in the dough thousands of small closed pockets filled with gas. As more gas is liberated, these pockets grow and are filled with more and more gas. This process causes the bread to rise. When the dough is placed in the oven, the yeast ceases to produce the carbon dioxide, but the dough continues to rise for the reason that the heat expands the carbon dioxide.

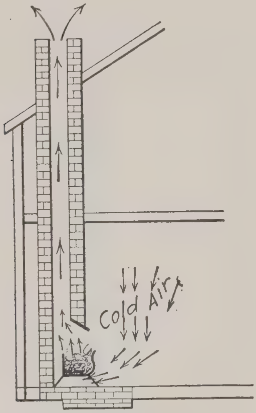


Fig. 150.—Rise of smoke in a chimney.

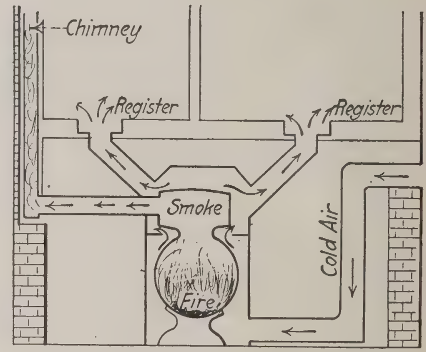


Fig. 151.—Hot-air heating system.

When eggs are beaten, small bubbles of air are included. As the eggs are being heated, these bubbles of air expand, and the volume of the egg increases, thus making the omelet rise. The eggs should be beaten in a cool place for the reason that they are more rigid and thus able to include more air, and for the further reason that the colder the air, the greater will be the change in temperature on heating and the greater the increase in volume.

207. Hot-air Heating System.—A hot-air heating system (Fig. 151) depends for its operation on the unequal heating of the air in the furnace and the expansion of the air resulting from this heating. The air directly over the fire box becomes heated and expands. On account of the decrease of density due to this cause, it rises and passes out through the registers into the rooms. Cold air from the outside enters through the cold air duct and is in its turn heated by the furnace. The process thus continues, giving a continuous supply of hot air to the rooms.

208. Ventilation.—Except where forced circulation is provided, most buildings depend for ventilation on the unequal heating of the air. The open fire place in Fig. 150 calls attention to the circulation of air resulting from the warming of the air in the chimney. The air to replace that which rises up the chimney comes from the room, and air from out doors comes in to take the place of that which has gone up the chimney.

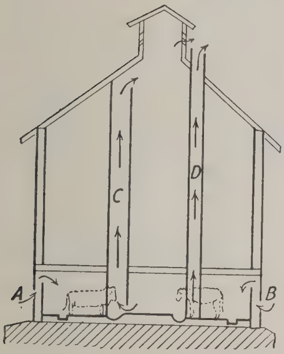


Fig. 152.—Ventilation of barns.

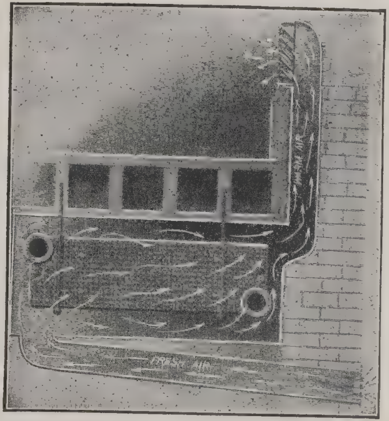


Fig. 153.—Ventilation by indirect heating.

The expansion of air when it is heated is utilized in the ventilation of barns as shown in Fig. 152. Warm air from the animal rises through C and D and passes up the ducts and out at the top of the barn. Colder air enters through A and B to replace the air which has risen. The animals in the barn are thus furnished with a continuous supply of fresh air. Fig. 153 shows a case of ventilation by indirect heating.

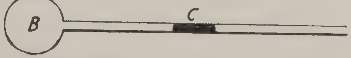


Fig. 154.—Expansion of gases at constant pressure.

209. Charles' Law.—It has been seen that gases, like solids and liquids, expand when the temperature is increased. In order to determine the rate of this expansion with accuracy, it is necessary to specify the way in which the gas is allowed to expand. Suppose that it is agreed that during the expansion the pressure on the gas will be kept unchanged.. It is then said to expand at constant pressure (Fig. 154). The coefficient of expansion can now be defined as it was defined for liquids.

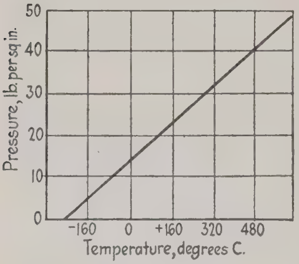


FIG. 155.—Relation between volume and temperature of a gas.

Let V_0 = original volume at 0°C.
V = volume at t°C.
t = temperature on Centigrade scale.
b = coefficient of cubical expansion.

then, $$b = \frac{V - V_0}{V_0 t}$$

or $$V = V_0(1 + bt).$$

The relation between the volume of a gas at constant pressure and its temperature is shown in Fig. 155.

By careful experiments Charles found that the coefficient of cubical expansion of all gases at constant pressure is the same and equal to 0.00367 or 1/273. This is known as Charles' law. It may be expressed as,

$V = V_0(1 + t/273)$ where t is measured on Centigrade scale.

Example.—The volume of a certain mass of gas at 0°C. is 350 cu. cm. Its temperature is raised to 175°C. What is the new volume and what is the change in volume?

$V = V_0(1 + t/273)$
$V = 350(1 + 175/273) = 574.3$ cu. cm.
Change in volume $= 574.3 - 350 = 224.3$ cu. cm.

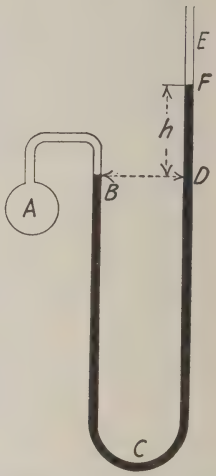


FIG. 156.—Heating a gas at constant volume.

210. Heating a Gas at Constant Volume.—A gas may also be heated in such a way that its volume remains constant. In this case the pressure exerted by the gas increases with rise of temperature. In order to show this, a bulb connected (Fig. 156) to a tube bent in the form of a U is immersed in a bath of water at room temperature. The mercury in the U tube is brought to a mark on the glass tube at B. The bulb is now heated, and mercury rises, showing that the pressure is increasing. If enough mercury is poured into the tube, or the right-hand side of the tube is elevated so that the mercury remains at B, the volume of the gas remains unchanged. The pressure increases and this rate of increase has been found to be constant for all gases. This is another form of Charles' law. In this form Charles' law states that, **if the volume of a gas is kept constant, the pressure increases 1/273 of its value at 0°C for every rise of 1°C in temperature.**

Let P_0 = pressure at 0°C.
P = pressure at t°C.
Then $P = P_0(1 + t/273)$.

Fig. 157 shows the relation between pressure and temperature of a gas at constant volume.

Example.—The pressure of a gas in a vessel was 75 cm. of mercury when the temperature was 0°C. After the gas was heated, the new pressure was found to be 125 cm. of mercury. If the volume remained constant during the heating, what was the new temperature?

$$P = P_0(1 + t/273)$$
$$125 = 75(1 + t/273)$$
$$t = \frac{125 - 75}{75} \times 273 = \frac{50}{75} \times 273 = 182°\text{C}.$$

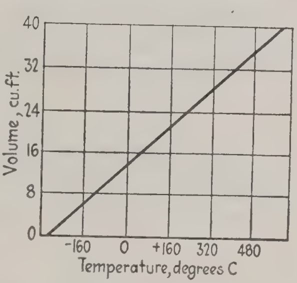


FIG. 157.—Relation between pressure and temperature of a gas.

211. Absolute Scale of Temperature.—In the study of the heating of a gas at constant volume, it was found that the pressure of a gas always increased 1/273 of its value for each degree rise in temperature above 0°C. If the pressure of the gas is measured at 0°C., and this rate of decrease in pressure continues as the temperature is gradually lowered, it is clear that when the temperature has become 273° below 0°C., the pressure will disappear and the gas will no longer be able to exert a pressure. From the standpoint of the kinetic theory of gases, a gas must always exert a pressure as long as the molecules are in motion. Hence, the temperature at which the molecules ceases to exert a pressure is the temperature at which these molecules cease to move. The temperature at which the molecules have no motion is known as the **absolute zero.** It is 273°C. below the freezing point of water. It is often convenient to measure temperatures from this zero instead of from the zero on the Centigrade scale. Temperatures on this scale are called **absolute temperatures** to distinguish them from those measured on the Centigrade scale. To find the absolute temperature in Centigrade degrees, 273° is added to the reading on the Centigrade scale. If we indicate temperatures on the Centigrade scale by t and those on the absolute scale by T,

$$T = t + 273$$

Example.—The reading on a Centigrade thermometer is 57°. What is the absolute temperature?

$$\begin{aligned} \text{Absolute temperature} &= \text{reading on Centigrade scale} + 273° \\ T &= t + 273 \\ &= 57 + 273 = 330° \text{ absolute.} \end{aligned}$$

212. The General Gas Law.—In Boyle's law it is assumed that the temperature of a gas remains unchanged while the pressure and the volume are varied. This law can therefore be applied only when the temperature is fixed. In that case

$$PV = \text{constant}$$

On the other hand, Charles' law assumes that the pressure of the gas does not change while the temperature and volume of the gas vary. In this case

$$V = V_0(1 + t/273) = \frac{V_0(273 + t)}{273} = \frac{V_0T}{273}$$

At another temperature

$$V' = V_0(1 + t'/273) = V_0T'/273$$

Hence $\frac{V'}{V} = \frac{T'}{T}$; i.e., the volumes are proportional to the absolute temperatures at constant pressure. If the volume remains constant, Charles' law states that

$$.P = P_0(1 + t/273) = \frac{P_0T'}{273}$$

$$\text{and } P' = P_0(1 + \frac{t'}{273}) = \frac{P_0T'}{273}$$

Hence $\frac{P'}{P} = \frac{T'}{T}$, i.e., the pressures are proportional to the absolute temperatures when the volume remains constant.

It would often be more convenient to have a law in which these restrictions with respect to temperature, pressure, and volume were removed. By combining Boyle's law and Charles' law these restrictions can be removed and the **general gas law** obtained. The general gas law expresses the relation between the volume of the gas, its temperature, and its pressure.

Let a given mass of gas having a volume V_0 and a pressure P_0 at 0°C. (Fig. 158) be heated until the temperature is t°C. Its new volume is

$$V_1 = V_0\left(1 + \frac{t}{273}\right) = \frac{V_0 T}{273}.$$

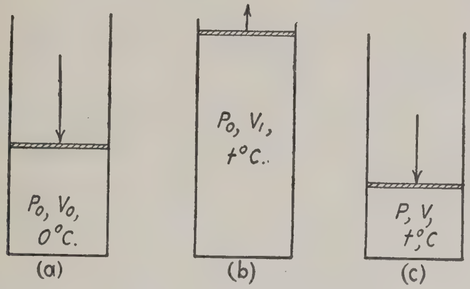


FIG. 158.—Combination of Boyle's and Charles' laws.

Keeping the temperature constant, compress this gas until its volume is V and its pressure is P. Then by Boyle's Law,

$$PV = V_1 P_0$$

Since $$V_1 = \frac{V_0 T}{273},$$

we have $$PV = \frac{V_0 P_0}{273}\, T$$

Let $$\frac{P_0 V_0}{273} = R = \text{Constant},$$

$$PV = RT,$$

$$\frac{PV}{T} = R = \text{Constant},$$

and $$\frac{P'V'}{T'} = \frac{PV}{T}$$

Example.—The volume of a gas at 25°C. atmospheric pressure is 1 liter. What is its volume when the pressure is 2 atmospheres and the temperature is 300°C.?

$$\frac{\text{Original pressure} \times \text{original volume}}{\text{Original absolute temperature}} = \frac{\text{final pressure} \times \text{final volume}}{\text{final absolute temperature}}$$

$$\frac{PV}{T} = \frac{P'V'}{T'}$$

$$\frac{1 \times 1}{298} = \frac{2 \times V'}{573}$$

$$V' = \frac{573}{298 \times 2} = \frac{573}{596} = .963 \text{ liters}$$

Problems

1. A gas at 0°C. and 76 cm. pressure has a volume of 350 cu. cm. Find its volume, when the temperature is −20°C. and the pressure is 74 cm.

2. A gas enclosed in a vessel is compressed to one-half of its original volume and the temperature is raised from 10° to 100°C. If the original pressure was 75 cm., what is the final pressure?

3. A certain volume of gas is enclosed in a vessel at 75 cm. and at a temperature of −35°C. The gas is then heated to 250°C. What is the pressure on the walls of the vessel?

4. The density of the air at 0°C. and under a pressure of 76 cm. is 0.00129 grams per cu. cm. What will be the mass of the air in a room which is 6 m. long, 5 m. wide and 3 m. high, when the temperature is 25°C. and the barometer reads 74 cm.?

5. A student in a chemical laboratory generates 40 liters of oxygen at 20°C. and at a pressure of 40 cm. Find the volume of this gas when the temperature is 0°C. and the pressure is 76 cm.

6. The cylinder of an automobile contains 38 cu. in. of air and gasoline vapor at atmospheric pressure at the beginning of the compression stroke. At the end of the stroke the volume is 5 cu. in. and the pressure is 15 atmospheres. If the temperature at the beginning of the stroke was 35°C., what is the temperature at the end?

7. A certain quantity of hydrogen occupies 450 cu. cm. at 25°C. under a pressure of 1 atmosphere. If the volume is allowed to increase until it is 600 cu. cm. to what temperature must the gas be raised in order to exert a pressure of 2 atmospheres?

8. An automobile tire is pumped up to 80 lb. per sq. in. at a temperature of 40°C. If the tire bursts when the pressure becomes 100 lb. per square inch, how hot may it be heated before bursting?

9. A gas engine having a clearance space equal to one-fourth the volume of the cylinder draws in a charge of gas at a pressure of 15 lb. per sq. in. and at a temperature of 30°C. When the charge is compressed into the clearance space, the pressure is 175 lb. What is the temperature?

CHAPTER XVII

FUSION

213. The Melting Point.—If a vessel of ice or snow is heated, the temperature at first rises until it is 0°C. and then remains stationary until all the ice is melted. After all the ice has been melted, the temperature of the water begins to rise. That temperature at which the solid changes into a liquid without a change of temperature is called the **melting point.** For ice this temperature is 0°C. and 32°F. At the melting point the addition of heat simply serves to hasten the melting process without any change of temperature.

If a pail of water is placed in a freezing mixture of ice and snow, the temperature of the water decreases until ice begins to be formed in the pail. After this temperature has been reached, the temperature of the water in the pail remains the same until all the water has become ice. That temperature at which the liquid changes into the solid state is its **freezing point.** The temperature at which the liquid freezes is ordinarily the same as the temperature at which the solid melts. For crystalline substances like ice or copper the freezing point or the melting point is sharply defined. For substances which are not crystalline like wax or glass, etc., the substance gradually softens in passing from the solid to the liquid state. Such substances do not have a definite melting point. In the cases of certain fats, the melting point is not the same as the freezing point. For example, butter melts between 28° and 32°C. and solidifies between 20° and 23°C.

214. Heat of Fusion.—In order to cause a solid like ice to change into a liquid, it is necessary to supply a given quantity of heat to each gram or each pound of it. This is true though the temperature of the ice at the beginning is the same as the temperature of the water at the end of the process. The ice has a crystalline structure, and the heat which is supplied is necessary to tear down this structure. Evidently the water in changing back to ice or in general the liquid in changing back to a solid will give up the heat which it absorbed in melting. **The heat of**

fusion of a substance is defined to be the number of calories necessary to convert one gram at the melting point into liquid at the same temperature. It may also be defined as the number of B.t.u. which must be supplied to change one pound of the solid to liquid without a change of temperature. To change 1 gram of ice to water at 0°C. requires 79.25 calories, and to convert one pound of ice to water at 32°F. requires 144 B.t.u.

The heat of fusion of a substance can easily be found by the method of mixtures.

Let M = number of grams of substance melted
t = melting point of the substance
t' = temperature of the calorimeter at the beginning
T = temperature of the calorimeter at the end
m' = initial mass of water in the calorimeter
m = the mass of the calorimeter
s' = the specific heat of the calorimeter
s = the specific heat of the substance in the liquid state
L = the latent heat of fusion

Heat gained by the water = $m'(T - t')$
Heat gained by the calorimeter = $ms'(T - t')$
Heat required to melt the substance = ML
Heat required to change the substance from the melting point to the final temperature $Ms(T - t)$
Hence:
Heat gained = heat lost

$$ML + Ms(T - t) = m'(T - t') + ms'(T - t')$$

In the case of ice this reduces to,

$$ML + M(T - 0) = m'(T - t') + ms'(T - t')$$

From this equation L can be found.

Example.—In making observations on the heat of fusion of ice, the following observations were taken:

Mass of ice melted = 50 grams
Initial temperature of the water in the calorimeter = 18.0°C.
Final temperature of water in calorimeter = 30.0°C.
Initial mass of water in calorimeter = 200 gram

Neglecting the heat given to the calorimeter, find the heat of fusion of the ice.

Heat given to the water = mass of water × change in temp. of water

$$= 200 \times (30 - 18) = 2400$$

Heat necessary to melt the ice = mass of ice × heat of fusion

$$= 50 \times L$$

Heat necessary to raise temperature of water from 0 to final temp.
= mass of melted ice × change in temperature
= 50(30 – 0)
Heat gained = heat lost
Hence,

$$200(30 - 18) = 5L + 5(30 - 0).$$

$$L = \frac{200 \times 120 + 50 \times 30}{50} = 78 \text{ calories}$$

215. Supercooling.—If a pure liquid is carefully protected from mechanical disturbances, it may be cooled below the temperature at which it normally solidifies. Thus water may be cooled to – 10°C. or lower without becoming ice. The liquid at such a temperature is in a state of unstable equilibrium and will immediately solidify if it is disturbed or if a crystal of the solid is dropped into it. This phenomenon is known as **supercooling.**

216. Change of Volume during Freezing.—It is a familiar fact that ice floats and that pipes or bottles filled with water burst when they freeze. This shows that water expands when it freezes. One cubic foot of water makes about 1.09 cu. ft. of ice when it is frozen. This expansion on solidification is due to the fact that the molecules of water form crystals leaving larger spaces between them than were present when the water was in the liquid condition. Cast iron behaves like water in this respect. It also expands when it solidifies, and for this reason is suitable for making castings in which it is desired to reproduce the detail of the mould. Most metals and other substances contract on solidification. For this reason gold and silver coins are stamped instead of being cast.

217. Forces Exerted by Freezing Water.—That the forces exerted by freezing water are very large may be seen from the fact that they are sufficient to burst the strongest water pipes on cold nights. Some idea of the magnitude of these forces may be obtained by filling a cast iron bomb with water and then putting it in a pail of ice and salt. If a screw plug has been placed in the bomb and care is taken to fill the bomb completely with water, the solidification of the water bursts the bomb.

The forces exerted by water on freezing are of much importance in the formation of soils from rocks. The water percolates into the crevices in the rocks and then freezes. The expansion which occurs during freezing breaks off small or large fragments, which by this process after a time are made into soil. The alternate freezing and thawing of soils tends to pulverize them. Ice forming in the interstices of the soil serve to loosen compact land and give it better tilth.

218. Effect of Pressure on the Melting Point.—Since the application of pressure tends to keep a body from expanding, the expansion which takes place during the solidification of water is partially prevented by pressure. It is therefore more difficult for the water to freeze, and its solidification is somewhat delayed. It therefore becomes necessary to lower the temperature below the normal freezing point before solidification takes place. Careful experiments show that the melting point of ice is lowered 0.0075°C. for an increase of one atmosphere in the outside pressure.

If on the other hand a substance expands upon melting, its melting point will be raised by the application of pressure. The pressure always favors that state in which the volume is the least.

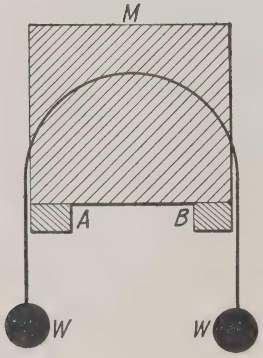


FIG. 159.—Effect of pressure on melting point.

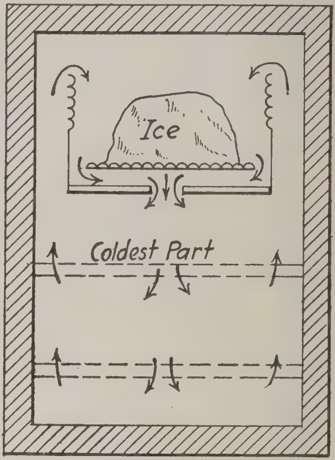


FIG. 160.—The refrigerator.

The effect of pressure on the melting point of ice may be shown by taking a piece of ice (Fig. 159) which is about 1.5 ft. long and 6 in. square and hanging over it a loop of wire from which a weight of 35 or 40 lb. is supported. The pressure of the wire on the ice causes the ice under it to melt and the wire cuts its way through. The water which is formed by the melting of the ice beneath the wire flows above the wire where it is again frozen because heat is taken from it in the melting of the ice beneath the wire. In order to melt the ice beneath the wire, it is necessary to supply to it 80 calories for each gram that is melted. This heat is taken from the water above the wire causing the water above the wire to freeze again. The wire thus cuts its way through the block of ice and leaves it as solid as it was at the beginning of the experiment. During the process there is a transfer of heat from the water above the wire to the ice below it.

219. The Refrigerator.—An important illustration of the absorption of heat in the melting of ice, is found in the ordinary refrigerator (Fig. 160). The ice which is placed in the upper part of the refrigerator, melts and thus absorbs heat from the air in contact with it. This air, being now heavier than the remainder of the air in the refrigerator sinks, through the openings in the bottom of the ice chamber and forces the warmer air upward. This warmer air is in turn cooled by the ice, and the process of circulation thus continues. The cool air in passing downward comes in contact with the food and takes heat from it. The temperature to which the air and the food in the refrigerator can be cooled in this way depends on the temperature of the outside air and the manner in which the inside of the refrigerator is insulated from the outside air. If the walls of the refrigerator were made of materials which would allow no heat to flow into the refrigerator, the temperature of the inside of the refrigerator might be reduced to 0°C. or 32°F. Some heat, however, always flows into the refrigerator and this amount is greater when the temperature of the outside air is high so that the temperature in the refrigerator is never as low as 0°C.

220. Freezing Point of Solutions.—When a solid like salt or sugar is dissolved in water, the freezing point of the solution is lower than the freezing point of water. This is true of any liquid in which some substance has been dissolved. The freezing point of the solution is always lower than that of the pure solvent. When such a solution begins to freeze, it is only the water or the solvent which freezes out. This makes the remainder of the solution more concentrated. This lowers its freezing point still further. This process continues until the solution becomes saturated. Then both the dissolved substance and the solvent freeze out in such a way that the concentration of the solution remains unchanged during solidification. For solutions which are not too concentrated, the lowering of the freezing point is proportional to the concentration of the dissolved substance.

221. Freezing Mixtures.—In making ice cream it is necessary to lower the temperature of the cream in the freezer somewhat below the temperature of melting ice. In order to produce such a temperature, it is customary to prepare a mixture of ice and salt. The usual proportion of salt to ice is one part of salt to three parts of ice or snow. This mixture has a lower freezing point than pure ice has. With this mixture it is possible to reach a temperature of −6°F. Other substances besides salt may be used for this purpose, but salt is the most convenient.

Problems

1. A mass of 400 g. of copper is heated in an oil bath and then placed on a mass of ice. It melts 100 g. of ice. What is the temperature of the copper?

2. A mass of iron weighing 200 g. is heated to 250°C. and then dropped on a cake of ice. If 75 g. of ice is melted, what is the specific heat of iron?

3. How much heat is necessary to heat up 8 lb. of ice from a temperature of 4°F. to its melting point, melt it and then heat the water to its boiling point? Specific heat of ice = 0.5 and heat of fusion of ice = 144 B.t.u.

4. How much water at 95°C. will have to be poured on a piece of ice weighing 1.5 kg. in order that the resulting temperature may be 24°C.?

5. Find the result of mixing 40 g. of ice at 0°C. with 80 g. of turpentine at 36°C. The specific heat of turpentine = 0.42 calorie.

6. In what proportions must ice at 32°F. be mixed with water at 180°F. to give water at 75°F.?

7. If the melting point of iron is 1,500°C. and its heat of fusion is 28 calories per gram, how much coal is necessary to melt 1 lb. of iron, the initial temperature of which is 30°C.? Assume that no heat is lost and that each pound of coal yields 12,000 B.t.u.

CHAPTER XVIII

VAPORIZATION

222. Evaporation.—In a preceding chapter attention was called to the fact that all liquids are made up of a large number of molecules which are held together by their mutual attractions. These molecules are in constant motion with varying velocities. When the liquid is heated, the velocities of the molecules are increased and the spaces between the molecules are increased.

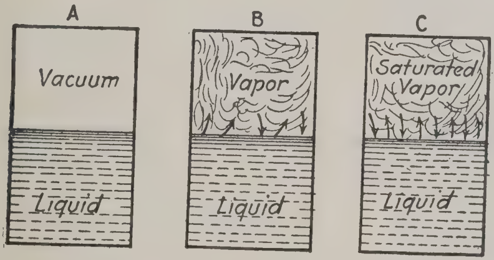


Fig. 161.—Evaporation of liquids.

Sooner or later, some of these molecules which are near the surface of a liquid move with sufficient velocity to escape from the surface (Fig. 161) and leave the neighboring molecules. Only the more rapidly moving molecules will escape, as the forces of attraction due to the other molecules will be sufficient to hold the more slowly moving molecules in the liquid. Those molecules which escape from the surface of the liquid move about in the space above the liquid. Since it is only the more rapidly moving molecules which escape from the surface, the average speed of the molecules in the liquid will decrease and the temperature of the liquid will drop during the evaporation.

223. Saturated Vapor.—If the space above the surface of the liquid is enclosed, some of these particles will again return to the surface of the liquid and be held fast. As more and more molecules escape from the surface of the liquid the number returning to the surface will also be increased. When the number of molecules in the space above the liquid has become so great that the number of molecules escaping from the surface of the liquid is just equal to the number returning to it, the space above the liquid will not gain more molecules at the expense of the liquid. The space above the liquid is then said to be saturated. The vapor which is above the liquid is called a **saturated vapor.** If the temperature of this vapor remains unchanged, it is not possible to increase the pressure on it. When an attempt is made to increase the pressure on a saturated vapor, the vapor condenses and changes into liquid. Fig. 162 shows that the vapor pressures of different liquids are not the same at a given temperature.

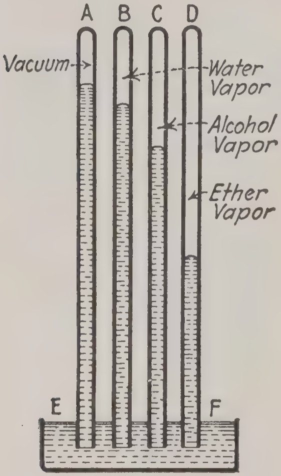


FIG. 162.—Vapor pressure of liquids.

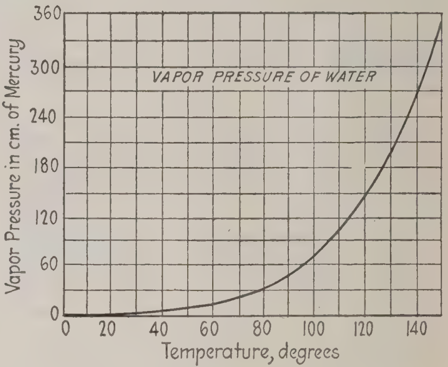


FIG. 163.—Vapor pressure curve for water.

224. Vapor Pressure Curve.—If a curve is plotted between temperatures as abcissae and the corresponding pressures of a saturated vapor, Fig. 163 is obtained. This curve shows the pressure of the vapor at each temperature. As the temperature of the liquid increases, the pressure of the saturated vapor also increases. In the case of water, when the temperature has become 100°, the vapor pressure is 76 cm. of mercury. For any lower temperature the pressure is less. Whenever the temperature of the liquid is known, the vapor pressure can be found from this curve. On the other hand, when the vapor pressure is known, the temperature at which the liquid has this pressure is obtained from this curve. Such a vapor pressure curve is obtained by observing the difference in level between the surfaces of mercury (Fig. 164) as the temperature is increased.

225. Heat of Vaporization.—Just as a certain amount of heat is required to convert a gram or a pound of ice into water without changing its temperature, so a certain amount of heat is required to change one gram or one pound of water into steam without changing the temperature. When the steam condenses, this heat is again liberated and becomes available for heating other bodies. The quantity of heat required to cause the liquid to become a vapor depends on the temperature at which the change takes place. The heat of vaporization is therefore said to vary with the temperature. The higher the temperature, the easier it is to cause the liquid to become a vapor. That is, the higher the temperature, the less the heat of vaporization.

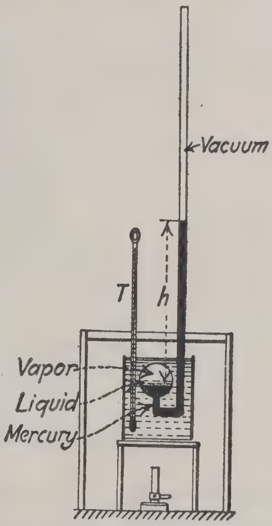


FIG. 164.—Measurement of vapor pressure.

The heat of vaporization is defined to be the heat necessary to change one gram of the substance from the liquid to the vapor state without any change in the temperature. For water at 100°C. it is found to be 538.0 calories. That is to say, it takes more than five times as much heat to change a gram of water at 100°C. into steam at 100°C. as it takes to heat that same gram of water from 0° to 100°C. For other temperatures the heat of vaporization of water is given by the equation,

$$H = 596.6 - 0.601t$$

where t is the temperature on the Centigrade scale.

To determine the heat of vaporization of water, take a calorimeter C (Fig. 165) filled with a known weight of water and a boiler B partly filled with water. Pass dry steam from the boiler into the calorimeter. In order to prevent water from being carried over, the steam passes through the trap D which catches the condensed steam. Note the temperature of the water in the calorimeter at the beginning and its temperature after a known quantity of steam has been condensed in it. The amount of condensed steam is found by weighing the calorimeter after the steam has been condensing in it for a sufficient time to produce the desired rise in temperature. The latent heat of vaporization of steam can now be calculated.

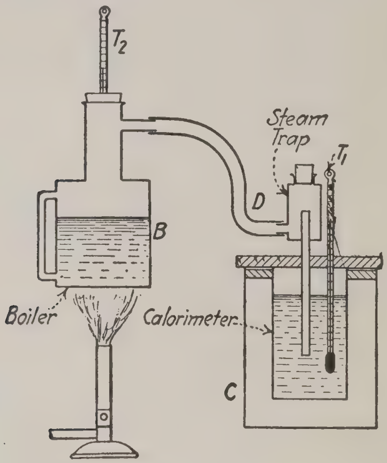


FIG. 165.—Heat of vaporization.

Let $L =$ the heat of vaporization.
$t =$ temperature of the steam.
$t' =$ the initial temperature of the water in the calorimeter.
$T =$ the final temperature of the water in the calorimeter.
$m =$ the mass of the steam condensed.
$M =$ the mass of the water in the calorimeter.
$C =$ the thermal capacity of the calorimeter.

Heat liberated by condensation of steam $= mL$.
Heat liberated by cooling water from temp. t to $T = m(t - T)$.
Heat given to calorimeter $= C(T - t')$.
Heat given to water $= M(T - t')$.
Heat gained = heat lost.

Hence $$mL + m(t - T) = M(T - t') + C(T - t')$$

$$L = \frac{M(T - t') + C(T - t') - m(t - T)}{m}$$

Example.—Mass of steam condensed = 1/4 lb.
Initial mass of water in calorimeter = 6.8 lb.
Initial temperature of water in calorimeter = 62°F.
Final temperature of water in calorimeter = 82°F.
What is latent heat of steam in B.t.u.?
Heat gained by water in calorimeter
= mass of water × change in temp.
= 6.8 lb. (82 – 42) = 272 B.t.u.
Heat lost by condensed steam in cooling from 212°F. to 82°F.
= mass steam × change in temp.
= 1/4 × 130° = 32.5 B.t.u.
Heat lost by steam in condensation = mass of steam × latent heat
= $1/4 \times L$ B.t.u.
Hence:

$$1/4 \times L + 32.5 = 272$$
$$1/4\, L = 239.5$$
$$L = 958 \text{ B.t.u. per lb.}$$
$$\text{Correct value} = 966 \text{ B.t.u. per lb.}$$

226. Cooling by Evaporation.—In considering the heat of evaporation, it was seen that, in order to cause a gram of liquid to change into the vapor state, it was necessary to supply a certain quantity of heat known as the heat of vaporization. If the liquid is evaporating, this heat will be taken from the surrounding bodies. The withdrawal of this heat from the surrounding bodies will cause their temperatures to decrease. For this reason the evaporation of the water sprinkled on the sidewalk causes the air to become cooler, and the evaporation of perspiration cools the body and makes sunstrokes less likely.

227. Evaporation from Soils.—If one of two soils which are exactly alike in their composition and structure is kept continuously wet so that large amounts of water are evaporating from its surface, while the other is kept as dry as possible, it will be found that the former soil is much cooler than the latter because of the fact that the heat necessary to evaporate the water is absorbed from the soil and the temperature is thus lowered. It often happens that in the spring differences of temperature of as much as 12°F. are observed in soils on account of differences in drainage. Since clay soils retain the moisture better than sandy soils, there will be a differences in the temperatures of two such soils which are equally well drained.

The amount of evaporation from soils may be much decreased by covering the soil with some sort of a protective coating as straw or dead leaves. Repeated shallow plowing by which a few inches of loose soil is always kept on the surface is very effective and is much followed in hot climates.

228. Evaporation from Leaves.—Evaporation takes place from the leaves of a growing plant whenever the amount of energy received from the sun and the state of the dryness of the atmosphere is such that water can be changed into vapor. This evaporation causes a stream of water to flow from the roots of the plant into its leaves. This water carries in solution the plant food obtained from the soil. These substances thus entering the plant with the water are retained and used for the production of organic matter in the plant.

To show evaporation from leaves, cut off a branch of a rapidly growing plant having on it a number of leaves. Insert the end of the stem into a rubber stopper (Fig. 166) through which passes a bent glass tube. Fill the bottle *C* with water which has been boiled to remove traces of occluded air. Take care that enough water flows up in the bent glass tube to fill it completely. The top of the stopper should be covered with some kind of soft wax which will prevent air from leaking in about the stem of the plant. After the glass jar *C* has stood long enough for a small bubble of air to form at the further end of the glass tube, immerse this end in another jar of water *A*. As water evaporates from the leaves of the plant, the air bubble *B* will travel along the horizontal part of the glass tube, showing the rate at which water is being supplied to the plant.

229. Iceless Refrigerator.—An iceless refrigerator consists of a wooden frame covered with screen wire. The top is covered with screen wire and the bottom is made of a solid board. One side is a door which is mounted on hinges and fits the frame closely. The frame is covered closely with cotton flannel or burlap. On the top of the frame is placed a large baking pan, and the frame rests in another large pan. Four double strips, which taper to 8 or 10 inches in width, are sewed to the upper part of the cover. These strips dip into the upper pan which is filled with water. The bottom of the cover extends down into the lower pan. The water is drawn by capillary attraction through the wicks and saturates the cover. Evaporation of water then takes place from the surface of the cover. In order to change water from liquid to vapor it is necessary to supply to each gram of water 538 calories of heat. The heat necessary to produce this evaporation is taken from the inside of the refrigerator, thereby lowering the temperature of the inside and the contents.

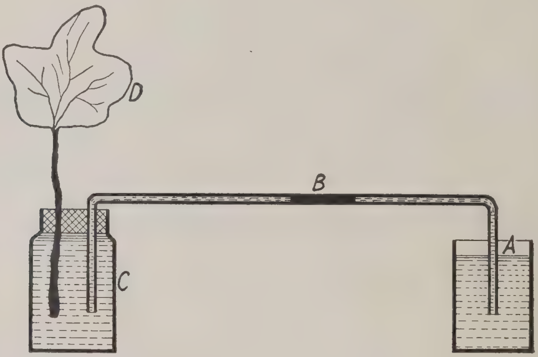


FIG. 166.—Evaporation from leaves.

When the air is warm and dry so that evaporation is most rapid, the lowering of the temperature is largest. When the air is damp and nearly saturated with moisture, the evaporation from the cover of the refrigerator will be slow, and the lowering of the temperature will be insufficient to make the refrigerator work satisfactorily.

Problems

1. How many grams of ice at 20°C. will be turned into water at 40°C. by 10 grams of steam at 150°C.? Specific heat of ice = 0.5; specific heat of steam = 0.48.

2. How many calories are required to change 10 grams of ice at −25°C. to steam at 175°C.?

3. How much steam must be passed into a vessel containing 200 grams of ice and 150 grams of water at 0°C. to bring the mixture to 60°C.?

4. How much melted zinc at 420°C. must be placed in 25 g. of alcohol at 0°C. to vaporize it?

Latent heat of fusion of zinc = 28 calories.
Melting point of zinc = 420°C.
Boiling point of alcohol = 78.2°C.
Specific heat of alcohol = 0.65.
Heat of vaporization of alcohol = 206 calories.

5. How much water must evaporate from the surface of an iceless refrigerator in order to produce as much cooling as would be produced by the melting of 50 lb. of ice? Neglecting all losses, if this evaporation takes place in a day of 24 hours at what rate is energy being supplied?

6. From the surface of a plant which weighed 3 kg., the evaporation amounted to 50 g. How much heat was necessary to produce this evaporation? If all this heat came from the plant, how much would its temperature be lowered? Assume the specific heat of the plant = 0.50 calories.

7. How much will the temperature of a soil be lowered by the evaporation from it of 1/10 lb. of water. Assume that the heat for this evaporation comes from an amount of soil which is 1 in. deep and has an area of 1 sq. ft. Take the specific gravity of the soil as 2.5 and its specific heat as 0.24.

230. Boiling Point of Water.—Fill a flask half full of water and insert a thermometer in one of the holes in the stopper. In the other hole insert a short glass tube through which the steam may escape. Heat the flask over a flame until the water boils. By reading the thermometer from time to time, it will be found that no matter how rapidly the heat is applied, the temperature does not rise above 100° C. During this process, which is known as boiling, bubbles form at the bottom of the flask and rise to the surface. They grow in size as they rise. That temperature at which the bubbles begin to reach the surface and beyond which the temperature does not rise is called the **boiling point or the boiling temperature.**

The boiling point can be defined in another way. The pressure on the surface of the liquid is the pressure of the atmosphere. This pressure is according to Pascal's principle, exerted throughout the liquid. Hence the pressure which a bubble of vapor inside the liquid must sustain is also equal to atmospheric pressure. Until the pressure of the vapor in the bubble becomes as large as atmospheric pressure the bubbles will collapse before they reach the surface of the liquid. But the pressure exerted by a vapor in contact with its liquid in a closed space is called the pressure of the saturated vapor or the vapor pressure of the liquid. This pressure increases rapidly as the temperature of the liquid is raised. Now at the boiling point, the bubbles of liquid exist under a pressure of one atmosphere. In order that this may be true, it is clear that the pressure of the vapor must also be equal to one atmosphere. The boiling point can then be stated as the temperature at which the pressure of the saturated vapor of the liquid is equal to the pressure of the atmosphere on the surface of the liquid.

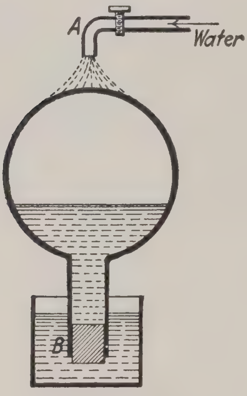


FIG. 167.—Effect of pressure on the boiling point.

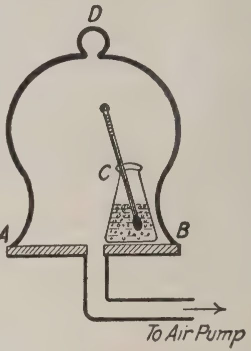


FIG. 168.—Boiling points at reduced pressure.

231. Effect of Pressure on the Boiling Point.—Since the boiling point of a liquid is the temperature at which the vapor pressure of the liquid is the same as the outside pressure on it, it follows at once that when the outside pressure is changed the boiling point will also change. This is easily understood if we recall that ordinarily the pressure of the atmosphere is about 15 pounds to the square inch. If this pressure is decreased it will not be necessary to raise the temperature so high in order to allow the bubbles to form. When the pressure is raised, it will be necessary to raise the temperature still higher in order to produce bubbles. The bubbles will form only when the pressure of the vapor in the bubble is equal to the pressure of the atmosphere on the surface of the liquid.

The influence of pressure on the boiling point can be shown by filling a flask (Fig. 167) half full of water and boiling it vigorously for some time to remove the air from above the water. Insert a rubber stopper in the flask, rendering the flask air tight. Remove the flask immediately from the flame. Invert the flask and pour cold water on the bottom. This cold water will cause some of the vapor in the flask to condense and the pressure on the hot water in the flask will be reduced sufficiently to allow the water to begin boiling again. The effect of pressure on the boiling point is also seen in Fig. 168. The water in the flask *C* under the bell jar boils at a lower and lower temperature as the air is more and more removed from the jar.

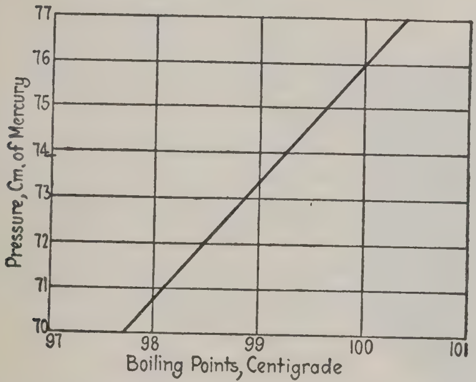


FIG. 169.—Change of boiling point with barometric pressure.

On the top, of high mountains the atmospheric pressure is less than at the base. The temperature to which the water can be raised before it boils will be lower on such mountains than at their bases. For this reason, vegetables like potatoes will cook more slowly at high elevations than at sea level. If the altitudes are very high, it may be impossible to cook such vegetables in open vessels because sufficiently high temperature can not be reached in this way.

In Fig. 169 is shown a curve giving the relation between the barometric pressure and the boiling point of water. The pressures have been plotted on the vertical axis and the temperatures on the horizontal axis.

232. Pressure Cooker.—In canning and also in cooking meats it is often desirable to cook at temperatures higher than 100°C. Pressure cookers are used for this purpose. They are essentially small closed steam boilers (Fig. 170) provided with a pressure gauge and a safety valve. As the temperature of the water in the cooker is raised the pressure of the water vapor increases just as in the ordinary steam boiler. By continuing to increase the temperature of the water, any desired pressure may be developed in the boiler. In this way any desired temperature can be obtained in the cooker so that foods may be cooked at temperatures above the boiling point of water under atmospheric pressure. The safety valve is adjusted to release at any pressure above that desired in the cooker. The pressure gauge indicates the pressure in the cooker in excess of the atmospheric pressure.

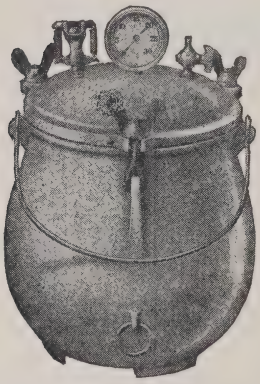


Fig. 170.—Pressure cooker.

233. Vacuum Pans.—In many cases substances which are to be vaporized are injured by heating to their normal boiling point. Such is the case with syrups or milks. These liquids must then be evaporated at temperatures below their normal boiling points. In order to accomplish this, use is made of vacuum pans which operate on the principle that by reducing the pressure on the liquid, its boiling point may be lowered to any desired temperature.

These pans, which are used in the manufacture of condensed milk, consist essentially of a closed boiler from which the air can be removed to any desired extent. As the pressure of the air on the liquid is reduced, the boiling is lowered and evaporation takes place at these low temperatures without heating the liquids to the higher temperatures that would be necessary under atmospheric pressure. By this evaporation the water contained in the milk or in the syrup is removed, and the solid matter remains behind in the pan.

234. Boiling Point of Solutions.—When water has some foreign substance like sugar dissolved in it, the temperature at which it boils is raised. The amount the boiling point is raised is proportional to the amount of the substance dissolved in the water. In making candies or syrups, the temperature is taken as a means of determining the concentration of the solution. By observing the temperature at which the solution is boiling, it is possible to know when the candy or syrup has been boiled sufficiently long. When vegetables, fruits, meats, etc., are boiled in water, some of their contents dissolve in the water. This raises the boiling point slightly so that the water boils above 212° F. The effect of a dissolved substance on the vapor pressure and the boiling point is shown in Fig. 171.

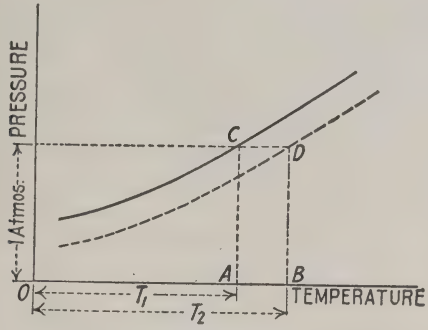


FIG. 171.—Boiling points of solutions.

235. Distillation.—When solids like salt or sugar are dissolved in liquids, the vapor which rises above the liquid does not contain any of the dissolved substance. In order to obtain pure water from water containing solids in solution, it is only necessary to first evaporate the water and then condense the vapor. The solids which were in solution will be left behind in the vessel. The way in which this is ordinarily done is represented in Fig. 172.

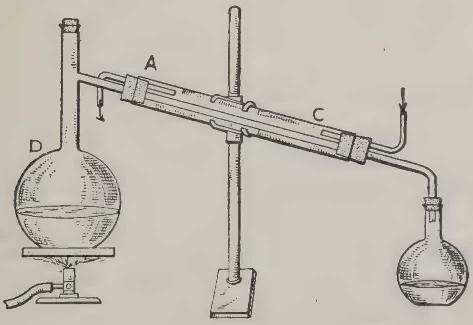


FIG. 172.—Distillation of liquids.

Pure water vapor rises from the water which is being boiled in the flask D. This vapor passes into the tube A where it is condensed by the water flowing in the tube C. The condensed water drops into the vessel placed under the end of the tube A. In the manufacture of sugar it is the solid sugar in solution which is of interest. The sugar would remain behind in the flask. It is not necessary in this case to condense the water vapor which rises from the flask or vessel containing the solution. The lower the pressure the lower the temperature at which the boiling takes place. In case it is not desired to heat the solution to the normal boiling point, the space above the liquid in the flask is kept exhausted by means of an air pump, as previously stated in the discussion of vacuum pans.

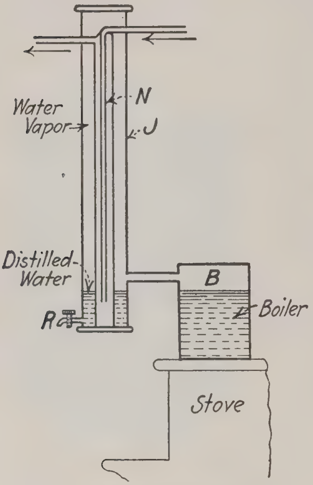


FIG. 173.—Household still.

236. Household Still.—Where it is desirable to have distilled water in the household, a form of still shown in Fig. 173 may be used. Water from the boiler B which is located on the stove is vaporized. The vapor passes into the condenser J. On the inside of this condenser there is a hollow tube N through which a current of cold water is flowing continuously. By this means the vapor is condensed and drops to the bottom of the condenser from which it may be taken through the tap R.

237. Sublimation.—If a substance like solid camphor or solid iodine be left for some time in a closed vessel, the sides of the vessel become covered with small crystals of the substance. This is due to the fact that vapor which is given off by the solid is afterwards condensed on the sides of the vessel. This formation of vapor directly from a solid without passing through the liquid state is called **sublimation.** In like manner, the vapor may go directly from the vapor state to the solid state without passing through the liquid state. For most solids, the vapor pressure is nearly zero at ordinary temperatures, but in these cases the sense of smell tells us that some vapor is being given off. For ice at 0°C, the vapor pressure is 4.6 mm. of mercury, but it increases rapidly with the temperature, becoming 47.5 mm. of mercury at 100°C.

Substances which sublime have a greater vapor pressure in the solid state than in the liquid state at the same temperature. Substances which sublime at atmospheric pressure can not be melted unless they are subjected to pressure greater than atmospheric pressure. A vapor pressure curve showing the vapor pressure when a solid is in equilibrium with its vapor may be plotted for this case as for the case of a liquid in equilibrium with its vapor. Such a curve may be called a **sublimation curve.**

238. Triple Point.—It has been seen that, under certain conditions of temperature and pressure, a liquid may be in equilibrium with its vapor. On a temperature-pressure diagram these conditions are represented by a curve called the vapor-tension curve. It was also seen that, under suitable conditions a solid may be in equilibrium with its vapor and that these conditions are represented by the sublimation curve. In like manner a curve may be drawn showing the relations between the temperatures and pressures at which a solid is in equilibrium with the liquid. If all of these curves are drawn for a substance like water (Fig. 174), they intersect at a point which is called the **triple point.** It indicates the temperature and pressure at which the solid, vapor and liquid can exist together without one of them gaining in mass at the expense of the others. At this temperature and pressure the solid, liquid and vapor are in equilibrium. There is only one such point for a simple substance like water. For water this point is at a pressure of 4.6 mm. of mercury and at a temperature of 0.0076°C.

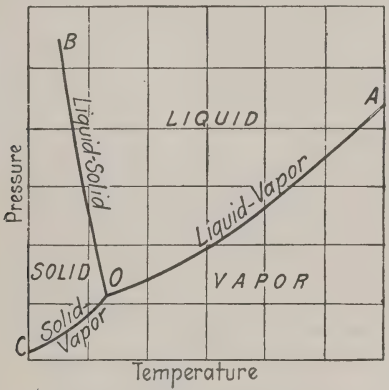


Fig. 174.—The triple point.

239. The Critical Point.—If a vapor is enclosed in a cylinder *HFEG* (Fig. 175) and the piston *M* forced in while the temperature of the vapor remains constant, the pressure of the vapor at first increases as the volume is decreased. The relation between the pressure and the volume is represented by the curve *AB*. When the volume and pressure corresponding to the point *B* are reached, the pressure of the vapor ceases to increase and liquefaction begins. As the piston is still further forced in the quantity of vapor decreases and the quantity of liquid increases. This continues at constant pressure until the volume corresponding to the point *C* is reached where liquefaction is complete. A further application of pressure to the piston causes a small decrease in the volume of the liquid, and the curve *CD* representing the relation between the pressure and the volume of the liquid, becomes very steep.

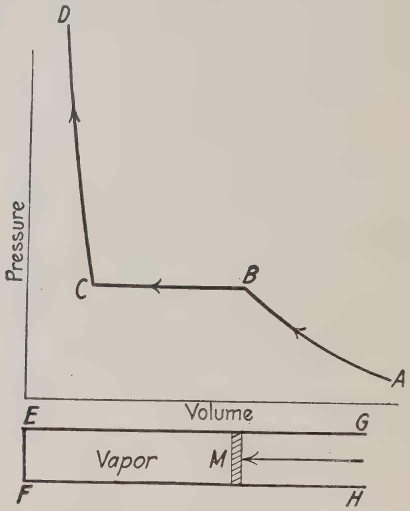


FIG. 175.—Isothermal of saturated vapor.

If this process be carried out at higher and higher temperatures (Fig. 176), the horizontal part of the curve becomes shorter and shorter until a temperature is finally reached at which the horizontal portion disappears and is replaced by a slight bend in the curve. At a temperature just below this it is possible to liquefy the vapor by the application of pressure alone, but at temperatures above this it is impossible to liquefy the vapor by the application of pressure however great that pressure. This highest temperature at which it is possible to liquefy a vapor by the application of pressure alone is called the **critical temperature.** The point *P* (Fig. 176) at which the horizontal part of the isothermal just disappears is known as the **critical point.** The pressure and volume of the vapor at the point *P* are known as the **critical pressure** and **critical volume** respectively.

The isothermals drawn in Fig. 176 refer to carbon dioxide. Its critical temperature is 31°.1C. and its critical pressure 77 atmospheres. Above this critical temperature, carbon dioxide can not be liquefied by the application of pressure alone. If the carbon dioxide vapor is at a temperature higher than 31°.1C., its temperature must be lowered as well as its pressure increased to produce liquefaction.

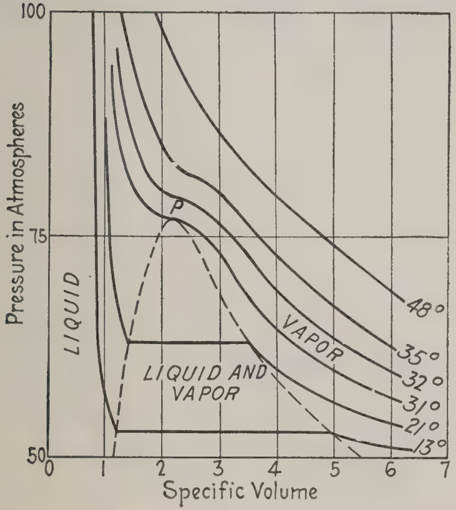


Fig. 176.—Isothermals of carbon dioxide.

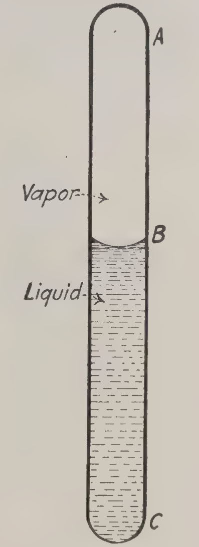


Fig. 177.—The critical point.

240. Transition through the Critical Point.—If a heavy walled glass tube (Fig. 177), closed at both ends is partly filled with liquid carbon dioxide, the remainder of the tube being filled with the saturated vapor of carbon dioxide, for any temperature there is equilibrium between the liquid and its vapor. As the temperature of the tube and its contents is increased, the volume of the liquid carbon dioxide increases and the volume of the saturated vapor decreases. The mass of the liquid, however, decreases while the mass of the saturated vapor increases. The density of the vapor increases and the density of the liquid decreases. The pressure exerted by the vapor also increases. As the temperature is raised more and more, the meniscus marking the boundary between the liquid and the vapor becomes more indistinct and finally disappears. The tube is now filled with a homogeneous substance. At this point it is impossible to distinguish between the liquid and vapor states. That temperature at which the meniscus disappears is the critical temperature.

When the temperature is again decreased, a hazy cloud forms in the upper part of the tube, the meniscus or boundary between liquid and vapor reappears and there are now again two states, liquid and vapor, in the tube. For carbon dioxide the temperature at which the meniscus disappears is 31°.1C and the pressure exerted by the saturated vapor is 77 atmospheres.

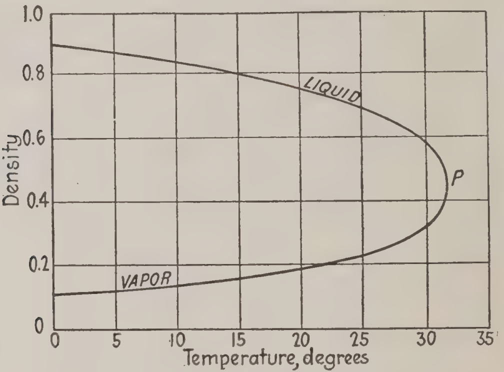


Fig. 178.—Densities near the critical point.

The behavior of the density of the liquid and vapor carbon dioxide is shown in Fig. 178. In this figure the densities of the vapor and liquid have been plotted as functions of the temperature. It is observed that at the point *P*, where the temperature is 31°.1C., the two densities have become equal. With lowering temperature the density of the liquid increases and that of the vapor decreases. Near the critical point *P* the change in the density of both the vapor and the liquid is very rapid. This makes the accurate determination of the critical volume difficult.

241. Liquefaction of Gases.—To liquefy a gas it is necessary to cool it below its critical temperature and then apply to it sufficient pressure to produce liquefaction. Most of the so-called permanent gases have low critical temperatures and require the application of large pressures. The necessary pressures are obtained by means of pumps which compress the gas to be liquefied in three or four stages so that the final pressure is between 170 atmospheres and 200 atmospheres.

Different methods have been used to produce the necessary lowering of the temperature to bring the gas below its critical temperature. The most common of these is the regenerative process. It is based on the fact that gases cool on expansion. The gas, having been compressed to about 200 atmospheres by means of the air pump *A* (Fig. 179), is led through the cooling bath *B* to the liquefier *C*. The liquefier, consisting essentially of three concentric copper pipes or tubes *F*, *G*, and *H*, is usually wound in the form of a compact spiral. The air passes from the central tube *F* through the valve *L*, and its pressure is reduced from 200 to 16 atmospheres. This expansion cools the gas. Part of it flows upward through *G* and further cools the stream of air flowing downward in *F*. This air then flows back through *I* to the pump *A* where it is again compressed. The remainder of the air which has escaped through the valve *L* flows directly through the valve *M*, and its pressure decreases from 16 atmos-

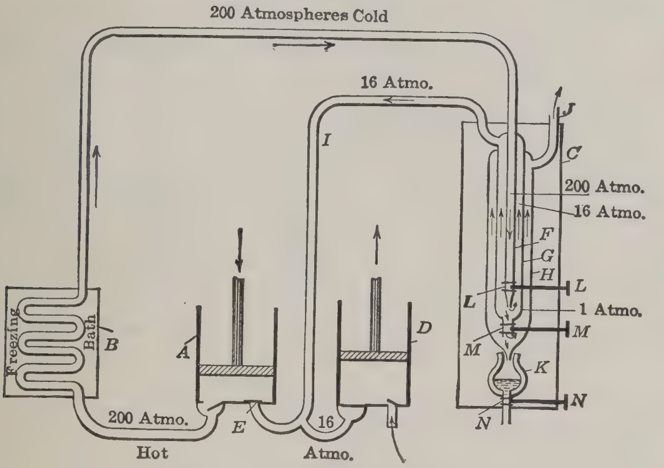


Fig. 179.—Liquid air apparatus.

pheres to 1 atmosphere. By this second expansion its temperature is still further decreased. The air thus passing through the valve *M* rises through the tube *H* and is led from *J* back to the pump *D* where it is recompressed. The air rising through the tube *H* cools the air flowing upward in *G*. This progressive cooling continues until the cooling of the air in passing the needle valve *M* causes part of the air to be liquefied. This liquefied air collects in the Dewar flask *K*, and the remainder is led from *J* back to the compressor *D*.

The following table shows the freezing point, boiling point and critical data of some of the common gases.

Name of gas	Freezing point	Boiling point	Critical temperature
	°	°	°
Helium	−272.0	−268.8	−268.0
Hydrogen	−259.0	−252.7	−234.5
Argon	−188.0	−186.0	−117.4
Nitrogen	−210.0	−195.7	−146.0
Oxygen	−219.0	−182.9	−118.0

CHAPTER XIX

ATMOSPHERIC HUMIDITY

242. Water Vapor in the Air.—If a jar filled with water be left open, the water evaporates after a time. The molecules of water escape from the surface and pass off into the air where they move about freely just as the molecules of air do. The molecules of vapor strike each other and also strike molecules of air. In this irregular motion, the molecules of vapor may occasionally strike the surface of the liquid and be held fast. Most of the molecules pass out of the jar and never return. The continual loss of liquid from open surfaces of water is compensated by the return of rain water from rivers and streams. When the temperature of the water is increased, the quantity evaporated in a given time increases. If the wind is blowing, it increases the rate of evaporation by decreasing the opportunity of the molecules to return to the surface of the liquid.

If the water is in a closed vessel none of the molecules can escape and more of them will strike back into the liquid and be held fast. After a while the number which is held fast will be equal to the number which escapes in the same time. The air above the water in the vessel is then said to be saturated. The number of molecules necessary to saturate the air will depend on the temperature of the liquid, for the number of molecules which can escape from the surface of the liquid in a given time is greater at high than at low temperatures. Hence the number of molecules of water vapor in air which is saturated is greater at high than at low temperatures. When warm saturated air is cooled, some of the water vapor condenses and reappears as water. The air will still be saturated, but not so much water vapor is necessary to saturate it.

243. Clouds, Fog, Dew, etc.—When the air above the surface of the earth is saturated and is then cooled, the water vapor in it forms minute drops about the dust particles which are present. As the air is still further cooled, these drops grow in size and fall to the earth as rain. When these drops in falling to the earth pass through a layer of air at a temperature below freezing, they are frozen into little spheres which fall to the earth as hail. If the temperature of the air is below freezing when the water vapor condenses from it, crystals are formed which fall to the earth as snow.

When the sun goes down at night and the surface of the earth begins to cool, some objects lose heat more rapidly than others. When warm moist air comes in contact with these objects, some of the moisture from the air is deposited. This moisture appears on the objects in the form of dew. If the air cools below the temperature at which it is saturated, moisture not only collects on objects, but forms minute particles in the air. Each of these particles of water has as its center a small particle of dust. This collection of water particles is called a fog. It does not differ from a cloud except that fogs are near the surface of the earth and clouds may have any height above the surface.

244. Dew Point.—The temperature to which it is necessary to cool the air before condensation begins is called the dew point. This temperature may be found by partly filling a polished vessel with ether and then inserting in the stopper a thermometer and a tube through which air can be bubbled. The ether is made to evaporate by blowing air through it, and in the process of evaporation it absorbs heat and cools the remaining ether and its containing vessel. By and by a temperature is reached at which moisture begins to form on the surface of the containing vessel. The temperature at which this deposit begins to form is called the **dew point.** The vessel is polished because it is easier to detect the formation of dew on a polished surface.

245. Relative Humidity.—It is often important to know the degree of saturation of the air. On such information depends the likelihood of rain or frost, the proper growth of plants and proper hygienic living conditions. **The degree of saturation or relative humidity is defined as the ratio obtained by dividing the amount of moisture actually present in the air by the amount of moisture necessary to saturate the air at that temperature.** Since the amount of water vapor is proportional to the pressure which it exerts, the relative humidity is also the ratio between the pressure of the water vapor in the air and its pressure if the air were saturated at that temperature.

Let $t =$ the observed temperature of the atmosphere.
$t' =$ the dew point.
$p =$ vapor pressure of water at t.
$p' =$ vapor pressure of water at t'.

Then,

$$\text{Relative humidity} = p'/p$$

Or

$$\text{Relative humidity} = \frac{\text{mass of water vapor per unit volume of air at observed temperature.}}{\text{mass of water vapor per unit volume of air when saturated at same temperature.}}$$

246. Measurement of Humidity.—For the measurement of the humidity of the air a number of different types of instruments is used. Some of these depend on the hygroscopic properties of hair or a thin flat strip of some material. This type of instrument depends for its action on the property of these substances to absorb moisture from the air and to change their length on account of this absorption. This change in length is indicated by a hand which moves over a dial. On this dial the words dry, moist, etc., are recorded, and the position of the hand on the dial is thus made to indicate the humidity of the air. These are very convenient but not very accurate instruments.

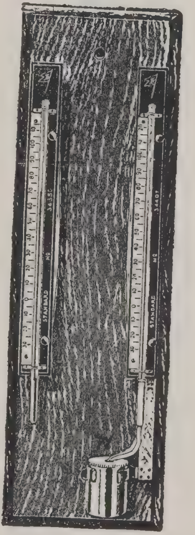


FIG. 180.—Wet and dry bulb hygrometer.

The wet and dry bulb hygrometer (Fig. 180) is the simplest of the reliable instruments. It consists of two thermometers, one of which gives the true temperature of the air. The other thermometer is covered with a wet cloth and gives a temperature that varies with the rate of evaporation of the water into the air. This wet bulb is generally kept moist by allowing the wick to dip into a cup of water. The drier the air, the greater is the evaporation from the wet bulb and the lower the temperature which is indicated by that thermometer. When the air is saturated, there is no evaporation from the wet bulb, and both thermometers indicate the same temperature. A chart showing the relation between the relative humidity and the difference between the readings of the two thermometers must be used with this instrument. This chart must be prepared by a calibration of this kind of instrument under the conditions under which it is to be used. In Fig. 181 provision is made for whirling the thermometers.

247. Chemical Hygrometer.—When it is desired to know the **absolute humidity or the mass of water vapor in unit volume of the air,** the chemical hygrometer is used. In this instrument (Fig. 182) a known volume of air is passed through a series of U-tubes which are filled with calcium chloride or some other substance which absorbs water vapor but does not absorb air. These tubes are weighed before and after passing the air through them. The increase in weight is the mass of water vapor absorbed by the substance in the tubes and is therefore the mass of water vapor originally in the volume of air which passed through the tubes. The amount of water vapor per unit volume is obtained by dividing the water vapor absorbed in the tubes by the volume of air passing through the tubes. This result gives the absolute humidity of the air. It is expressed in grams per cubic centimeter or in pounds per cubic foot.

FIG. 181.—Psychrometer of U. S. Weather Bureau.

248. Importance of Atmospheric Humidity.—The moisture in the air plays an important part in the health and comfort of every one. It also has a marked effect on the physical conditions and behavior of many materials such as wood, wool, cloth, etc. The lack of moisture often causes woodwork and furniture to become damaged. On the other hand an excessive amount of moisture is just as serious. The low humidity which is often found in houses in winter is in-

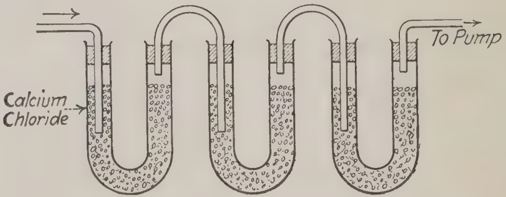


FIG. 182.—Chemical hygrometer.

jurious to health. It seems a well established principle that the extremes of heat and cold are felt less and are less injurious where the humidity is low. In many forms of manufacture it is necessary to control by artificial means the humidity in the factory. Thus in the manufacture of bread or in the manufacture of cigars it is desirable that the humidity remain constant and that it be neither too high nor too low.

CHAPTER XX

TRANSFER OF HEAT

249. Convection.—The simplest way in which heat may be transferred from one place to another is by the motion of the heated substance. Such a movement of heat from one place to another by the movement of the heated substance is known as **convection.** The movement of the heated substance depends in this case on the change in density which takes place when the substance is heated. For example, when a gas or a liquid is heated it expands and becomes lighter than the cold gas or liquid. When water is heated in a vessel on a stove the liquid in the bottom of the vessel is hotter than that on the top. The density at the bottom is less than that near the top. The cool liquid sinks down and forces the warmer liquid to a higher level. The currents of water thus set up (Figs. 183 and 184) in the liquid are known as convection currents. In case a block of ice is placed on the top of a vessel of warm water the convection currents are reversed from what they were in the preceding case.

Consider the convection currents in a lake as winter approaches. The layers of water near the surface are first cooled and these layers, being heavier than the lower layers, sink. Convection currents are set up which persist until all the water in the lake has been cooled to 4°C, the temperature at which the density of the water is greatest. A further cooling of the layers of water at the surface will make the water at the surface lighter than that at the bottom. Since in this case the lighter water is already at the surface no convection currents will be set up. The colder water thus tends to remain at the surface. The deeper layers of liquid are thus protected from further cooling. This fact keeps the water some distance below the surface from becoming cold enough to destroy all forms of animal life in winter.

250. Draft in a Stove.—The draft in a stove or a chimney (Fig. 150) is produced by convection. The air in the neighborhood of the fire is heated and expands. It thus becomes lighter than the air higher up in the chimney or stove. The heavier air sinks and forces the lighter air up the chimney. Since there is a greater difference of pressure between the top and the bottom of a chimney when it is high than when it is low, tall chimneys draw better than low ones.

251. Trade Winds.—The trade winds which are found in regions a few degrees north and south of the equator are convection current in the atmosphere. These winds blow in the same direction for a long time. They are produced by the considerable heating of the atmosphere in the neighborhood of the equator. This heated atmosphere rises and the colder atmosphere from the north and south of the equator flows in to take its place. If the earth did not rotate on its axis and if it were perfectly smooth these winds would blow from the north toward the equator in the northern hemisphere and from the south toward the equator in the southern hemisphere. On account of the rotation of the earth these currents are somewhat deflected.

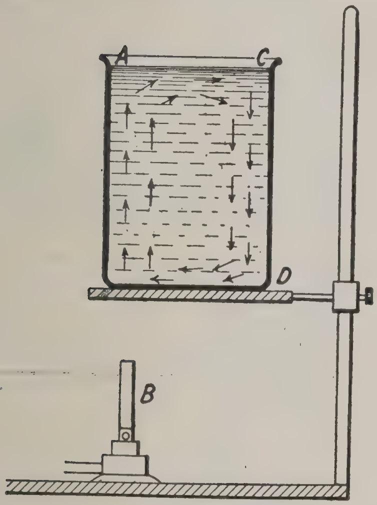


Fig. 183.—Convection currents in a beaker.

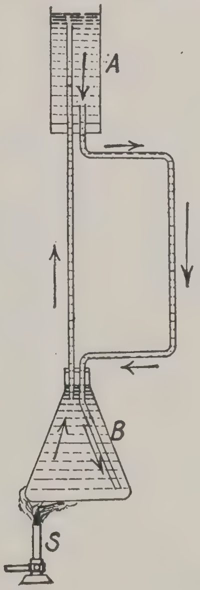


Fig. 184—Convection currents in connected vessels.

252. The Hot-Water Tank.—The hot-water tank (Fig. 185) is heated by convection currents in the water. The water in the portion of the heater which is within the stove becomes heated and rises to the top of the tank. Cold water comes in from the water mains and replaces the water which has risen after being heated. A continuous flow of water is kept up by this unequal heating. The direction of this flow is indicated by the arrows in Fig. 185.

253. The Hot-Water Furnace.—One of the best methods of heating a house is by means of a hot-water heating plant. (Fig.

186.) Such a plant transfers heat from the furnace to the radiators by means of convection. The plant consists of a number of pipes holding water. These pipes surround the fire box and the water in them is heated and rises from the top of the furnace through a supply pipe to the radiator. It there loses some of its heat, cools and thus contracts. The increase in density arising from this cooling causes the water to be heavier than the incoming water. It therefore sinks to the bottom of the furnace through the return pipe where it is again heated and the process is repeated. In order to take care of the excess expansion arising out of the expansion of the water in heating, an expansion tank (Fig. 147) is provided. This tank is usually located in the attic at a height greater than the height of the highest radiator.

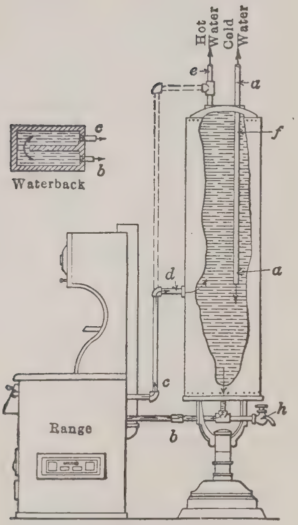


Fig. 185.—A range boiler for heating water.

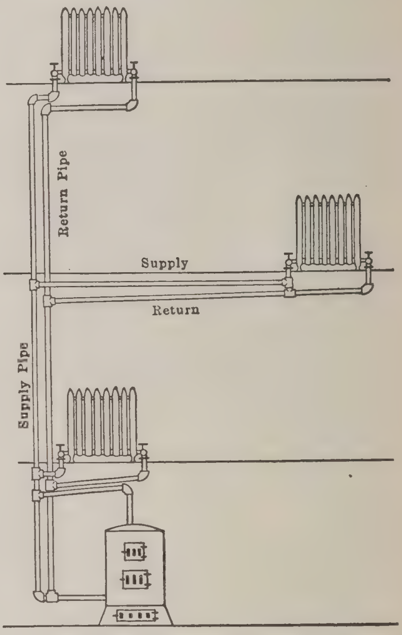


Fig. 186.—Hot water furnace.

254. Ventilating System.—In schools and homes as well as elsewhere a constant supply of fresh air is important for the health of those occupying the rooms. A common method of admitting cold air to rooms to produce ventilation depends entirely on convection currents in the air. The hot air enters the room through the radiator in the floor. The cold air entering through an opening in the side of the house rises to the ceiling through a channel provided for that purpose. The cold air being heavier than the warm air, sinks to the floor while the warm air rises to the ceiling. This provides a continuous circulation of air in the room.

255. Radiator of a Gasoline Engine.—A gasoline engine, when in operation, becomes very hot. The most common method of cooling the engine is by means of circulating water. The cylinder is surrounded by a hollow chamber filled with water. In this water jacket the water is heated by contact with the hot cylinder. After being thus heated the water rises through a pipe at the top of the cylinder and is replaced by cold water which enters through a pipe at the bottom of the water jacket. The hot water after being cooled in the radiator again passes around the circuit and serves again to cool the engine.

256. Conduction.—When a metal rod is held in the fire the heat travels along the rod and after a time the rod becomes too hot to hold. If a rod of wood or glass is held in the flame the heat travels along it much less readily. In this process the heat which consists of the vibrations of the molecules of which the body is made, is handed on from particle to particle. The layer of molecules in contact with the fire is heated first and thus made to vibrate more rapidly. This layer hands the motion on to the adjacent layer because each layer is bound to the adjacent layer by certain cohesive forces. It is thus impossible for the molecules in one layer to vibrate without setting the molecules in the neighboring layers in vibration. As this process goes on the entire medium is heated after a time. When heat, as in this case, is transferred from one part of the body to the other without any visible motion of the parts of the substance, the heat is said to be transferred by **conduction.**

To show that all metals do not have the same power to conduct heat, take four heavy wires, one of copper, another of zinc, another of iron and another of lead. Fasten them all into a solid piece of brass and then heat this piece of brass in a Bunsen burner. If the wires are painted with a soft wax which melts easily it will be found that the wax will be melted to different distances on the wires. If on a particular wire the wax is melted far from the end which is being heated it means that that wire is a good conductor. Of these four metals copper is the best conductor.

Liquids are poor conductors of heat. This fact can be shown in the following way. Nearly fill a test tube with water. Into the bottom of it drop a piece of ice. Hold the ice at the bottom of the tube with a weight of some kind. By holding the top of the tube in the flame of a Bunsen burner the liquid on the top of the tube may be made to boil without melting the ice in the bottom of the tube.

Let a spherical bulb filled with air be inserted in a stopper and the bulb then covered with liquid. The end of the tube to which the spherical bulb is attached is immersed in a beaker of water. If ether is poured on the top of the water and lighted it will burn but the air in the bulb will not expand sufficiently to force any air out of the bulb through the water into the beaker.

257. Measurement of Thermal Conductivity.—In order to study more carefully the law by which heat is conducted, consider the case in which heat flows down a uniform bar whose ends are kept at uniform temperatures so that the fall of temperature in the bar remains the same. The quantity of heat Q which will be conducted down this bar depends on the folllwing factors:

(1) **The quantity of heat passing through the conductor in a given time is in direct proportion to the area of the conductor.** In this case just as in the case of a water pipe the larger the area through which the flow takes place, the greater the quantity of water or heat which flows.

(2) **The quantity of heat is also directly proportional to the difference in temperature between the ends of the bar.** This difference in temperature may be compared to the difference in pressure which forces the water through the pipe. No flow of water occurs unless there is a difference in pressure and the greater the difference in pressure the greater the flow. Likewise in conduction there is no flow of heat unless there is a difference in temperature between the ends of the bar and the greater this difference the greater is the flow of heat.

(3) **The quantity of heat decreases as the length of the path or the distance between the ends of the bar is increased.** This is also analogous to the flow of water in a pipe in which it is found that the quantity of water flowing is inversely proportional to the length of the pipe under a given set of conditions.

(4) **The quantity of heat is directly proportional to the time of flow.** These facts may be expressed compactly in the following equation,

$$Q = \frac{kA(t' - t)T}{d}$$

where Q = the quantity of heat flowing through the bar.
A = the area of cross section of the bar.
d = distance between the ends of the bar.
T = time during which heat flows.
t' = temperature of hot end of bar.
t = temperature of cold end of bar.
k = constant called the coefficient of thermal conductivity.

The physical meaning of the coefficient of thermal conductivity is seen by considering a special case in which it is assumed that area of cross section of bar = 1 sq. cm.; length of bar = 1 cm.; the difference in temperature between the ends of the bar = 1°C. and the time 1 sec. In this case

$$Q = k$$

Hence k the coefficient of thermal conductivity **may be defined as the quantity of heat which will flow through unit cross section of the bar in one second when the difference of temperature between the ends of the bar is 1°C. and the bar is 1 cm. long.**

In the metric system,

$$\text{Coefficient of thermal conductivity} = \frac{\text{calories per sec.} \times \text{thickness in cms.}}{\text{area of surface in sq. cms.} \times \text{temp difference in degrees C.}}$$

In English system of units,

$$\text{Coefficient of thermal conductivity} = \frac{\text{B.t.u. per hour} \times \text{thickness in inches.}}{\text{area of surface in sq. ft.} \times \text{difference in temp. in degrees F}^\circ\text{.}}$$

An apparatus for measuring the coefficient of thermal conductivity is represented in Fig. 187. It consists of the copper rod *AB* the coefficient of thermal conductivity of which is to be determined. This bar should be about 15 cm. long and have a diameter of about 3 cm. On one end is soldered a copper box *C* through which steam may be passed and the temperature of that end of the rod made about 100°C. Over the other end of the rod is soldered a second copper box through which water is made to circulate from some supply of water at constant pressure. The temperature of this water as it enters this box is found by means of the thermometer T_3 and its temperature as it leaves the box is found from the reading of the thermometer T_4. The difference in temperature between the ingoing and the outgoing water multiplied by the weight of water gives the quantity of heat which has flowed down the rod. The boxes and the rod are carefully lagged with felt to prevent losses of heat. Two small holes drilled at right angles to the axis of the rod contain thermometers by which the fall of temperature in the rod is measured. From the distance between these holes and the readings of the thermometers in them the fall in temperature per cm. is at once obtained.

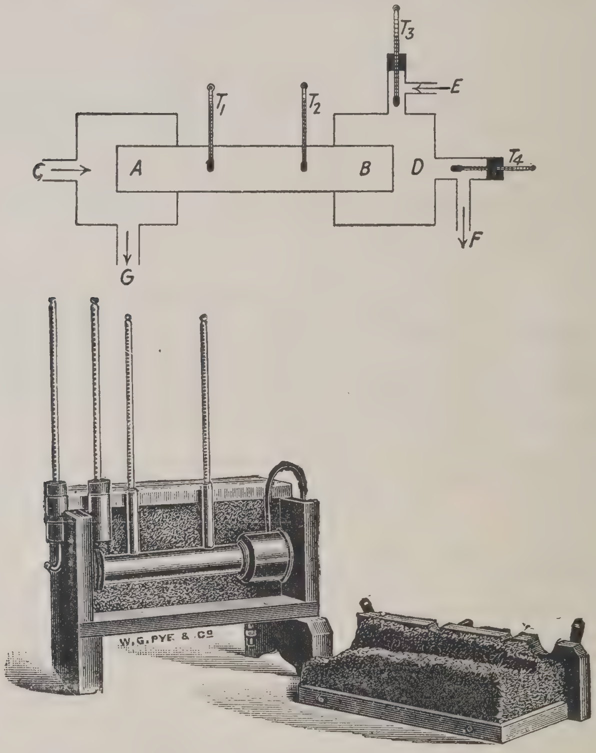


Fig. 187.—Method of measuring thermal conductivity.

Let W = weight of water delivered in T seconds.
T_3 = temperature of ingoing water.
T_4 = temperature of outgoing water.
T_1 = higher temperature measured on the rod.
T_2 = lower temperature measured on the rod.
d = distance between the thermometer inserted in the holes in the rod.

$$Q = W(T_4 - T_3) = k\frac{T_1 - T_2}{d}AT$$

Example.—In measuring the thermal conductivity of a rod the following data were obtained. Find the coefficient of thermal conductivity.

Temp. of ingoing water = 20.0°
Temp. of outgoing water = 30.0°
Higher temperature on rod = $T_1 = 80.0°$.
Lower temperature on rod = $T_2 = 60.0°$
Area of rod = 20 sq. cm.
Distance between thermometers = 10.0 cm.
Weight of water flowing through box = 650 g.
Time of flow = 3 min. = 180 sec.
Heat gained by water = mass of water × change of temp.
= 650 × 10 = 6,500 calories

$$\text{Temp. fall per cm.} = \frac{80 - 60}{10} = 2.0°\text{C. per cm.}$$

$$Q = kA\,\frac{T_1 - T_2}{d}\,T$$

$$6500 = k \times 20\,\frac{20}{10} \times 180 = k \times 20 \times 2 \times 180$$

$k = \dfrac{6{,}500}{20 \times 2 \times 180} = \dfrac{6{,}500}{7{,}200} = 0.9$ calorie per sq. cm. per sec. when temp. dif. is 1°C. per cm.

258. Thermal Conductivity of Soils.—The temperature of the subsoil and the soil at the surface depends in part on the thermal conductivity of the soil. If the thermal conductivity of the soil is small the surface of the soil will become hot during the day when it is exposed to the sun's rays. Since the heat will not flow easily into the layers below the surface these layers will not be much warmed by the solar radiation. During a clear night the surface will loose heat rapidly by radiation and since heat does not easily flow through the soil, heat will not be supplied to the surface from the lower layers. A soil which conducts heat more readily will be heated to a greater depth by the sun's rays and the surface will not be so hot. At night the surface will lose heat less rapidly and heat will be supplied more readily to the surface from the lower layers. Hence the soil will not be cooled so much at night as in the former case. Good conductivity tends to make the subsoil have a temperature more nearly equal to that of the soil on the surface.

The thermal conductivity of a soil is greatly lessened by making it in the form of a powder and the finer the powder the less readily the soil conducts heat. The air which fills the spaces between the grains in the powder is a very poor conductor of heat. The water which replaces the air in wet powders is a much better conductor of heat. A fine, dry loose soil will conduct heat much less rapidly than a compact, moist soil conducts heat. The surface of a fine, dry soil will be hot in the day time but this heat will not flow far below the surface. In a compact, moist soil the temperature will be more evenly distributed and whatever heat is supplied to the surface will flow to greater depths below the surface.

259. Poor Conductors.—The handle of a tea kettle is made of wood because the wood is a poor conductor of heat and will not become so hot when the tea kettle is heated. For the same reason the handle of a coffee-pot is made of ivory or porcelain. Steam pipes (Fig. 188) are lagged with felt or other poor conductors to help retain the heat.

When a farmer stores vegetables in the ground for winter, he is availing himself of the fact that the earth is not a very good conductor of heat. A thick layer of earth as shown in Fig. 189 is covered over the vegetables. This is made in the form of a cone so that it will drain well. Dry earth is a poorer conductor of heat than wet earth. Of course there is in this case another reason for keeping the earth about the potatoes dry. If the pit is also covered with straw, the vegetables will not freeze in severely cold weather. The heat from the interior of the pit flows out slowly to the outside air. The more protracted the cold weather the greater is the danger of the vegetables freezing.

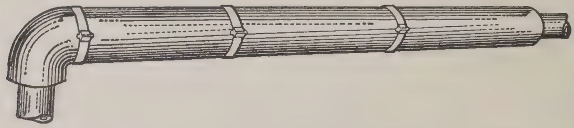


FIG. 188.—Covering of pipe to prevent loss of heat.

Sometimes this same protection is accomplished in a better way by making a concrete storage cellar. Here again the protection from freezing arises from the fact that the earth is not a very good conductor of heat and thus prevents the outward flow of heat from the fruits or vegetables until the winter is passed.

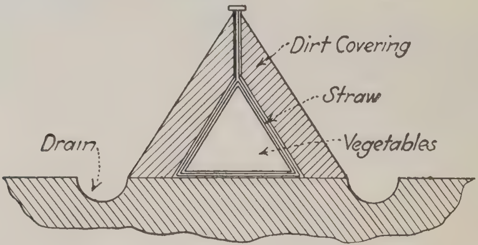


FIG. 189.—Pit for protection of vegetables.

260. Ice House.—In storing ice for the winter it is placed in a house (Fig. 190) leaving a space of about one foot between it and the sides of the house. This space is filled with dry sawdust. The ice is also covered with sawdust and between it and the floor is a heavy layer of sawdust. Now this sawdust is a poor conductor of heat and thus prevents the heat from the outside of the house from flowing through the walls and melting the ice. The thicker the layer of sawdust, the greater the protection given the ice and the less will be the losses.

261. Silo.—To prevent the ensilage in a silo from freezing in the winter the farmer often jackets his silo (Fig. 191) with a layer of stone. Stone is a rather poor conductor of heat. If a sufficiently thick layer of stone is placed around the silo the heat on its inside will not have much opportunity to flow out and the contents of the silo will not be frozen. The thicker the layer of stone placed around the silo the greater will be the protection against freezing.

262. Concrete Floors.—Concrete floors are colder than wood floors because the former are better conductors of heat than the latter. A concrete floor will therefore carry the heat away from the body more rapidly than a wood floor. For this reason when concrete floors are used in barns animals must be provided with an adequate supply of bedding between the body of the animal and the floor to prevent in a measure the flow of heat from the body of the animal to the floor. This layer of non-conducting material between the animal and the floor tends to conserve the heat of the animal.

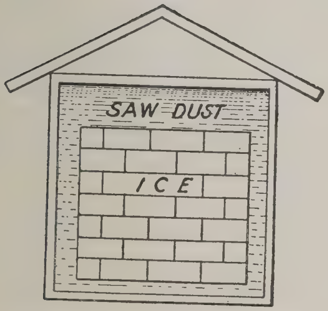


Fig. 190.—Ice house.

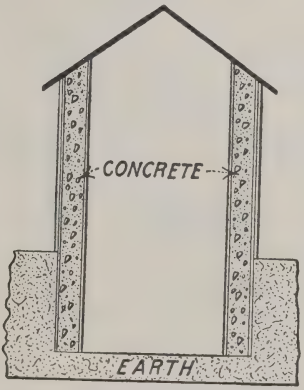


Fig. 191.—Silo with thick walls to prevent freezing.

263. Non-Conductors in Building.—When it is desired to construct a room which will be warm in winter and cool in summer, the same principles must be employed in each case. In the first case it is desired to prevent the heat generated in the room from flowing out through the walls and in the second place it is desired to prevent the heat on the outside from flowing into the room through the walls. To meet this condition the wall must be as nearly air tight as possible and as poor a conductor as possible. When stone or brick walls are made hollow they are much warmer in winter and cooler in summer than when built solid because the air is a poor conductor of heat. The air in the cavities of the stones or bricks prevent the flow of heat through the walls to a greater extent than an equal quantity of brick or stone would prevent it.

Wood is a poor conductor of heat. Hence houses made of heavy hewn logs should be warm in winter and cool in summer. In these cases, however, it is not often possible to make the house sufficiently air tight to secure the necessary conditions for protection. Where heavy log houses have been made nearly air tight by modern methods of papering and plastering these houses are very warm in winter and cool in summer.

264. Fireless Cooker.—A fireless cooker depends for its operation on the poor conductivity of certain substances. Such a cooker consists of an outer vessel (Fig. 192) and an inner metal lining *C* which is usually an aluminum can. The space between the outer vessel and the aluminum can is packed carefully with some kind of non-conducting material. A soapstone *B* is heated to the desired temperature and placed in the aluminum can. On this soapstone is placed the vessel containing the food to be cooked. When the cooker has been covered with a non-conducting lid, the opportunity for the heat to flow away is not large. The heat in the stone is thus available for the cooking of the food in the vessel. Without the careful protection of the vessel and stone from the loss of heat the stone would soon cool and the food be insufficiently cooked.

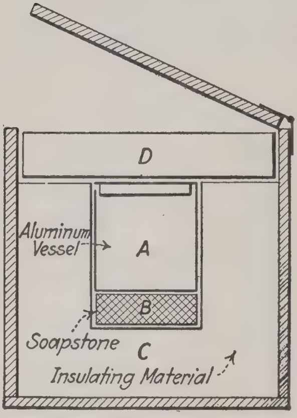


Fig. 192.—Fireless cooker.

265. Radiation.—A person sitting in front of a stove receives heat from the stove although the air in the room is cold. In like manner sunlight falling on a body will warm it above the temperature of the surrounding air. This method of transfer of heat is distinguished from convection and conduction by the fact that the medium through which the transfer occurs is not heated. Thus the earth receives great quantities of heat from the sun although the space which separates the sun from the earth is very cold. The fact that the earth receives such quantities of heat from the sun shows that this heat can pass through the empty space between the sun and the atmosphere which surrounds the earth. Now it is difficult to see how energy can be transferred from one object to another unless some medium connects these objects. For this reason it is customary to assume that there is a medium known as the **ether** which fills all space, even those portions of space which are occupied by ordinary matter. It is through this medium that the energy travels from the sun to the earth. The parts of the atoms of the sun or any other hot body are in rapid motion. The motion of these parts of the atoms set up waves in the ether. These waves travel from the sun to the earth. When these waves fall on the earth they cause the molecules of the body on which they fall to vibrate more rapidly and the body is thus heated. This movement of heat from one place to another by means of waves in the ether is known as **radiation.** These waves are similar to light waves and travel with the velocity of light. They do not heat the medium through which they pass but they heat any substance on which they fall and by which they are absorbed. The transfer of heat by radiation is therefore a two-fold process: **the conversion of the heat energy of the hot body into a wave motion of the ether and a reconversion of the wave motion into heat by the body on which it falls.**

266. Exchange of Radiations.—All bodies whether cold or hot will radiate some heat. If the bodies are exactly alike in every particular the hot bodies will radiate more heat than the colder ones. Whatever the temperature and whatever the surroundings a certain amount of heat will be radiated by each body. At the same time each body will be receiving heat from the neighboring bodies. Some of this heat will be absorbed and thus go to raise the temperature of the body absorbing it. Whether a body will decrease or increase in temperature under these conditions will be determined by the amount of heat which it radiates in comparison with the amount which it absorbs. If the amount radiated exceeds the amount absorbed the temperature will decrease. If on the other hand the amount absorbed exceeds the amount radiated, the temperature will increase. If the heat radiated is equal to the amount absorbed, the temperature will remain unchanged.

267. Character of the Radiating Surface.—The amount of heat radiated depends not only on the temperature of the body but also on the character and area of the radiating surface. Some surfaces are good radiators, while others are poor radiators. Generally black surfaces which are rough radiate best, but polished surfaces are poor radiators. For example lampblack is an excellent radiating surface as compared to other surfaces. This surface is such a good radiator that it is customary to regard it as a perfect radiator and use it as a standard of comparison for other radiating surfaces. The amount of heat radiated from a surface increases with the area of the radiating surface. For this reason milk coolers are sometimes built of a series of hollow discs.

268. Absorption of Radiation.—The facility with which a surface radiates heat depends on the character of the surface. The same is true of the facility with which it absorbs heat. There is an intimate relation between the rate at which a surface radiates heat and the rate at which the same surface under the same conditions absorbs heat. **This fact is stated by saying that a good radiator of heat is also a good absorber of heat.** In other words surfaces which radiate slowly will also absorb slowly. A surface covered with lampblack was found to be an excellent radiator. If this same surface is exposed to radiations they will be readily absorbed by it.

269. Absorption of Radiations by Soils of Different Colors.—If sunshine falls on a number of bodies of different colors the amount of it which is absorbed by these bodies is determined largely by the color of the bodies. This fact has an important application in the temperature of soils. The amount of heat absorbed by a soil exposed to the solar radiation depends much on the color of the soil. If the same soil is mixed in one case with lampblack and in the other case with powdered chalk so that the one soil is black and the other is as white as possible, the black soil absorbs more radiation than the white one and for that reason becomes 10° or 12° hotter. On the other hand at sunset when radiation is no longer being received by the soils, the black soil will lose its energy more rapidly than the white one and on this account cool more rapidly. This is because bodies which are good absorbers of energy are also good radiators of energy.

270. Absorption of Radiation by the Air.—The radiations emitted by the warm earth are almost completely absorbed by the moist atmosphere above the earth. Hence a moist atmosphere above a portion of the surface of the earth prevents it from cooling by radiation. For this reason frosts occur only when the air is clear and dry. Since the humidity of the air is greater in the neighborhood of a body of water there is less danger of frost in the regions which lie near bodies of water.

The high temperatures obtained in glass houses arise from the fact that the sun's rays pass freely through the glass and are then absorbed by the earth in the greenhouse. The radiation which the warmer earth sends out is almost completely absorbed by the moist air in the greenhouse. Hence the radiation is imprisoned in the greenhouse.

One reason for the diminishing heating effect of the sun's rays as the sun approaches the horizon is the fact that at the horizon the rays must travel through a greater layer of the earth's atmosphere. There occurs in the atmosphere a considerable absorption of radiation and the thicker the layer of air through which these rays travel the greater is the absorption. In Fig. 193 is shown a representation of the air surrounding the earth. From this figure it is seen that the nearer the sun is to the horizon, the greater is the thickness of the layer of air through which the rays pass and the greater the absorption.

271. Reflection of Radiations.—The wave motion which constitutes radiation is reflected by certain surfaces in the same way in which light is reflected. The amount of radiation which is reflected from a surface depends on the character of the surface. Polished surfaces reflect much of the radiation while rough or blackened surfaces reflect little.

Of course surfaces which are good absorbers would be poor reflectors. In order to prevent a body from losing heat by radiation and to prevent it from gaining heat by absorption it is often silvered. The surface is thus made into a mirror which reflects most of the radiation which falls on it.

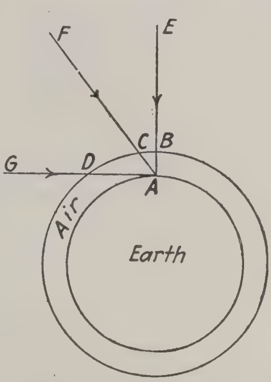


Fig. 193.—Absorption of radiation by the atmosphere.

272. Dewar Flask or Thermos Bottle.—A useful device for keeping things either hot or cold is the modern vacuum or thermos bottle. (Figs. 194 and 195.) This bottle is made up of one silvered glass bottle inside a second glass bottle which is also silvered. The space between the two bottles is free from air or any other gas. Since the space between the glass walls of the bottle is a vacuum there is little opportunity for heat to flow across this space. The residual air in this space is a very poor conductor of heat. When the surfaces are silvered they become good reflectors of whatever radiation falls on them. This very much decreases the amount of energy which might enter the flask by radiation and also the amount of energy which the flask might lose by radiation. If hot bodies are placed in such a bottle there is little chance for the heat to escape from it either by conduction, radiation or convection. Hence the liquid in the bottle remains hot for a long time. On the other hand if a cold body is placed in the bottle there is little chance for heat to flow into it from the outside and the substance inside the bottle will remain cool for a long time.

273. Radiation at Different Temperatures.—The total radiation sent out by a body increases with the temperature. The law governing this increase is only known for a perfect radiator, that is, for a body which has the power to absorb all the radiation falling on it. The total energy emitted by such a radiator is proportional to the fourth power of the absolute temperature. This law can be expressed as,

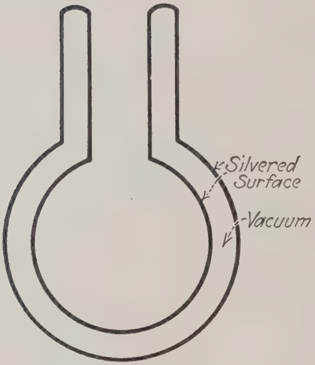


FIG. 194.—Dewar flask.

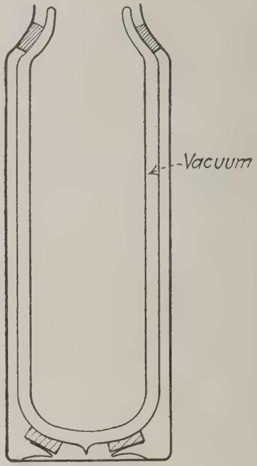


FIG. 195.—Thermos bottle.

$$E = CT^4$$

where E is the energy emitted per unit area of the black body; C, a constant; and T the absolute temperature. If one black body at a temperature T_1 sends radiation to another surrounding it at a temperature T_2, the net radiation sent by the first body to the second is,

$$E = C(T_1^4 - T_2^4).$$

As the temperature of the radiating surface is increased the energy emitted in every wave-length increases but not in the same proportions. Light from a hot radiating surface changes its color as the temperature of the surface is raised. When the body becomes hot enough to be visible, it first appears red. With further rise of temperature the color changes to yellow and then to white. At extremely high temperatures the color is blue-white. The curves in Fig. 196 show the relation between the energy emitted and the wave length. The intensity of the energy is small for radiations of short wave lengths and also for those of long wave lengths. For intermediate wave lengths the energy has its maximum value. As the temperature is increased the height of the maximum is increased and at the same time shifted toward the short wave lengths. This shift of the energy curve produces an increase in the proportion of blue in the emitted light, thus producing the change of color to which reference has already been made. Because the intensity of the visible radiations in an incandescent body is greater at high than at low temperatures, for great efficiency the filaments of incandescent lamps are heated to as high temperatures as possible. Hence the modern tungsten lamp is much more efficient than the carbon incandescent lamp.

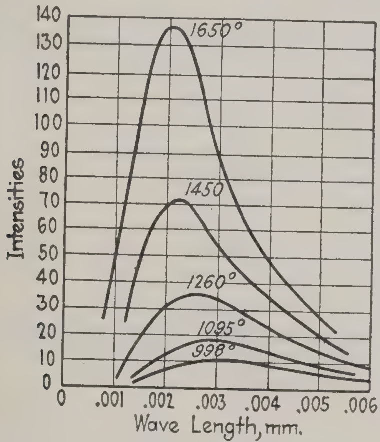


Fig. 196.—Change of intensity with temperature.

274. Solar Constant.—The heat received by the earth from the sun is measurable. It is agreed to call **the solar constant the amount of heat received per square centimeter per minute on a surface which is perpendicular to the rays from the sun.** Some of this radiation is absorbed by the earth's atmosphere so that the amount received on a surface above the earth's atmosphere is greater than the amount received at the surface of the earth. It has been found that about one-third of the total amount of radiation received is absorbed by the atmosphere of the earth. The latest determinations of the quantity received per square centimeter on a surface above the atmosphere is 2 calories per minute when the surface is perpendicular to the direction of the rays.

275. Influence of Slope on Temperature of Soils.—If the rays from the sun strike the surface of the earth at an angle other than 90° the energy in a given bundle of rays is spread out over a larger surface so that the amount of energy received per unit of area is less than it would have been if the rays had been perpendicular to the surface of the earth. This spreading of energy in the rays of the sun over a larger area and the consequent decrease in the amount received per unit area is illustrated in Fig. 197. This fact is of much consequence in determining the temperature of soils which are differently exposed to the sun's rays. Consider the northern and the southern exposure shown in Fig. 198. If a bundle of rays from the sun be represented in width by the line *DG* and this bundle be then divided into two equal parts *DE* and *EG*, it is seen that the southern slope *BH* receives more rays than the equal northern slope *HC*. For this reason southern exposures warm up earlier in the spring than do northern exposures.

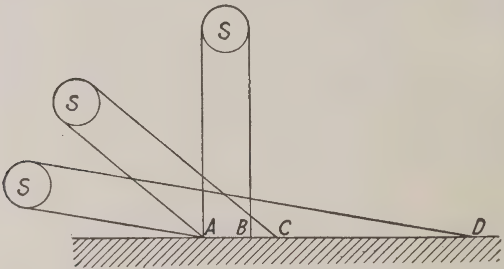


Fig. 197.—Position of sun and intensity of radiation.

Problems

1. If sawdust has a coefficient of thermal conductivity in B.t.u. of 0.4 what is the weight of ice melted per day in an ice house which is 100 × 50 × 50 ft. assuming that the thickness of the wall is 3/4 ft. and the outside temperature is 75°F.?

2. How much water would be evaporated per hour per square foot by

the heat which flows through a boiler plate which is made of iron 1/4 inch thick, when there is a difference of temperature of 125°F. between the faces of the plate? Coefficient of thermal conductity of iron is 160 B.t.u.

3. A refrigerator has in it a glass window having an area of 4 sq. ft. If the window is 1.8 in. thick and the inside temperature is 36°F. and the outside temperature is 70°F., what loss in ice occurs each day?

4. A refrigerator has an area of 100 sq. ft. Its walls are 2/3 ft. thick. The coefficient of thermal conductivity of the walls is 2.2 B.t.u. The temperature of the inside is to be kept at 40°F. and the outside at 80°F. How much heat will flow through the wall in 24 hours?

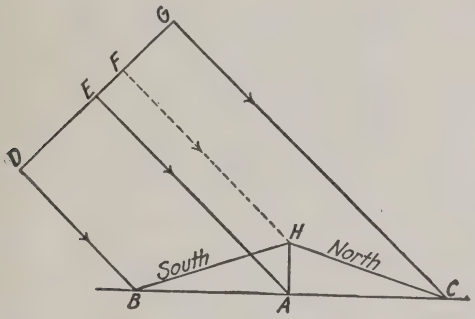


FIG. 198.—Effect of slope on intensity of radiation.

5. A silo has a radius of 10 ft. and a height of 30 ft. It is made of wood which is 2 inches thick. If the temperature outside is 0°C. and the temperature inside is 40°C., how much heat will flow out in one day? Take the thermal conductivity of wood to be 0.002 calories per sq. cm. per sec. per degree C. difference in temp.

6. The temperature inside a fireless cooker is 200°C. and that outside is 20°C. The thickness of the insulating material of the cooker 15 cm. and the walls are made of material having a coefficient of thermal conductivity = 0.0030. If the cooker has an effective area 1,200 sq. cm. how much heat is lost through the wall in 1 hr.?

CHAPTER XXI

HEAT AND WORK

276. Nature of Heat.—It is now believed that heat is the energy which a body possesses by virtue of the fact that its molecules are in motion. In solids these molecules do not much alter their relative positions but vibrate back and forth through positions of equilibrium. The distances through which these molecules vibrate are increased by the addition of heat and the energy of the molecules is also increased. There are ways by which these molecules can be made to vibrate more rapidly and to increase their energy. If two sticks are rubbed together vigorously they become hot. Drills or augurs become too hot to hold in the hand when used to bore hard wood or metal. A grindstone is kept wet with water to keep the tools which are being ground from becoming too hot. In all such cases work is done on the body and heat is developed in consequence of this work. From such experiments it is possible to conclude that work can be transformed into heat.

277. Mechanical Equivalent of Heat.—Since heat is a form of energy it may be converted into other forms of energy and other forms of energy may be converted into heat. According to the law of conservation of energy it is not possible to create or destroy energy. Hence a definite quantity of work is required to produce a given amount of heat. Experiments have shown that this amount of work is the same under all conditions.

The following experiment first performed by Joule shows one method of determining the amount of mechanical work necessary to generate one unit of heat. The apparatus which was used in this experiment is given in Fig. 199. It consists of a calorimeter *C* in which revolves a series of paddle wheels. The calorimeter was fitted with baffle plates having spaces cut in them to allow the paddle wheels to pass. By this means the water in the calorimeter was thoroughly churned. The paddle wheels were driven by two weights which were hung over two pulleys. When the weights were allowed to descend through a given distance the paddle wheels revolved and did work on the water in the calorimeter. This work caused the water in the calorimeter to increase in temperature. By observing the weights and the distance through which they descend, the work done on the water is easily calculated from the relation,

Work = force × distance = weight in pounds × ft.

By measuring the rise in the temperature of the water in the calorimeter, the heat generated is calculated from this rise in temperature together with the mass of water and the thermal capacity of the calorimeter.

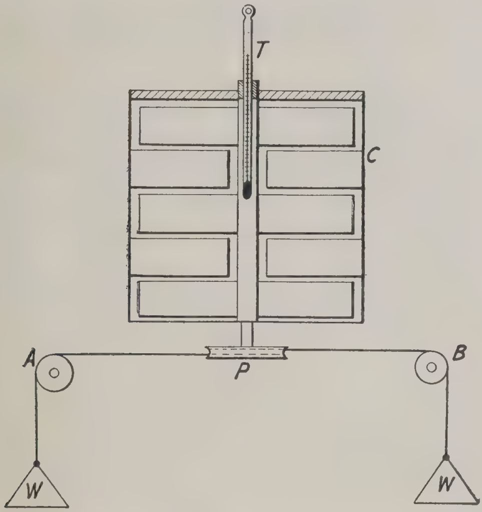


FIG. 199.—Apparatus for mechanical equivalent of heat.

Heat generated = (mass of water + water equivalent of calorimeter) × change of temperature.

Let M = mass of water in pounds.

m = mass of calorimeter.

s = specific heat of material out of which calorimeter is made.

t = initial temperature of water.

T = final temperature.

h = distance through which weights descend.

$2W$ = sum of weights driving the paddle wheels.

Work done on paddle wheels = $2W \times h$ ft.-lb.

Heat generated = $M(T - t) + ms(T - t)$ in B.t.u.

$$\text{Work to generate one B.t.u.} = \frac{2Wh}{M(T - t) + ms(T - t)} \text{ ft.-lb.}$$

Example.—In an experiment on the determination of the amount of work necessary to generate one B.t.u. the following data were taken. Find the number of ft.-lb. to generate 1 B.t.u. Neglect heat given to calorimeter.

Weight hanging over each pulley = W = 50 lb.
Distance each weight descended = 20 ft.
Initial temperature of water = 65°F.
Final temperature of water = 75°F.
Mass of water = 0.52 lb.
Work done = $2 \times 50 \times 20 = 4{,}000$ ft.-lb.
Heat generated = $0.52(75 - 65) = 5.2$ B.t.u.

$$\text{Work per B.t.u.} = \frac{4{,}000}{5.2} = 770 \text{ ft.-lb.}$$

Mechanical equivalent of heat is defined as the number of units of work necessary to generate one unit of heat. In the English system the mechanical equivalent of heat is the number of foot-pounds necessary to generate one B.t.u. In the metric system it is the number of gram-centimeters or kilogram-meters required to develop one calorie.

To generate one B.t.u. according to the latest determinations requires 778 ft.-lb. and to generate one calorie requires 4.26×10^4 gram-cm.

Problems

1. A tank filled with water contains 4 gallons. It is fitted with a stirrer which takes a 0.5 horsepower engine to drive it. If all the work were converted into heat how long will it take to raise the temperature of the water from 30° to 65°F.?

2. A train has a mass of 300 tons. It is running at the rate of 30 miles per hour when the brakes are applied. Find the heat generated if all the work done against the frictional resistance is converted into heat.

278. Work from Heat.—More important for us than the transformation of work into heat is the transformation of heat into work. On this transformation depends all the heat engines which play such a large part in modern life. This is exactly the reverse operation to that which was described in Joule's experiment. The heated steam in the cylinder of a steam engine does work in pushing the piston back. This work is available for driving the machinery connected to the engine. A gasoline engine can drive an automobile or a tractor only when it is supplied constantly with heat from the exploding gasoline in the cylinders. In these cases heat is transformed into work.

279. Steam Engine.—The simplest form of the cylinder of a steam engine is that shown in Figs. 200 and 201. It is a cylinder in which moves a closely fitting piston. This cylinder is connected to the steam chest by means of two pipes *A* and *B*. These pipes which are provided with valves serve alternately as inlet and exhaust for the steam. As the piston moves forward steam enters through *A* and the used steam is forced out through *B*. When the piston moves in the opposite direction, steam enters the cylinder at *B* and used steam is forced out at *A*. With this simple arrangement the steam would leave the cylinder on exhaust

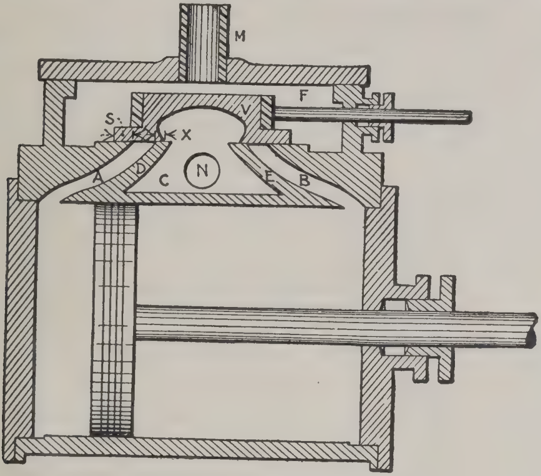


Fig. 200.—Engine cylinder and plane slide valve.

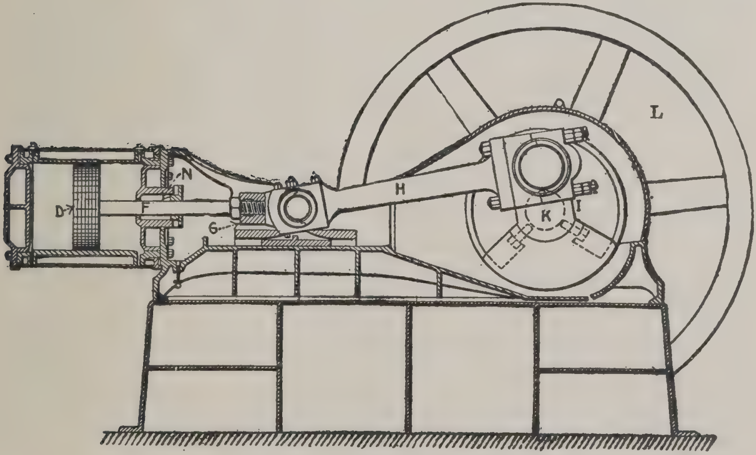


Fig. 201.—Steam engine.

at a temperature nearly as high as that at which it entered it. A considerable quantity of heat would thus be carried to the condenser or the outside air and lost so far as useful work is concerned. The efficiency of the engine is thus decreased. In order to prevent as far as possible this waste, an automatic cut-off is provided. When the piston has moved through about one-fourth of its stroke, this slide valve automatically cuts off the supply of steam.

After this cutting off of the steam from the steam chest, the steam that has already entered the cylinder expands and pushes the piston forward, through the remainder of the stroke. During this expansion the piston is doing work, the pressure of the steam is being reduced and its temperature lowered. The heat contained in the steam is thus converted into useful work and the efficiency of the engine is increased. For it must be remembered that this heat would otherwise have been lost to the outside air without producing any useful work.

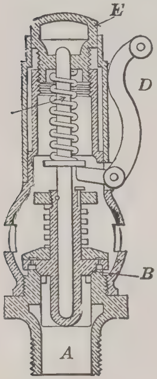


Fig. 202.—Cross section of pop valve.

When the piston has reached the end of this stroke the slide valve opens *A* and connects *B* to the steam chest. Live steam is now again admitted to the cylinder behind the piston and it pushes the piston toward the left. The dead steam in front of the piston is forced through *A*. When as before the piston has made about one-fourth of its stroke, the slide valve closes *B* and the steam behind the piston expands until the piston has reached the end of its stroke. The cycle is then repeated.

The pressure of the steam in the boiler is regulated by means of a pop valve (Fig. 202) which allows the steam to escape when the pressure exceeds a certain value.

280. Gas Engine.—Gas engines and gasoline engines operate on the same principle. In each case the energy is derived from the explosion of a mixture of air and gas or gasoline vapor. The gasoline engine is now the most common type of engine. It is used to drive motor cars, motor boats, tractors, etc.

A diagram of the common four-stroke cycle gasoline engine is shown in Fig. 203. This engine makes four strokes or two revolutions of the fly wheel for each power stroke. The first is the charging stroke, the second the compression stroke, the third the power stroke and the fourth the exhaust stroke. The charging stroke is represented in (1). The fly wheel pulls the piston forward and a mixture of gasoline vapor and air is taken in through the intake valves *A*. During this stroke the cylinder is filled with a mixture of air and gasoline vapor. When the cylinder has been filled the valve closes and the piston moves back and compresses the mixture of air and gasoline vapor in the cylinder to about one-fifth of its original volume. When the compression stroke is near its end a spark from an induction coil or magneto ignites this mixture and causes an explosion which forces the piston forward with considerable velocity. The valve *B* is opened and the burnt gases are forced out through this valve. The momentum of the fly wheel is sufficient to carry the piston through three of these strokes. After the burned gases have been forced out of the cylinder the cycle is again repeated. Out of four strokes such an engine gives only one working stroke.

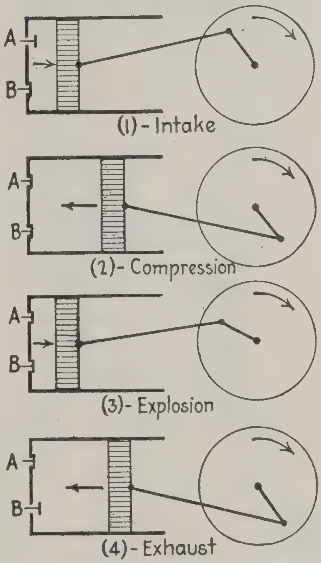


Fig. 203.—Four-stroke cycle gas engine.

281. The Two-stroke Cycle Engine.—The principle of the two-stroke cycle internal combustion engine is shown in Fig. 204. On the upward stroke of the piston *P* the pressure is reduced in the crank case *C* and the explosive mixture of air and gasoline vapor is drawn in through the valve at *A*. At the same time a quantity of this mixture previously taken into the space above the piston *P* is compressed. Near the end of this compression the explosive mixture is fired by means of the spark from the magneto. The spark plug is shown at *S*. This explosion produces an increase of pressure in the cylinder behind the piston *P*. This pressure drives the piston down and this is the working stroke of the piston. As the piston descends it compresses the mixture in the crank case. During this downward stroke the admission valve *A* is closed and the exhaust valve *E* is also closed. Near the end of the downward stroke the exhaust valve *E* is opened and the burned gases are allowed to escape to the atmosphere. The piston continues its downward stroke and soon opens the port *I* through which the slightly compressed gases in the crank case are forced into the space *W* above the piston.

The port *I* and the exhaust port *E* are opened and closed by the piston. The inlet valve in the crank case is operated as the valves in the other type of engine.

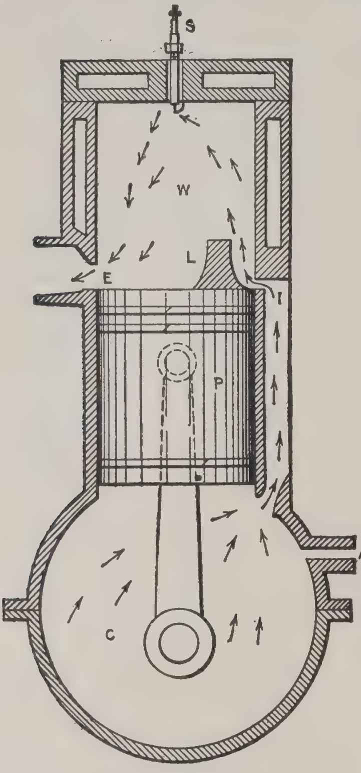


Fig. 204.—Two-stroke cycle engine.

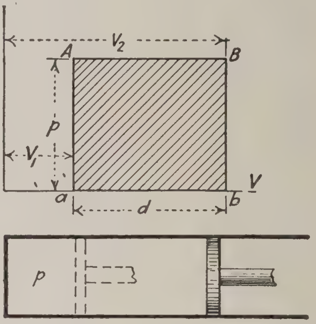


Fig. 205.—Work done by a gas at constant pressure.

282. Work Done by a Gas Expanding at Constant Pressure.—The work done by a gas expanding at constant pressure affords an easy illustration of mechanical work derived from heat. Let a volume of gas be enclosed behind a piston (Fig. 205) which is air tight and moves without friction. Let the pressure acting on the piston per unit area be denoted by p and the area of the piston by A. The total force acting on the piston pushing it backward is,

$$F = pA$$

If now the gas in the cylinder is heated without allowing any change in its pressure, the piston moves back through a distance d.

Work done on piston = force × distance

$$= p \times A \times d$$

Now $A \times d =$ increase of volume of the gas during expansion Hence

$$\text{Work} = p \times \text{change in volume}.$$

It is convenient to represent the work done by the gas by plotting the pressure of the gas on the vertical axis and the volume of the gas on the horizontal axis. In this case the pressure is constant for all volumes. Hence AB represents the relation between the volume and the pressure. If V_1 denotes the original volume and V_2 the final volume, the length of the line ab represents the change in volume during expansion. The product of the change in volume and the pressure is represented by the rectangle $ABba$. This area then stands for the work done by the gas during its expansion and since this work is equal to the heat supplied to the gas during expansion, this rectangle also represents the heat taken in by the gas during its expansion.

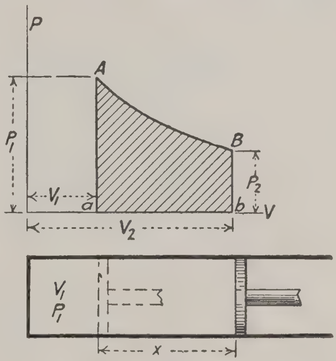


FIG. 206.—Work done by a gas at variable pressure.

283. Work Done by a Gas Expanding at Variable Pressure.—If the gas expands under variable pressure, the work which it performs may be represented in the way in which it was represented for a gas expanding at constant pressure. In this case the line AB (Fig. 206) instead of being horizontal as in the preceding case slopes toward the horizontal axis along which the volumes are plotted. Nevertheless the area under AB will represent the work done by the gas as it expanded under the action of a changing pressure. It would be possible to construct a rectangle having the same base as the figure $ABba$ and the same area. The height of this rectangle would be the average pressure of the gas during its expansion. If the varying pressure acting on the piston has been replaced by a constant pressure equal to the average pressure, the work done by the gas would remain unchanged. **The average pressure may then be defined as the constant pressure by which a varying pressure may be replaced without changing the amount of work done on the piston for the same stroke.**

284. Positive and Negative Work.—In the preceding cases we have discussed the work done on a piston by the expanding gas. Such work must be considered **positive** for it is work delivered by the engine.

In order to force the dead steam or the burned gases out of the cylinder the piston on its return stroke must exert a pressure and must therefore do work. This is the work done to return the piston to its initial position ready for a new forward stroke. It is work done by the engine on the useless gases. It represents a waste or loss which must take place in order to make the working stroke of the piston possible. To distinguish this work from the work done on the piston by the gases in the forward stroke it is called **negative.** This work done on the gases by the piston must be subtracted from the work done on the piston by the gases in order to get the useful work done by the piston in one complete cycle.

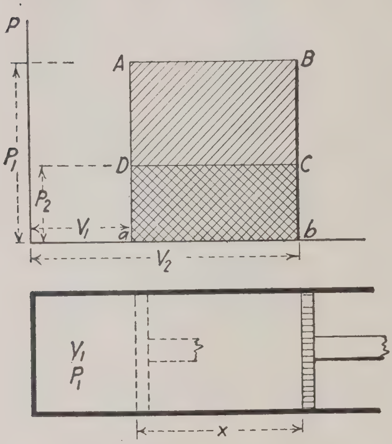


Fig. 207.—Positive and negative work.

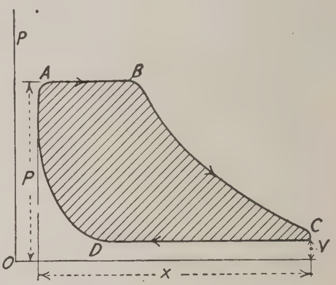


Fig. 208.—Indicator card for steam engine.

This work done on the gases by the piston may also be represented by an area such as that used to represent the work done by the gases on the piston. One of these areas, however, represents the work done on the piston and the other represents work done by the piston. The net work will be the difference between these areas. If the gas expands at constant pressure and is later compressed at a lower constant pressure, the net work done by the piston is represented by the difference between the areas *ABba* and the area *DCba*, i.e., by the rectangle *ABCD*. (Fig. 207.)

285. Indicator Card.—If the gases expand and are compressed under varying pressures, the net work may still be obtained by this method. A case of this kind is shown in Fig. 208. The area under the line *ABC* represents the work done on the piston during the expansion and the area under the line *ADC* represents the work done by the piston on the gases during compression. The useful or net work obtained from the cycle is represented by the area of the irregular figure *ABCDA*. It is the work which is left after the work of returning the piston is deducted. Such a figure is called an **indicator card.** The indicator card for a steam engine is essentially like Fig. 208 and in Fig. 209 is given an indicator card of a four-stroke gas engine. It gives a measure of the work performed by the engine in one cycle.

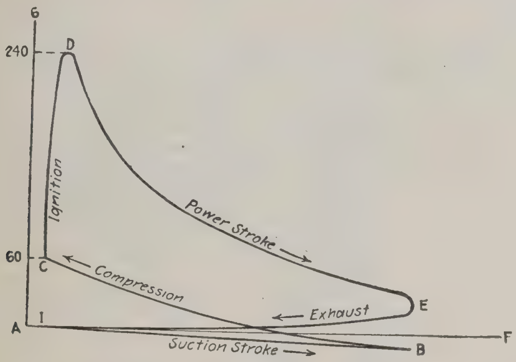


Fig. 209.—Indicator card for gas engine.

286. Indicator.—In order to make an indicator card from which to determine the horsepower of an engine a device known as an **indicator** is used. The essential part of such an instrument is a small vertical cylinder (Fig. 210) containing a movable piston. This cylinder is in communication with the steam in the cylinder of the engine. This piston is held down by a spring which will be compressed as the steam exerts its pressure on the piston. The compression of the spring is made to raise an arm at the end of which is a pencil which records its movements on a revolving drum. The drum is covered with paper and its motion is controlled by the motion of the piston rod. In this way a diagram showing the unbalanced pressure in the cylinder of the engine for each part of the stroke is obtained. From the average pressure and the length of the stroke the horsepower of the engine can be calculated.

287. Indicated Horsepower of an Engine.—In order to measure the horsepower of an engine it is necessary to know, (1) the length of the stroke in ft., (2) the number of revolutions made each minute, (3) the area of the piston, (4) the average unbalanced pressure in pounds per square foot on the piston. The length of the stroke and the area of the piston are easily measured and the number of revolutions per minute counted. The average unbalanced pressure of the steam in the cylinder is found from the indicator card in the following manner. Take the area of the figure *ABCDA* (Fig. 208) and find its mean height. This is equivalent to finding a rectangle having the same base as *ABCDA* and the same area. The height of this equivalent rectangle is is the mean effective pressure which means that if the pressures actually operating in the cylinder had been replaced by a constant pressure equal to this mean effective pressure, the work done by the engine would have been unchanged. To find the horsepower multiply the mean effective pressure in pounds per sq. ft. by

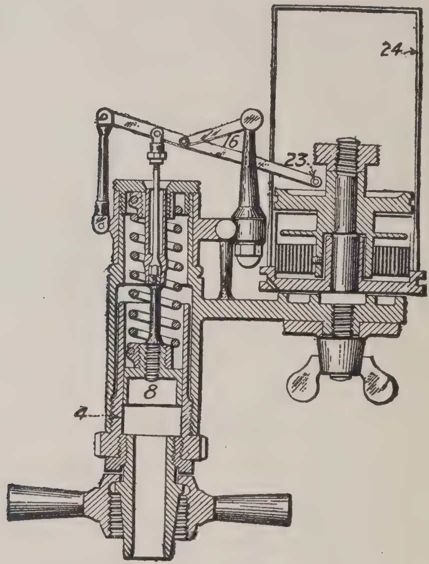


FIG. 210.—Steam engine indicator.

the area of the piston in sq. ft. This gives the average force acting on the piston. Multiply this force by the length of the stroke. This gives the work done in each stroke in ft.-lb. Multiply the work done in each stroke by the number of strokes per minute. The result is the work done per minute. By dividing this work by 33,000, the number of ft.-lb. per minute to make 1 horsepower, the indicated horsepower of the engine is obtained.

$$\text{Horsepower} = \frac{\text{Average pressure} \times \text{length of stroke} \times \text{area of piston} \times \text{No. rev.}}{33{,}000}$$

$$= \frac{PLAN}{33{,}000}$$

Example.—In a steam engine the average pressure was 40 lb. per sq. in., the length of the stroke was 12 in. and the number of revolutions per minute 300 and the area of the piston 125 sq. in. Find the horsepower of the engine.

Force on piston = pressure × area = 40 × 125 = 5,000 lb.
Work per stroke = force × length of stroke = 5,000 × 1 ft. = 5,000 ft.-lb.
Work per minute = work per stroke × no. strokes per minute
= 5,000 × 300 = 1,500,000 ft.-lb.

$$\text{Horsepower} = \frac{1{,}500{,}000}{33{,}000} = 45.3 \text{ horsepower}$$

288. Brake Horsepower.—The brake horsepower measures the power which the motor or engine can deliver at the shaft or belt for work such as driving a threshing machine or pulling a tractor. This power is measured by what is known as a Prony brake. This brake which is shown in Fig. 211 consists of two wooden blocks *BB* which are clamped to the pulley *P* by means of thumb screws. From one of these blocks projects a lever *L* which rests on a pair of platform scales. When the shaft *P* is rotating there is a tendency to drag the blocks with it and the lever presses on the platform scales with a certain force which

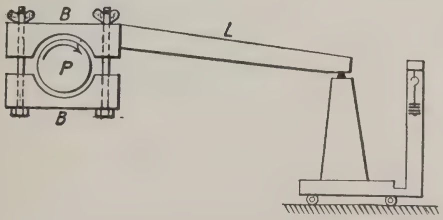


FIG. 211.—Prony brake for measuring horsepower.

depends on the force exerted on the blocks by means of the pulley. The force of the pulley on the blocks is better seen in Fig. 212 which represents a belt passing around the pulley. The difference between the readings on the spring balance is equal to the force of friction between the belt and the pulleys. The horsepower of the engine is calculated from the weight registered on the scales in the following way. Let L be the length of the brake arm in feet, W the weight registered on the scale expressed in pounds and N the number of revolutions which the engine or motor makes per minute.

$$\text{Brake horsepower} = \frac{2\pi L \times W \times N}{33{,}000}$$

Example.—The reading of the scale of a Prony brake when the engine was making 300 revolutions per minute was 75 lb. The length of the brake arm was 4 ft.

Find the horsepower of the engine.

$$\text{Brake horsepower} = \frac{2 \times 3.1416 \times 4 \times 75 \times 300}{33{,}000} = 1.71 \text{ horsepower}$$

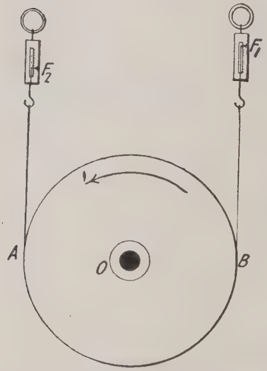


FIG. 212.—Forces operating on a Prony brake.

289. Efficiency.—Since heat and work are convertible, the most important thing to know about any device for this purpose is its efficiency which gives a measure of the amount of heat which can be transformed into work under a given set of conditions. **The ratio of the heat put into the machine to the work obtained from it is called the efficiency of the machine.** In finding, for example, the efficiency of a burner used to heat a kettle of water it is necessary to find the amount of gas consumed by the burner and the amount of heat thus developed. It is next necessary to find the amount of heat which gets into the water in the kettle. The ratio obtained by dividing the heat which gets into the water by the heat developed by the burning of the gas is called the efficiency of the burner. The efficiency may be defined as the fraction which tells what portion of the total heat supplied is used for the purpose for which it was intended,

$$\text{Efficiency} = \frac{\text{heat used}}{\text{heat supplied.}}$$

The operation of transforming heat into work is inefficient because only a small fraction of the heat developed by the combustion of the fuel is converted into work. In a steam engine a part of the heat must pass over with the exhuast steam and be lost in the condenser. This loss is so great that even the best steam engines do not use more than about 18 per cent of the heat of combustion of the fuel. An ordinary locomotive uses about 8 per cent of the heat generated by the combustion of its fuel. In a gas engine provision must be made for removing the excess heat. This is done by circulating air or water. An air-cooled engine is represented in Fig. 213.

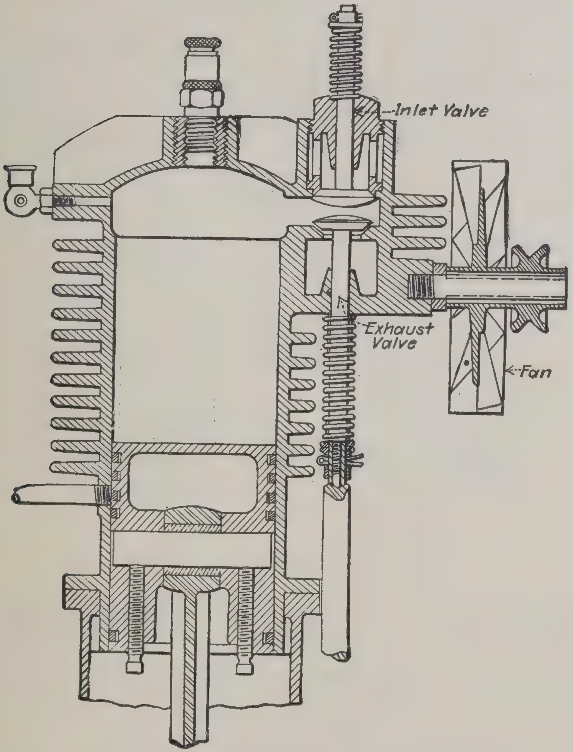


Fig. 213.—Air-cooled engine.

Example.—A pound of coal which has a heating value of 8,000 B.t.u. was burned in a machine which raised 500 gallons of water 100 ft. What percentage of the heat from the coal was converted into useful work?

Heat supplied = 8,000 B.t.u.
Work done = 500 × weight of water per gallon × 100
= 500 × 8.3 × 100 = 415,000 ft.-lb. = 533 B.t.u.

$$\text{Efficiency} = \frac{\text{heat used}}{\text{heat supplied}}$$

$$= \frac{533}{8{,}000} = 6.7 \text{ per cent}$$

290. Refrigerating Machines.—The process of reducing the temperature of a body below that of its surroundings is essentially the reverse of the process employed in the steam engine or in the gasoline engine. In the engine the steam passes from the boiler to the cylinder at high temperature. In the cylinder it gives up some of its heat. This heat is transformed into work which drives the piston forward. In a refrigerating machine the working substance is taken into the cylinder at a low temperature. Here it is compressed and thus heated in consequence of the work done on it during compression. This heat is removed by water circulating around the cylinder in which the compression takes place. The working substance then expands or vaporizes and during this expansion or vaporization it takes heat from the surrounding bodies and thus cools them below their original temperatures.

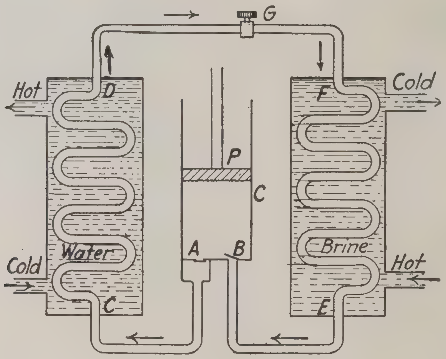


Fig. 214.—Refrigerating machine.

One of the common forms of refrigerating machines is one in which ammonia is used as the working substance. The cooling effect is in this case obtained from the absorption of heat by the vaporization of the ammonia. A simple diagrammatic sketch of this type of machine is shown in Fig. 214. A force pump *C* is closed by two valves *A* and *B*. One of these opens outward from the cylinder and the other opens into the cylinder. On the upward stroke of the piston the valve *B* is opened and ammonia gas is drawn into the cylinder from the pipes immersed in the cooling chamber *FE*. On the downward stroke of the piston the valve *B* closes and the valve *A* is opened. The ammonia gas is compressed and on account of this compression the vapor is heated. In spite of this heating some of the vapor is changed into the liquid condition. The mixture of liquid and vapor is forced into the coils *CD* where it is cooled by water circulating around these coils. Here all of the vapor becomes a liquid. After this cooling the liquid ammonia is allowed to escape through a regulating valve *G* into the coils *FE* which are immersed in the brine to be cooled by the refrigerating machine. In these coils the pressure is low because this space is continually being exhausted by the pump. The liquid ammonia under a reduced pressure in these pipes evaporates and changes back to vapor. In this process of evaporation heat is absorbed. The heat necessary to evaporate the ammonia comes from the coils *FE* and whatever liquids may surround them. The cooling effect in the case described is sufficient to reduce the temperature of the water below the freezing point of water. By immersing the coils *FE* in brine and then placing in the brine vessels filled with pure water, ice may be frozen. It is in this way that artificial ice is manufactured.

Problems

1. How many hours will a 50 horsepower gasoline engine with an efficiency of 20 per cent run on 8 gallons of gasoline?

2. A heat engine has an efficiency of 15 per cent. It burns 250 lb. of coal in one hour. What horsepower does it develop if each pound of coal gives 14,500 B.t.u.?

3. The flow of water from the tail race of a mill is 1,500 cu. ft. per minute. If the head of water is 20 ft. what horsepower will a water wheel give which uses 9 per cent of the energy supplied by the water?

4. How many cubic feet of water must be held at a height of 300 ft. to store an amount of energy which is as large as that contained in a ton of coal which yields 14,000 B.t.u. to the pound?

5. It requires about 400 B.t.u. per hour for the natural processes of the human body. Considering the body as a power plant with an efficiency of 25 per cent how many B.t.u. must be supplied to the human body each day?

6. How far will a pound of coal containing 13,500 B.t.u. propel a tractor weighing 10 tons against a rolling friction of 16 lb. per ton, if 25 per cent of the heat energy is used up in work?

7. If a field containing one acre receives energy at the average rate of 4 B.t.u. per square foot per minute for 10 hours per day, how many tons of coal each yielding 12,000 B.t.u. would be required to supply an equal amount of energy?

8. If each square centimeter of the earth's surface receives 3.5 calories per minute from the sun, how many horsepower does a farmer receive who owns a farm containing 100 acres?

9. Ice is used to cool a Prony brake which is absorbing 7.5 horsepower. How much ice must be supplied per hour if the water escapes at 10°C.?

10. A saw used to cut marble takes 10 horsepower. How much water must be supplied per minute to keep the work and saw at a temperature of 60°F., if the temperature of the water is 40°F.?

11. A Prony brake is found to be absorbing energy at the rate of 5 horsepower. Cold water is run in continuously at 40°F. and leaves the brake at 120°F. How much water flows through the brake in order to keep its temperature constant?

PART III.—ELECTRICITY AND MAGNETISM

CHAPTER XXII

MAGNETISM

291. Natural Magnets.—A peculiar mineral called lodestone was found in early times in the neighborhood of Magnesia in Asia Minor. This mineral has the power of attracting small particles of the same mineral and of setting itself in one particular direction when suspended. When a piece of this lodestone is dipped into iron filings they adhere to it. It is found that there are two places on each piece of this mineral at which the iron filings adhere in greatest quantities. If such a piece of lodestone is suspended by means of a silk thread, it is found that the line joining the places at which the iron filings adhered in greatest quantities points north and south.

292. Artificial Magnets.—When a steel knitting needle is stroked from one end to the other with a piece of lodestone, using for point of contact one of the points at which the iron filings adhered most freely, the needle acquires the property of attracting iron filings and of setting itself north and south when suspended. Such a needle is called an **artificial magnet.** There are other ways in which more powerful artificial magnets can be made. The properties of these magnets do not differ in any way from those of the needle, except that they are much more powerful.

293. Magnetic Poles.—On dipping a magnetized needle into iron filings it is seen that the iron filings adhere most freely at the ends of the needle. These places at which the tendency of the iron filings to cling to the needle is largest, are called **poles.** If that end of the needle which points north is marked, it will be found that however the needle may be disturbed it will come to rest with the marked end pointing north. For convenience it is desirable to call the end of the magnetic needle which points north **the north-seeking pole or N-pole.** The other end is called the **south-seeking pole or the S-pole.**

294. Force between Poles.—The first law of magnetism states that **like poles repel each other but unlike poles attract each**

other. Thus two N-poles repel each other and two S-poles repel each other but a N-pole and a S-pole attract each other.

295. Molecular Theory of Magnetism.—Every substance which is capable of being rendered a magnet consists of a very large number of small parts which are very small magnets. These parts are sometimes called molecular magnets for they are probably no larger than the molecules out of which the substances are made. When the substance is unmagnetized these molecular magnets are not arranged in any particular direction but are oriented indiscriminately. When the substance is magnetized a larger number of these little magnets are made to point along the axis of the magnet than point in any other direction. In the interior of the magnet the little north poles lie so close to the little south poles that each destroys the influence of the other. At one end of the bar are free north poles and at the other end are free south poles. The sum of all these little north poles makes the N-pole of the magnet and the sum of all the little south poles makes the S-pole of the magnet.

Take a knitting needle which has been magnetized and after marking its north pole break it into two nearly equal parts. Suspend each of these parts in turn and test their polarity. It will be found that each part has become a magnet and that two new poles, a N-pole and a S-pole have been developed by breaking the magnet. A further breaking will produce additional poles and this process may be continued until the magnets become as short as we please. It would seem from such an experiment that a magnet is made up of a large number of very small magnets which are directed along the axis of the magnet.

Evidence in favor of the molecular theory of magnetism is also seen in the following experiment. Place a bar magnet on a piece of wood sufficiently large to float it in a vessel of water. The magnet is free to move in any direction but it only rotates until it points north and south. If one of the poles of the magnet were stronger than the other there would be a force tending to move the magnet bodily either toward the north or toward the south. There is no such tendency but only a rotation around an axis through the center of the magnet. This shows that the total amount of magnetism on the north pole is just equal to the total amount on the south pole.

296. Law of Force between Magnetic Poles.—Experiments show that the **force with which the poles of two magnets attract or repel each other, in air, is equal to the product of the pole strength divided by the square of the distance between the poles,** or expressed mathematically,

$$F = mm'/d^2.$$

where F = force in dynes. (See Appendix C.)
d = distance in centimeters between the poles.
m = strength in unit poles of one pole;
m' = strength in unit poles of the other pole.

297. Unit Pole.—**A unit pole is a pole of such strength that it will repel a similar pole of equal strength with a force of one dyne when placed one centimeter away from it in air.** An isolated north pole is not a physical possibility since every magnet must have an equal south pole for every north pole. It is possible to realize nearly enough the conditions assumed in this definition by supposing that the magnets are very long so that the south poles are so far away that their influence is small. The number of unit poles on the end of a magnet is measured by number of dynes of force which it will exert on a unit pole 1 cm. from it in air.

Example.—Find the force on a N-pole of strength 40 units when placed in air 30 cm. from a like pole of strength 20 units.

$$F = m'm/d^2 = \frac{40 \times 20}{302} = \frac{800}{900} = 8/9 = 0.89 \text{ dynes}$$

298. Magnetic Field.—The space outside the magnet in which its influence can be detected is called the **magnetic field.** In the case of powerful magnets this space extends far from the magnet but in the case of feeble magnets the magnetic field is confined to a small region near the magnet.

At every point near a magnet or a system of magnets a free magnet pole would experience a force tending to drive it in a definite direction. The direction in which this free pole would move is called the direction of the magnetic field at that point. The force which unit pole experience at that point is known as the **intensity of the magnetic field.** **The intensity of the magnetic field at a point may also be defined as the force necessary to hold unit magnetic pole in position at that point.** The intensity of the field is measured in dynes.

For example if the intensity of the magnetic field at a point is 1,000 dynes, it will take 1,000 dynes of force to hold a unit magnetic pole in position at that point.

299. Magnetic Lines of Force.—In order to make it easier to understand the way in which the magnetic field changes from point to point, it is convenient to draw certain lines (Fig. 215) which by their direction represent the direction of the magnetic field and by their number represent the intensity of the magnetic field. Such lines are called **lines of magnetic force.** Such a line of magnetic force is the path along which a perfectly free N-pole would travel when left alone in the magnetic field. Since such a free N-pole can not be obtained in practice, a small compass needle is used to determine the direction of the magnetic field at a point. When the compass needle comes to rest its axis points in the direction of the magnetic field at that point.

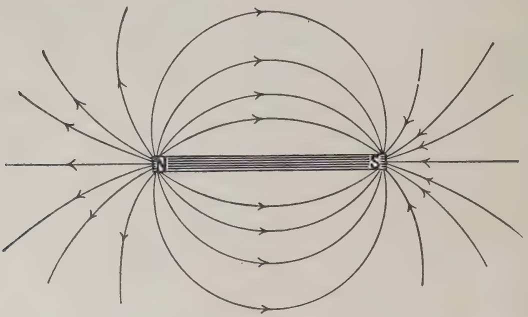


Fig. 215.—Magnetic lines of force.

300. Earth's Magnetic Field.—The action of the compass needle in pointing north and south is explained by the fact that the earth itself acts as if it were a great magnet with a S-pole near the geographical north pole and a N-pole near the geographical south pole (Fig. 216). It is only the horizontal part of the field from this large magnet that is effective in turning the compass needle. The needle itself remains horizontal because it is

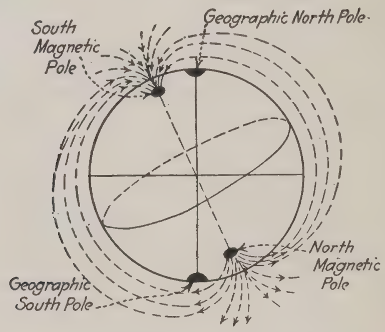


Fig. 216.—The earth—a magnet.

ordinarily adjusted in a stirrup or on a needle point until it assumes a horizontal position.

If a needle is carefully balanced (Fig. 217) so that it is free to rotate in a vertical plane and is then magnetized, it will be found that in the northern hemisphere the N-pole of the needle dips downward and in the southern hemisphere the S-pole dips downward. The earth's magnetic field is therefore inclined to the horizontal and the amount of this inclination varies from point to point on the surface of the earth.

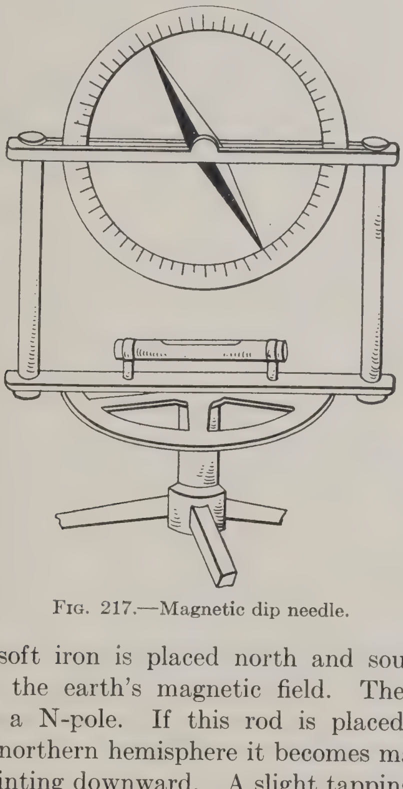


Fig. 217.—Magnetic dip needle.

If a rod of soft iron is placed north and south it becomes magnetized by the earth's magnetic field. The end pointing north becomes a N-pole. If this rod is placed in a vertical position in the northern hemisphere it becomes magnetized with a north pole pointing downward. A slight tapping assists in the production of this magnetization.

301. Declination.—The early users of the compass were aware that it did not always point exactly north and south but that the direction of the needle changed as it is moved from point to point on the surface of the earth. The reason for this variation lies in the fact that the earth's magnetic poles do not coincide with the geographical poles (Fig. 216). There are also local causes which influence the direction in which the compass needle points. **The number of degrees between the true north and south line and the axis of the needle is called the declination of the needle.** To determine the declination at any place, it is first necessary to determine the geographical meridian and then the magnetic meridian. The former is determined from astronomical observations and the latter by noting the direction in which the axis of a compass needle points at that place. The angle between these two directions is the declination of the compass needle at that place.

302. Magnetic Dip.—The angle which a magnetic needle free to move about a horizontal (Fig. 217) **axis makes with the horizontal is called the angle of dip.** This angle varies from place to place on the earth. At the earth's magnetic equator it is zero and at the earth's magnetic north pole it is 90°. This angle is measured by means of a dip circle. This consists of a vertical circle at the center of which is suspended a magnetic needle so that it is free to rotate about a horizontal axis. When the plane of the circle is placed in the magnetic meridian, the magnetic needle dips below the horizontal. The angle of this dip is read on the divided circle.

Problems

1. A magnetic pole whose intensity is 15 units acts with a force of 8 dynes on another pole at a distance of 12 cm. in air. Find the strength of the second pole.

2. With what force will a N-pole of strength 8 attract a S-pole of strength 15 when the poles are 4 cm. apart in air?

3. Calculate the force in dynes with which a N-pole having 800 units of pole strength attracts a south pole of 500 units of pole strength, when the two poles are 20 cm. from each other in air.

4. Find the force necessary to hold a magnetic pole having a pole strength of 5 units in a magnetic field whose intensity is 5,000 dynes.

5. A magnet has a pole strength of 500 unit poles. It is placed in a magnetic field having an intensity of 7,000 dynes. What is the force acting on it?

6. A magnet pole of 25 units strength is 30 cm. from a similar pole of 30 units strength. Find the force which one pole exerts on the other.

CHAPTER XXIII

ELECTRICAL NATURE OF MATTER

303. Two Kinds of Electricity.—If a pith ball is hung from a support by a silk thread, and a rubber rod which has been stroked with cat's fur and thus electrified is brought near it, the pith ball is at first attracted to the rod. If the pith ball is allowed to come in contact with the rod, some of the electricity on the rod is imparted to the ball. It is now found that the ball is repelled by the rod. When a glass rod which has been rubbed with silk is brought near the same pith ball carrying the charge which it received from the rubber rod, the charged pith ball is attracted by the electrified glass rod. There are then two kinds of electricity, that which is generated by rubbing an ebonite rod with cat's fur and that which is generated by rubbing a glass rod with silk. These charges differ in one important respect. **Charges which are alike repel each other and charges which are unlike attract each other.** That kind of electricity which is generated when a glass rod has been rubbed with silk is called **positive electricity** and the kind which is generated when an ebonite rod is rubbed with cat's fur is called **negative electricity.**

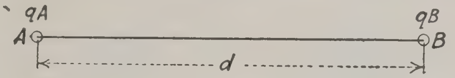


FIG. 218.—Law of electric force between charges.

304. Law of Electric Force.—If two point charges of electricity of opposite sign are in the neighborhood of each other (Fig. 218), they will exert attractive forces on each other. If they are of the same sign they will exert repulsive forces on each other. The force which one exerts on the other is determined by the distance between the charges and the magnitude of the charges. Experiments have shown that the force is inversely proportional to the square of the distance between the charges and directly proportional to the product of the charges. Thus,

$$F = K \frac{q_A q_B}{d^2}$$

where F = the force B exerts on A and also the force which A exerts on B,
d = the distance between the charges and q_A and q_B the charges of electricity located at A and B respectively. If these charges are the same, that is both positive or both negative, the force will be a repulsion pushing the charges apart. If the charges are unlike, that is one positive and the other negative, the force will be an attraction pulling the charges together.

This law of force gives a convienent method of defining the unit of electrostatic charge. **Unit electrostatic charge is defined to be that charge which when placed one centimeter from an equal charge of like sign in a vacuum, will repel it with a force of one dyne.**

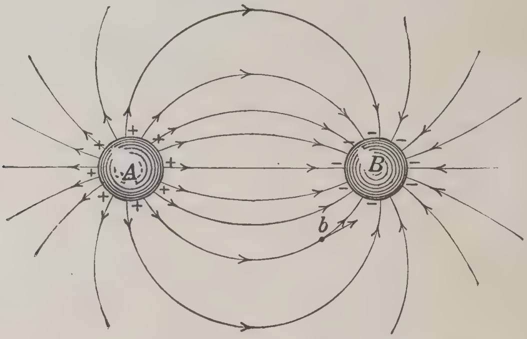


Fig. 219.—Lines of force between unlike charges.

305. Electrical Field of Force.—It has just been seen that if one charged body is brought into the neighborhood of another, there is a force of attraction or repulsion between them and that this force depends on the distance between the bodies, being largest where the distances are least and least where the distances are greatest. However large the distance, there is always some force from a charged body though this force may be quite small. The region surrounding one or more charged bodies is known as the **electrostatic field.** It is frequently represented by drawing lines which represent the direction in which the force on unit positive charge acts at different points in the neighborhood of the bodies. Such lines are called **lines of forces.** They show the direction in which a positive charge would move if it were placed in the field of force. Lines showing the field of force due to two unlike charges and to two like charges are shown in Fig. 219 and Fig. 220 respectively.

The intensity of an electric field is determined by the force which unit positive charge experiences when placed in it. Suppose that *A* (Fig. 221) is a charged sphere having an electrostatic field about it. The intensity of this field at *P* is the force which would be required to hold unit positive charge in position at *P*. The force on *q* units of electricity at *P* would be *q* times as great as the force on unit charge. If *E* denotes the intensity of the electric field at *P* and *q* the number of unit charges located at *P*, then the force *F* on this charge is,

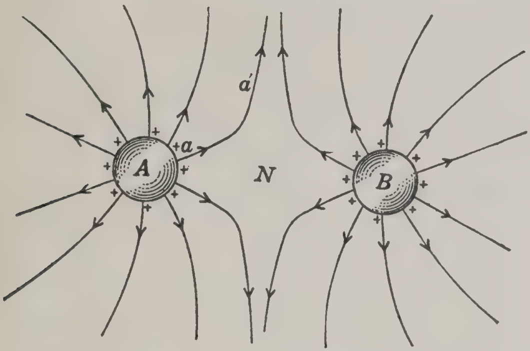


FIG. 220.—Lines of force between like charges.

$$F = Eq \text{ dynes.}$$

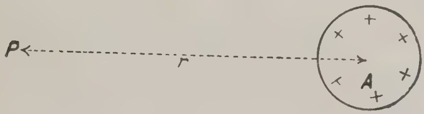


FIG. 221.—Definition of the intensity of an electric field.

An electrostatic field has an intensity of unity when it exerts a force of one dyne on unit charge at that point.

306. The Electron.—The preceding elementary experiments showed that there are two kinds of electricity,—positive and negative. Later investigations have showed that these two kinds of electricity are the most important entities in nature, for every atom is built up of a certain number of positive units of electricity together with an equal number of units of negative electricity. It is a matter of first importance that negative electricity is found in matter in very definite, small, indivisible units. It is possible to have any number of these units but it is never possible to subdivide one. This is analogous to a monetary system in which the money is made up of a large number of small units, the smallest in America being a penny. But there can be no subdivision of the penny. This requires us to think of electricity as granular in its structure. The size of the units are extremely small. The grains are very fine but they are all the same size and absolutely indivisible. This result has been arrived at by most careful experiments which have confirmed each other in every particular.

It is not possible to have electricity apart from matter and this elementary charge is always associated with matter. It has been most carefully studied in the case of negative electricity, for we know much more about negative than about positive electricity. It is found that the elementary charge of negative electricity is always associated with a particle of matter which has a mass which is one eighteen hundreth of the mass of a hydrogen atom. When this particle is torn off from an atom it is always accompanied by the same negative charge of electricity. That particle of matter which is always associated with the elementary charge of negative electricity has been named **the electron.** In whatever kind of atom this electron is found it is always the same in size and has the same charge. In this respect it differs much from atoms which are normally uncharged and differ from substance to substance, existing in about one hundred different varieties. Means have been found for detaching electrons from a great number of different kinds of atoms and carefully measuring their mass and their charge. The answer is always the same from whatever source the electrons are obtained and it becomes possible to say that one of the important constituents of all matter is the electron which is common to all kinds of atoms and is of unvarying charge and mass.

307. Structure of the Atom.—It is now believed that the atom is built up somewhat as a miniature solar system with an extraordinary minute nucleus corresponding to the sun of our solar system. The diameter of this nucleus is never more than one hundred thousandth part of the diameter of the atom. Around this nucleus probably in orbital motion there are a certain number of subsidiary bodies,—the electrons. These electrons correspond to the planets of our solar system. On the nucleus is one or more charges of positive electricity according to the kind of atom. Hydrogen has one positive charge on its nucleus and helium has two positive charges. Other atoms have a larger number. Each of these electrons revolving about the nucleus carries the same charge of negative electricity and their mass is also always the same. The number and position of the electrons in an atom varies from atom to atom but the properties of the electron remain unchanged.

Negative electricity and electrons are inseparable. The only way to put a negative charge on a body is to add to it some electrons and the only way to put a positive charge on it is to take away some electrons, leaving an excess of positive electricity. Each atom in the normal state contains enough positive electricity to just balance the negative electricity on its electrons. Positive electricity never leaves the atom but negative electricity on the electrons may be taken away from the atom or added to it. In the former case the atom becomes charged positively and in the latter case it becomes charged negatively. To charge a body positively then is to take away some of its electrons and to charge it negatively is to add to it some additional electrons. **The negative electricity is mobile. The positive electricity is fixed.** When the rubber rod was charged negatively by rubbing with cat's fur, some electrons passed from the cat's fur to the rubber rod leaving the cat's fur charged positively and the rubber charged negatively. On the other hand when the glass rod was charged positively by rubbing with silk, some electrons passed from the glass to the silk, leaving the glass rod charged positively and the silk charged negatively.

In the normal condition the amount of positive electricity in the atom is just equal to the amount of negative electricity on all the electrons revolving about its center. One or more of these electrons may be detached from the atom leaving it with an excess of one or more positive charges of electricity. In such a case the residue which is left after detaching these electrons is what is called a **positively charged ion.** On the other hand an atom may gain one or more electrons in excess of its quota. It has on it then one or more negative charges and becomes a **negatively charged ion.** For example, when hydrochloric acid dissociates in solution to form hydrogen ions and chlorine ions, the chlorine takes one more electron than its quota, thus giving it one negative charge and making it a negatively charged ion. Since the molecule of hydrochloric acid is originally neutral, the hydrogen is left with one less electron than its quota and is thus charged positively and becomes a positive ion.

308. Evidence for the Structure of the Atom.—The evidence for believing that all atoms are built up of positive and negative charges of electricity and that these charges enter into the atom in quantities in which the charge on the electron is the unit, is both varied and conclusive. The nature of this evidence can not be discussed here. All that can be done is to mention some of the sources of such evidence.

It has been found possible by various agents such as X-rays, ultra-violet light, high temperatures, etc., to detach from atoms infinitesimally small particles. When these particles are examined, they are always found to carry a negative charge which has the same magnitude whatever the source. When the masses of these particles are determined, they are found to be always about one eighteen hundredth that of the hydrogen atom. This is true whether the particles come from oxygen, helium, zinc or any other element. Thus observations of the most careful kind have shown that electrons of unvarying mass and charge can be torn from all elements. It thus becomes necessary to conclude that these electrons are one of the constituents of the atoms.

309. Radioactivity.—It has been found that certain substances, notably radium, thorium and uranium, are in a process of spontaneous disintegration. An examination of the changes taking place in these substances has shown that during the process of disintegration there are emitted two kinds of particles which differ much in their properties. One of these particles is called the **alpha** particle. It has an atomic weight of about 4 and carries two elementary charges of positive electricity. By collecting these particles it has been possible to show that each of them is an atom of helium which has lost two electrons. The residue is the **alpha** particle which has on it two units of positive electricity. A second particle known as the **beta** particle is given off in some of these radioactive changes. The **beta** particle is an electron. It consequently carries one elementary charge of negative electricity and its mass is about one eighteen hundredth of that of the hydrogen atom. These **beta** particles are ejected from the parent atoms with very large velocities. Beside these two material particles, very penetrating electromagnetic waves known as **gamma** rays are emitted during many of these radioactive changes. These waves correspond somewhat to the sound waves which accompany the discharge of a gun.

On account of the emission of these **alpha** and **beta** particles the atoms of radium, thorium and uranium are disintegrating into other elements of lower atomic weights. The case of radium is represented in Fig. 222. Each time an atom of radium loses one **alpha** particle its atomic weight is decreased by 4 units. During its entire life radium has been found to lose 5 **alpha** particles, thus decreasing its atomic weight from 226 to 207, the atomic weight of lead. There is then in these changes direct evidence that one atom can disintegrate to form other atoms of lower atomic weights. The fact that the particles emitted by the atom during this disintegration are electrons and positively charged atoms of helium furnishes direct proof that atoms are built up of positive and negative charges of electricity. The heavy atoms like radium have many electrons and are in unstable equilibrium. They break down into more stable atoms like the atoms of lead. One element then differs from another in the number and distribution of the electrons and in the number of units of positive electricity. There are two building materials out of which all elements are made. These building materials are positive and negative electricity.

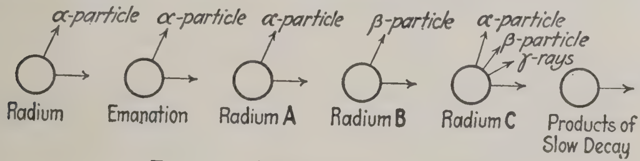


Fig. 222.—Disintegration of radium.

310. Conductors and Insulators.—The electrons are more or less loosely bound to their parent atoms. In some atoms the forces holding the electrons to the atoms are not very large and some of the electrons may be detached temporarily from the atoms and wander about in the vacant space between the atoms. Thus in copper and silver some of the electrons become detached from the atoms and are free to wander about for longer or shorter times in the interstices between the atoms. Under the action of an electric force these electrons migrate through the metal. In some cases the electron may jump from atom to atom and in others they may travel through the vacant spaces between the atoms. Such substances in which there are free electrons which can be made to migrate through the substance under the action of an impressed electric force, are called **conductors** because they allow the passage of electricity through them. To this class of bodies belong the metals.

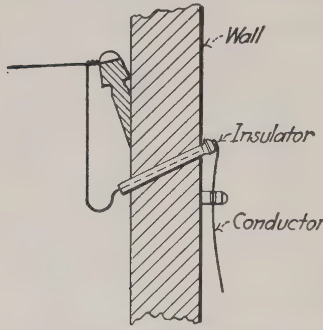


FIG. 223.—Use of conductors and insulators.

If the electrons are more rigidly bound to the atom so that they do not become free except under the action of very large forces, no free electrons will be found in the vacant spaces between the atoms. Quartz and sulphur are substances of this kind. If an electric force is applied to such a substance, it can not cause the electrons to migrate through the substance and there is no flow of electrons from one end of the substance to the other. If an excess number of electrons be placed on one end of such a substance, they will remain there without wandering to other parts of the body. The most that an impressed electric force can do in such a case is to cause a limited displacement of the electrons within the atom without causing the electrons to jump from atom to atom. Such substances in which the electrons are rigidly bound to the atom and which do not, therefore, have the power to transfer electrons through itself are called **non-conductors or insulators.** To this class of substances belong mica, porcelain, quartz, glass, wood, ebonite, etc. Fig. 223 shows an illustration of the use of conductors and insulators.

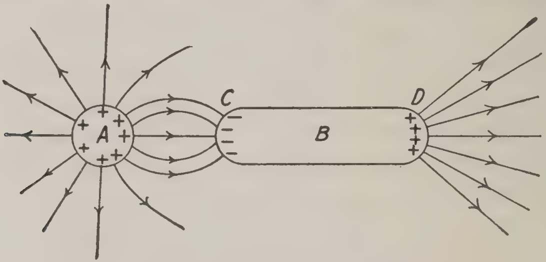


FIG. 224.—Electrostatic induction.

311. Electrostatic Induction.—If an uncharged conductor *B* (Fig. 224) is brought into the field of force of a conductor *A* charged positively, that is a conductor in which there is a deficit of electrons, the attractive forces due to the excess positive charges in *A* will cause some of the electrons in *B* to be pulled toward *C* leaving the further end of *B* with deficit of electrons and therefore charged positively. Hence that end of *B* nearest *A* is charged negatively and the other end is charged positively. Since *B* was originally uncharged, that is contained as much positive as negative electricity, this displacement of electrons will still leave it with as much positive as negative electricity on it, no charges of either kind having been added to it. Hence the positive charge on one end is just equal to the negative charge on the other end.

If the conductor *B* is connected to the earth by means of a wire, enough electrons come from the earth to the conductor *B* to just neutralize the positive charge at *D*. Meanwhile the electrons on the other end *C* are held fast by the attractive forces due to the positive charges on *A*. If the connection to the earth is now broken, and *B* then removed from the presence of *A*, it will of course have an excess of electrons and so be charged negatively. If *B* is now connected again to the earth, this excess of electrons will flow to the earth, leaving *B* uncharged.

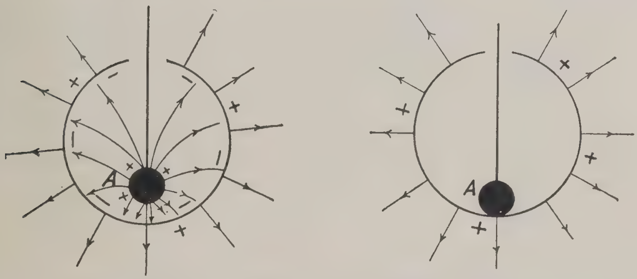


Fig. 225.—Induction in a hollow sphere.

Another illustration of electrostatic induction is seen in Fig. 225. If a metal sphere charged positively is introduced into an uncharged hollow sphere which is insulated, some of the electrons of the hollow sphere are drawn to its inner surface leaving the outer surface charged positively. If now the metal sphere *A* is placed in contact with the inner surface of the hollow sphere, the electrons from the inner surface of the hollow sphere go over to the sphere *A* and just compensate the deficit of electrons which caused its positive charge. This leaves both the sphere

A and the inner surface of the hollow sphere without a charge. The outer surface of the hollow sphere has too few electrons and is therefore charged positively. If the outer surface is now connected to the earth it will gain a sufficient number of electrons to compensate for its deficit and it will be left without charge.

312. The Electroscope.—One form of gold leaf electroscope consists of a metal sphere (Fig. 226) which is fastened by means of a metal rod to two thin gold leaves. The metal rod passes through a sulphur plug by which it is insulated from the glass jar in which the leaves are mounted. When the brass sphere and the leaves are uncharged, the leaves collapse and hang together. When some electrons are removed from the sphere leaving the sphere and the leaves charged positively, the leaves diverge because of the repulsion between the like positive charges on them. If part of the positive charge is neutralized by adding some electrons, the gold leaves partly collapse. If all of the positive charge is neutralized, the leaves collapse completely. If an excess of electrons is now added to the sphere some of them will migrate to the leaves charging them with negative electricity. This causes the leaves to again diverge because of the repulsive forces between the like charges.

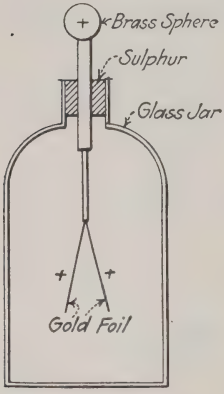


FIG. 226.—Gold leaf electroscope.

313. Lightning.—Lightning is a case of electrostatic induction on a very large scale. There is present in the atmosphere a number of electrons which have become freed from their parent atoms in various ways. These electrons furnish nuclei about which the water vapor in the air can condense. As this condensation progresses and the cloud accumulates, there is collected together not only a large quantity of water but also a large quantity of electricity. When a cloud thus charged comes near the surface of the earth, an electric charge of opposite sign is induced in the surface of the earth due to the repulsion of electrons out of the surface, leaving the earth charged positively. The positive charge on the earth and the negative charge on the cloud try to rush together through the air. When this disruptive discharge occurs there is a flash of lightning and the accompanying thunder. This is the simplest possible case. Discharges from cloud to cloud also occur and in some cases the cloud may be changed positively instead of negatively.

314. Difference of Potential.—If a sphere (Fig. 227) has had removed from it a given number of electrons so that it is charged positively, and it is desired to carry away from it more negative electricity, the positive charge on the sphere attracts the negative charge which is being removed so that work must be performed to remove the negative electricity from the positive. If on the other hand there is on the sphere an excess of electrons so that it is charged negatively and it is desired to bring up additional electrons, the negative electricity on the sphere repels the negative charge on the electrons, making it necessary to perform work to bring up the additional charge. Since the force which one charge exerts on another increases as either of the charges are increased, the amount of work necessary to bring up an additional charge depends on the amount of charge already present on the sphere. As more and more charge is added to the sphere, it requires more and wore work to bring up additional charges of electricity of the same kind. **When it requires work to carry a charge from one body to another or when work is done in allowing a charge to pass from one body to another, these bodies are said to differ in potential** and the work is used as a measure of this difference of potential. **The difference of potential between two bodies is unity when it requires one erg of work to move unit charge of electricity from one of these conductors to the other.** If work is obtained when a unit of negative electricity is carried from B to A, then the potential of A is said to be greater than that of B. If, on the other hand, work must be

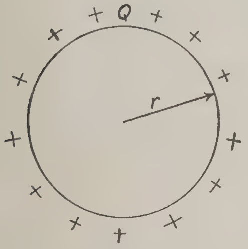


FIG. 227.—Potential of a sphere.

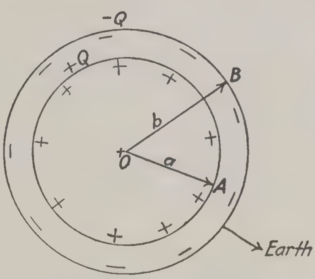


FIG. 228.—Difference of potential between concentric spheres.

done in carrying a unit of negative electricity from B to A, then A is said to have a potential which is less than that of B.

315. Concentric Spheres.—In Fig. 228 is represented two concentric spheres A and B. The outer sphere is connected to earth and the inner sphere is charged with positive electricity. The outer sphere is then charged with an equal amount of negative electricity. Electrons will be repelled by B and attracted by A and hence go from B to A with the performance of work. The potential of A is therefore higher than that of B. The difference of potential between these spheres is measured by the work required to carry unit charge from one to the other. If the quantity of electricity on A and B is increased, the difference of potential between these spheres will be increased, because the potential is proportional to the charge. If the radii of the spheres become more and more nearly equal the difference in potential becomes less and less for a given charge on the spheres. By inserting some medium other than air between the spheres the difference of potential between them may also be changed. Hence the difference in potential between the spheres depends on the distance between their surfaces, the area of the spheres and the medium which fills the space between them.

316. Capacity.—In the concentric spheres just discussed it was seen that the greater the charge on one of the spheres, the greater the difference of potential between the spheres. This is true in all cases. The greater the charge on the conductor, the greater is its potential. In general other things being equal the potential is proportional to the charge. That is,

$$Q = CV \text{ or } V = \frac{Q}{C}$$

where Q = the charge on the conductor in coulombs,
C = the factor of proportionality depending on the shape and size of the body and its position with respect to other bodies.
V = the potential of the conductor in volts.

The factor C is a measure of the work required to charge the conductor. **It is called the capacity.** When the work to give a conductor unit charge is little, the capacity of the conductor is small. The capacity is determined by the size and shape of a conductor and its position with respect to other conductors.

317. Unit Capacity.—The farad is the most familiar unit of capacity. **A conductor has a capacity of one farad when the addition of one coulomb of electricity to it will change its potential by one volt.** (For units see Appendix C.) If the body is charged positively and one coulomb of negative electricity is added the potential is lowered by one volt but if one coulomb of negative electricity is taken away the potential is raised one volt. This is a large unit of capacity because unless the body is very large its potential will be raised or lowered a large number of volts when a coulomb of electricity is added to it or taken away from it. For this reason a smaller unit is chosen. This unit is known as the **microfarad.** It is one millionth of a farad and is therefore the capacity of a conductor when one millionth of a coulomb of electricity changes its potential one volt.

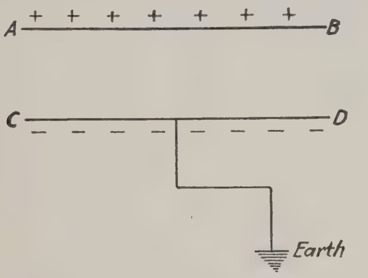


Fig. 229.—Plate condenser.

318. Plate Condenser.—When two parallel plates (Fig. 229) are near to each other and one of these plates is connected to the earth, their capacity depends on, (1) the area of the plates, (2) the distance between the plates, (3) the character of the medium by which the space between the plates is filled. The capacity of such a condenser is,

$$C = \frac{A}{4\pi kd}$$

where C = the capacity in farads
A = the area of the plates.
d = the distances between the plates.
k = a constant depending on the medium between the plates.

In order to make such a condenser have a large capacity the area of the plates should be large and the distance between them small.

319. Dielectric Constant.—If a plate condenser of given area with the plates a fixed distance apart is filled with air and its capacity determined and if it is then filled with some other medium like paraffin, it is found that the capacity is about twice as great in the latter case as in the former. If other media are substituted for the paraffin still other values will be found for the capacity. **The ratio of the capacity of a condenser filled with air to the capacity of the same condenser filled with some other medium is called the dielectric constant or specific inductive capacityof the medium in comparison with air.**

320. Condenser.—It was seen above that in order to construct a large capacity, it was necessary to have plates of large area and have them close together. This can be most easily accomplished (Fig. 230) by putting thin sheets of mica or oiled paper between sheets of tin foil. The sheets of tin foil are the plates of the condenser and the strips of mica or oiled paper are the insulators. The alternate plates of tin foil are joined together on one side and the remainder of these plates on the other side. This gives a large area for both the positive and negative plates. The condenser then becomes essentially a plate condenser of large area or a large number of small plate condensers connected together in parallel. To find the capacity of such a condenser it is only necessary to count the number of effective small plate condensers out of which the large condenser is composed and then add together all of their capacities. Such condensers may be connected in parallel or in series (Appendix D).

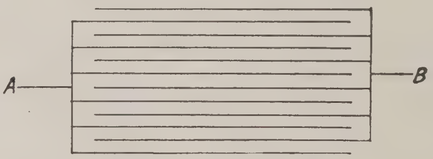


Fig. 230.—Condensers.

321. The Electric Current.—In a metal there is a large number of more or less free electrons which are capable of migrating through the metal under the action of an impressed electric force. When such an electric force is applied to the metal and a stream of these electrons migrate from one part of the metal to another, there is said to be **an electric current** in the conductor. Whether the electrons jump directly from atom to atom or pass through the vacant spaces between the atoms we do not know with certainty. Whether the electrons are free from the atom for a long or for only a short time is not yet settled. It is, however, quite certain that the current of electricity consists of the movement of these electrons through the conductor.

This flow of electricity through a conductor, for example through a wire, is very analogous to the flow of water through a pipe. The quantity of water which flows depends on the size of the pipe and on the pressure driving the water. In like manner a driving force is necessary to send these electrons through the wire and the greater this force the greater the quantity of electricity which flows in unit time. In the flow of water the pump supplies the driving force and in the flow of electrons the driving force is supplied by a battery or dynamo.

CHAPTER XXIV

THE CHEMICAL EFFECT OF AN ELECTRIC CURRENT

322. Electrolysis of Copper Sulphate.—If a beaker is filled with a solution of copper sulphate and two copper plates are inserted in it (Fig. 231) and these plates connected to the terminals of a battery, it will be found that when the plates are removed from the solution, one of them is bright and the other tarnished. By weighing the plates before and after placing them in the solution it can be determined that the bright plate has gained in weight and that the other plate has lost in weight. There has been a transfer of electricity through the solution and it has carried copper from one plate to the other.

The chemical formula for copper sulphate is $CuSO_4$ and when this substance goes into solution it splits up or dissociates into $\overset{++}{Cu}$ and $\overset{--}{SO_4}$ ions. The copper atom losing two electrons carries two elementary charges of positive electricity and the $\overset{--}{SO_4}$ ion retaining these two electrons carries two elementary charges of negative electricity. When the current passes through the solution from one copper plate to the other, the positive copper ions travel in the direction in which the current is flowing and are deposited on the copper plate at which the current leaves the solution. The $\overset{--}{SO_4}$ ion carrying negative electricity travels in the opposite direction and when it reaches the plate by which the current enters the solution, it gives up its negative charge and combines with an atom of copper to form copper sulphate. This copper sulphate goes into solution and dissociates as the other copper sulphate had done. The amount of copper sulphate in the solution thus remains the same. The electrode by which the current enters the electrolytic cell is called the **anode** and that by which it leaves is called the **cathode.** The copper goes to the cathode and adheres to it and the $\overset{--}{SO_4}$ ions go to the anode and cause it to lose as much copper as was gained by the cathode. Thus the gain of copper by the cathode is equal to the loss of copper by the anode.

323. Electrolysis of Water.—Instead of taking a solution of copper sulphate in water, take a dilute solution of sulphuric acid in water. The current enters the solution by means of a platinum or silver strip and leaves by means of another platinum or silver strip. Over each of the metal strips or electrodes by which the current enters or leaves the solution there is inverted (Fig. 232) a closed tube which is filled with some of the solution. The sulphuric acid in the solution splits up into $\overset{+}{H}$ ions and $\overset{--}{SO_4}$ ions. There are two $\overset{+}{H}$ ions for each $\overset{--}{SO_4}$ ion. Each of the hydrogen ions having lost one electron carries one elementary charge of positive electricity and each of the $\overset{--}{SO_4}$ ions having an excess of two electrons carries two elementary charges of negative electricity. Under the action of the electric force arising from the cells connected to the electrodes, the hydrogen ions go to the cathode or electrode at which the current leaves the solution. The $\overset{--}{SO_4}$ ions go to the other electrode. The hydrogen ions lose their charge at the cathode and collect at the top of the tube as hydrogen gas. The $\overset{--}{SO_4}$ ions give up their negative charge at the anode and then unite with two

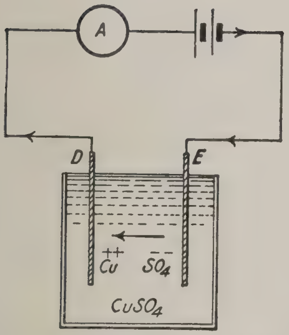


FIG. 231.—Electrolysis of copper sulphate.

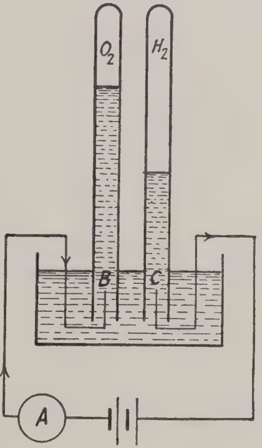


FIG. 232.—Electrolysis of water.

atoms of hydrogen to form sulphuric acid. These atoms of hydrogen are taken from a molecule of water and one atom of oxygen is thus set free. The atom of oxygen thus set free unites with another atom of oxygen to form oxygen gas and this gas arises to the top of the tube inverted above the anode. The sulphuric acid formed at the anode goes into solution again and consequently the amount of sulphuric acid in the water does not change. The water, however, is decomposed into its constituents, **hydrogen** and **oxygen.** The volume of the hydrogen is twice that of the oxygen and the weight of the oxygen eight times that of the hydrogen. The hydrogen goes in the direction of the positive current, because it carries a positive charge of electricity.

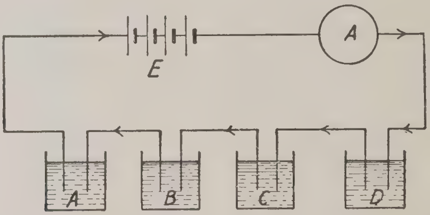


Fig. 233.—Relation of chemical equivalent to mass deposited.

324. Copper Plating.—The deposition of metal by means of an electric current is often used in a commercial way to cover one metal with another. Suppose that it is desired to cover a metal with a coating of copper. A solution of copper sulphate is prepared and the metal to be coated is placed in this solution. There is also inserted into the solution a plate of pure copper. The two metals, the copper plate and the metal to be electroplated, are then connected to the terminals of a battery. The copper plate is made the electrode by which the current enters the solution and the plate to be covered is the electrode by which the current leaves the solution. The copper traveling in the solution in the direction of the current is deposited on the metal to be covered. This metal receives the desired coating and the copper plate loses an equal quantity of copper. The amount of copper sulphate in the solution remains unchanged.

325. Nickel Plating.—If it is desired to nickel plate a metal a solution of nickel salt and a plate of nickel are chosen instead of a solution of copper sulphate and a copper plate. Otherwise the arrangement of the apparatus is the same. As before the solution does not change in strength. The nickel plate loses in weight and the metal to be plated gains an equal weight.

326. Faraday's Laws of Electrolysis.—To understand the quantitative laws of electrolysis, consider a number of electrolytic cells connected in series (Fig. 233). Assume that all of the electrodes are made of platinum so that there are no secondary reactions between the electrodes and the ions in solution. Now let these cells be connected to a battery so that the same current flows through each cell for the same time.

Suppose *A* contains a solution of silver nitrate; *B* a solution of hydrochloric acid; *C* a solution of copper sulphate and *D* a solution of sodium chloride. There will be liberated at the respective cathodes silver, hydrogen, copper and sodium. By determining the amount of substance liberated at each cathode by a given current in a given time, it is possible to establish the two laws of electrolysis.

1. **The mass of any substance liberated is proportional to the current flowing and the time during which it flows.** Hence the mass liberated is proportional to the product of the current and the time.

Suppose that in cell *B* the current flows until 1 gram of hydrogen is liberated, then in cell *A* 108 grams of silver will have been deposited; in cell *C* 31.5 grams of copper and in cell *D* 23 grams of sodium. Whatever current is chosen and whatever the length of time it is allowed to flow, it is found that the masses deposited in these cells always bear the same ratio to each other. **This result may be stated by saying that the masses deposited are always proportional to the quotient obtained by dividing the atomic weight by the valence.** The ratio of the atomic weight to the valence of an element is called the **chemical equivalent.** Where a substance is monovalent the chemical equivalent is equal to the atomic weight. If the substance is divalent the chemical equivalent is equal to one-half of the atomic weight. The second law of electrolysis may be stated as follows:

The masses of different substances liberated by a given current in a given time are proportional to the chemical equivalents or combining weights of the substances.

327. Ampere.—In order to compare currents of electricity it is necessary to have a unit of electric current. This unit must be defined from one of the effects produced by the electric current. In electrolysis we have an effect of an electric current which is easily measured. According to Faraday's first law of electrolysis the current is proportional to the amount of copper or silver or other element which it deposits in a given time. By weighing the amount of metal deposited by the current in a specified time it is thus possible to compare different currents. This gives a convenient method of defining unit current. The practical unit of current is called the **ampere. It is defined to be a current which when flowing for one second will deposit .0011183 grams of silver from a silver solution of specified temperature and concentration.**

328. Coulomb.—If a current of water is flowing in a pipe the quantity is measured in gallons and the current is expressed as the number of gallons which flows in one second. In electricity the current is measured in amperes and unit quantity is said to be that quantity which is carried by unit current in one second. This unit of quantity is called a **coulomb. A coulomb is defined as the quantity of electricity which is carried by a current of one ampere flowing for one second.**

329. Electrochemical Equivalent.—The ampere has been defined in terms of the quantity of silver which it can deposit in one second. From Faraday's law of electrolysis it is evident that the mass of any other substance liberated by one ampere in one second can be calculated as soon as the mass of silver liberated by one ampere is known, for this law states that masses of different substances deposited by equal currents in equal times are proportional to the chemical equivalents of the substances. Knowing then that one ampere deposits .0011183 grams of silver in one second, to find the amount of copper deposited by one ampere in one second, divide the amount of silver deposited by the combining weight of silver and multiply the quotient by the combining weight of copper.

$$\begin{array}{l}\text{Amount of copper deposited}\\ \quad\text{by 1 ampere in 1 sec.}\end{array} = \frac{.0011183}{107.88} \times 31.5 = .0003295 \text{ grams.}$$

The amount of substance deposited in one second by a current of one ampere is defined as the electrochemical equivalent of that substance. (For table of electrochemical equivalents see Appendix G.)

Example.—How much copper will be deposited from a solution of copper sulphate in 12 hr. by a current of 2 amperes?

$$\begin{aligned}\text{Weight deposited} &= \text{electrochemical equiv.} \times \text{current in amps.} \times \text{time}\\ &= 0.0003295 \times 2 \times 12 \times 60 \times 60\\ &= 28.5 \text{ g.}\end{aligned}$$

330. Effect of Electrolysis on Water Mains.—Electrolysis is an important factor in determining the life of underground pipes which are laid near electric railways. It is intended that the track should offer a path by which the dynamo is connected to the wheels of the car. In case the joints of the track are not intimately united, the current instead of traveling from the generator to the car through the track, in part finds its way into the water mains as indicated in Fig. 234, and then returns to the track and the car. Where the current leaves the water main to return to the track and car or to return to the generator, the metal forming the pipe is eaten away as was the anode in the case of the electrolysis of copper sulphate. The electrochemical action is somewhat complicated but it is a case of electrolysis in which iron is removed from the positive electrode to make iron sulphate and the weight of the iron thus reduced, the pipe weakened and its life shortened.

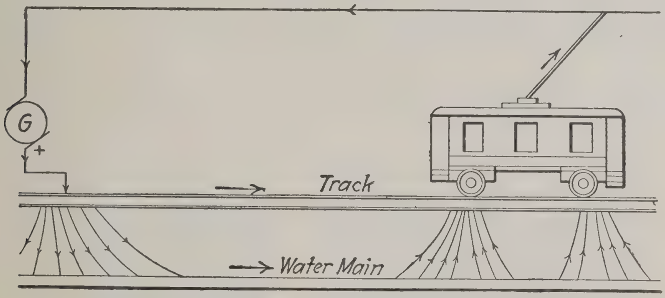


Fig. 234.—Electrolysis of water mains.

Problems

1. How long will it take 5 amperes to deposit 32 g. of copper from a solution of pure copper sulphate?

2. What current is necessary to deposit 18 g. of silver in one hour and thirty minutes? How much sodium would be deposited by this same current in this time?

3. A current of 8 amperes flows for 5 hours through a cell containing copper sulphate, another containing nickel nitrate and a third containing silver nitrate. Find the amount of copper, nickel and silver which is deposited.

4. An iron sheet which is 5 ft. square is to be coated with zinc. How thick will the deposit be when a current of 15 amperes has been flowing for 24 hrs.?

5. Two electroplating vats are arranged in series. One vat contains a solution of a gold salt for plating. The other contains the solution of a nickel salt. A certain current flowing through the vat for gold plating, deposits 1.5 g. of gold. How much silver is deposited in the other vat in the same time?

331. Primary Batteries.—In the preceding paragraphs it was seen that an electric current can produce certain chemical actions like the decomposition of copper sulphate or the decomposition of water into its elements. The reverse effect also takes place.

Certain chemical actions when properly arranged can produce currents of electricity. Indeed this was the first method by which currents of electricity were obtained. These sources of current in which chemical action was directly responsible for the flow of electricity, have been called **primary batteries.** Except in one or two cases they have ceased to be of any practical importance. They have been replaced by storage batteries or by dynamos. Their action will be understood from a study of the Leclanche cell and its modification, the **dry cell.**

332. Leclanche Cell.—The Leclanche cell (Fig. 235) is important because a later modification is now widely used. This cell consists of a zinc rod which dips into a solution of ammonium chloride or salammoniac. The other electrode of the cell is a carbon rod which is surrounded by a cup filled with powdered manganese dioxide and graphite to reduce the polarization as much as possible. The ammonium chloride in the electrolyte dissociates into $\overset{+}{NH_4}$ and $\overline{Cl}$. The $\overset{+}{NH_4}$ carries one positive charge and $\overline{Cl}$ carries one negative charge. While the current is flowing the $\overset{+}{NH_4}$ goes to the carbon plate and there gives up its charge. It then breaks down into $\overset{+}{NH_3}$ and $\overset{+}{H}$ which after combining with another atom of hydrogen collects on the electrode except for the action of the polarizer. Since the action of the polarizer is slow the cell is adapted to open circuit work in which it is used

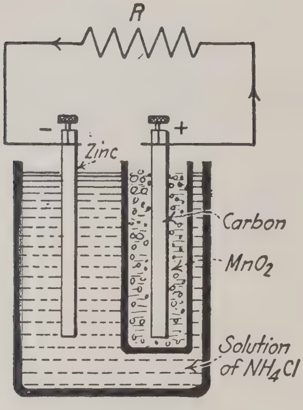


Fig. 235.—Lecanche cell.

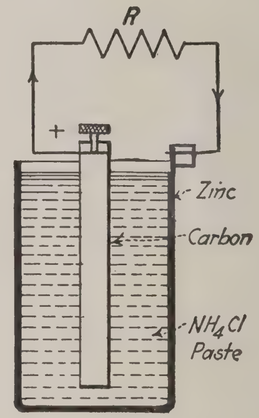


Fig. 236.—Dry cell.

for a short time and then allowed to stand for some time. The cell is entirely free from local action and may be allowed to stand for an indefinite time without deterioration.

333. The Dry Cell.—A type of this cell now widely used (Fig. 236) is called the **dry cell.** It differs from the Leclanche cell just described only in the fact that the electrolyte is in the form of a paste instead of the solution of ammonium chloride. The negative electrode is the zinc can which contains the carbon and the paste. The zinc on the inside of the can is covered with several layers of blotting paper and the space around the carbon rod which forms the positive electrode is filled with a mixture of carbon and manganese dioxide and sawdust saturated with a solution of ammonium chloride. The top is sealed with wax to prevent the evaporation of the moisture in the paste.

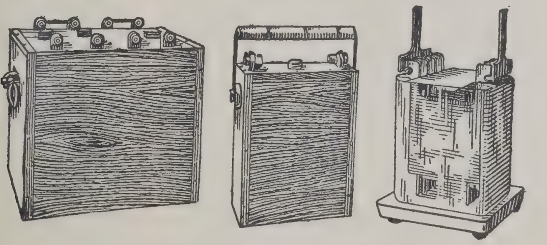


FIG. 237.—Lead storage cell.

This type of cell is now much used in ringing door bells and in flash lights.

334. Lead Storage Cell.—In the lead storage cell (Fig. 237) both the positive and negative plates are made of heavy lead grids full of holes or grooves filled with the active material. The positive plates are made of lead peroxide and the negative plates are made of spongy lead. A cell is formed of a number of such plates, alternately negative and positive. They are set in a glass jar filled with dilute sulphuric acid. The negative plates are connected together and form one effective plate of the battery and the positive plates are connected together to form the other plate. There is always one more negative plate than positive plates so that a positive plate always lies between two negative ones. In this way both sides of the positive plates are charged or discharged. The formation of the oxide which takes place during charging is accompanied by an increase in volume causing a swelling of the plate. Since this swelling takes place equally on both sides of the positive plate, there is little tendency to buckle or warp the plates.

The nearness of the plates together and the large area obtained by using a number of plates causes the cell to have a small resistance. By using a large number of plates and making the areas as large as possible, the carrying capacity of the cell is increased.

When a lead storage cell is delivering a current, the lead peroxide on the positive electrode gives up a part of its oxygen while the spongy lead of the negative electrode becomes oxidized. Finally the two electrodes become very much alike and the current ceases to flow. The battery is now said to be discharged.

In order to charge the battery and make it ready for further use it is only necessary to drive an electric current through it in a direction opposite to that in which the current flows when the cell is in use. This process is known as charging. During this process the oxygen is removed from the negative plate and restored to the positive plate. The positive plate again becomes lead peroxide and the negative plate becomes spongy lead. After a sufficient time of charging the original condition of the battery is restored.

The chemical action taking place during the process of charging and discharging may be represented by the following equations:

Charging:

At positive plate, $PbSO_4 + SO_4 + 2H_2O = PbO_2 + 2H_2SO_4$

At negative plate, $PbSO_4 + H_2 = Pb + H_2SO_4$

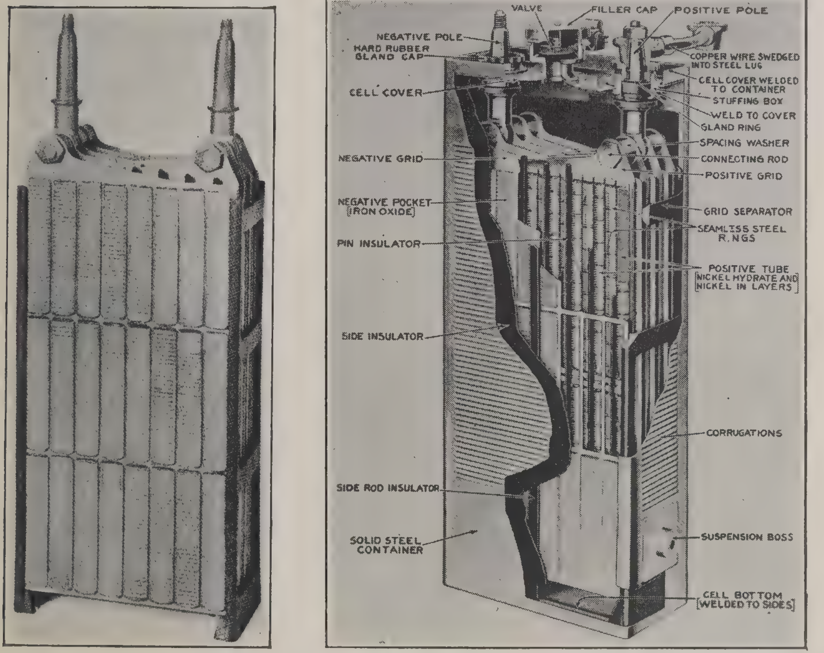
Discharging:

At positive plate, $PbO_2 + H_2SO_4 + H_2 = PbSO_4 + 2H_2O$

At negative plate, $Pb + SO_4 = PbSO_4$

It is seen from these equations that during the process of charging sulphuric acid is formed. Consequently the density of the electrolyte rises when the batteries are being charged. During discharge sulphuric acid disappears and water is formed. For this reason the density of the electrolyte decreases during discharge. By observing the density of the electrolyte it is possible to find how nearly the battery is discharged. When the battery is fully charged the density of the acid should be about 1.21.

335. Edison Storage Cell.—In the Edison storage cell the positive plate is a nickel plated steel grid with perforated steel tubes which are filled with alternate layers of nickel hydroxide and flaked nickel. The nickel hydroxide is changed by electrochemical action into nickel peroxide (NiO_2). The flaked nickel is added to reduce the internal resistance of the cell. The negative plate (Fig. 238) is also made of nickel plated steel grid with a large number of rectangular pockets filled with powdered iron oxide (FeO). In charging the cell the iron oxide is changed into metallic iron. The positive plate of the cell thus becomes nickel peroxide (NiO_2) and the negative plate metallic iron (Fe). The electrolyte is a 21 per cent. solution of caustic potash (KOH).


VALVE
FILLER CAP
POSITIVE POLE
NEGATIVE POLE
HARD RUBBER GLAND CAP
COPPER WIRE SWEDGED INTO STEEL LUG
CELL COVER
CELL COVER WELDED TO CONTAINER
STUFFING BOX
WELD TO COVER
GLAND RING
SPACING WASHER
NEGATIVE GRID
CONNECTING ROD
POSITIVE GRID
NEGATIVE POCKET [IRON OXIDE]
GRID SEPARATOR
SEAMLESS STEEL RINGS
PIN INSULATOR
POSITIVE TUBE [NICKEL HYDRATE AND NICKEL IN LAYERS]
SIDE INSULATOR
CORRUGATIONS
SIDE ROD INSULATOR
SOLID STEEL CONTAINER
SUSPENSION BOSS
CELL BOTTOM [WELDED TO SIDES]

a b

FIG. 238.—Edison storage cell.

When the cell is delivering current, the nickel peroxide is reduced to a lower oxide (Ni_2O_3) and the iron oxidized to form iron oxide (FeO). The reaction is:

$$2NiO_2 + Fe = N_2O_3 + FeO$$

nickel peroxide + iron = nickel oxide + iron oxide

When the cell is being charged by driving a current through it in the opposite direction to that in which the current flows when the cell is discharging, the reaction is reversed and becomes:

$$Ni_2O_3 + FeO = 2NiO_2 + Fe$$

nickel oxide + iron oxide = nickel peroxide + iron

The electrolyte does not enter into either of these reactions. Its density changes only slightly during the reactions. The effect of charging and discharging is to transfer oxygen from one plate of the cell to the other. The normal voltage is about 1.2 volts.

336. Weston Standard Cell.—Standard cells are not used to furnish current. They offer a means of obtaining definite and constant electromotive forces. They are the concrete standards in terms of which differences of potential are measured. The most widely used of these standard cells is the **cadmium** or **Weston cell.**

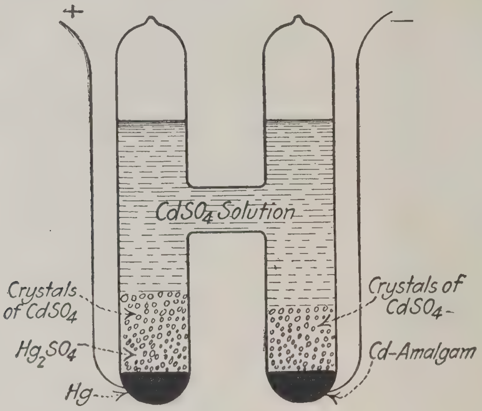


FIG. 239.—Weston cell.

This cell (Fig. 239) consists of an H-shaped, hermetically sealed glass tube containing pure mercury as the positive electrode in one leg and a cadmium amalgam in the other leg as the negative electrode. Platinum wires sealed through the glass connect the electrodes with the circuit. Above the mercury is placed a paste of mercurous sulphate. The electrolyte is a saturated solution of cadmium sulphate. In order to insure that this solution remains saturated at all temperatures, an excess of crystals of cadmium sulphate is added to the solution.

The electromotive force of this cell changes very little with the temperature. For this reason it is considered the best standard available. At 20°C its electromotive force is 1.0183 volts. For other temperatures the electromotive force is given by the equation.

$$E_t = 1.0183 - 0.00004\ (t - 20) \text{ volts.}$$

CHAPTER XXV

OHM'S LAW AND ITS APPLICATION

337. Resistance.—A stream of water flowing through a pipe is held back by the friction of the pipe. The amount of this friction depends on the smoothness of the inner surface and on the length and size of the pipe. In like manner every conductor offers some opposition to the flow of electricity through it. The opposition which a current of electricity encounters in flowing through a wire depends on the length of the wire, on its size and on the material of which it is made. This opposition to the flow of an electrical current is **electrical resistance.** Its value, which is least in silver, varies greatly from substance to substance.

338. Unit of Resistance.—In order to measure this resistance it is necessary to have some fixed unit. The practical unit of resistance is the **ohm. It is defined as the resistance which a thread of mercury having a cross sectional area of one square millimeter and a length of 106.3 cm. has at 0°C.** This thread of mercury should carry an unvarying current. Mercury was chosen as the metal in terms of which to define this standard because it is easy to obtain in great purity and it is easy to measure with great accuracy the cross section of a tube containing it. The resistance of mercury changes rapidly with the temperature and for this reason the temperature of the mercury must be kept constant. Since a column of mercury is not convenient to use as a standard it is customary to use standard coils (Fig. 240) made of wire which changes its resistance slightly with the temperature. This is a great convenience for it is not necessary to keep the temperature of the coil so nearly constant.

339. Electrical Pressure.—In order to cause water to flow in a pipe it is necessary to have a difference of pressure applied to the two ends of the pipe. This pressure may be obtained by means of water in a stand pipe. The weight of the water in the stand pipe develops the pressure which causes the water to flow. In order to cause a flow of electricity through a wire, it is necessary to have a difference of electrical pressure between the ends of the wire. This difference of electrical pressure bears the same relation to the flow of electricity that the difference in the hydrostatic pressure bears to the flow of water. In order to obtain this difference in electrical pressure a number of methods have been adopted. The oldest and most familiar for small currents is the use of a battery.

340. Volt.—In hydraulics the number of gallons of water flowing per second through a pipe depends to a large extent on the pressure under which it flows. This water pressure may be measured in pounds per square foot. In the same way the number of amperes flowing through a wire depends in a large measure on the electrical pressure driving the current through the wire. The unit in which this electrical pressure is measured is the **volt.** It is the measure of the electrical pressure which the battery or dynamo can exert in driving the current around the circuit. **A volt is the electromotive force or electrical pressure necessary to drive a current of one ampere through a resistance of one ohm.** A dry cell gives an electromotive force of about 1.4 volts and a lead storage cell about 2 volts.

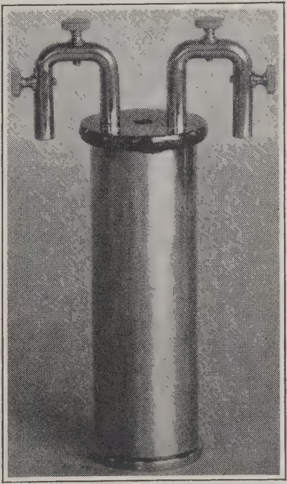


FIG. 240.—Standard resistance.

341. Laws of Resistance.—Just as in the case of the flow of water the resistance of a pipe of large cross section is less than the resistance of a pipe of small cross section, so in electricity the resistance of a wire having a large area is less than the resistance of a wire of small area. In exactly the same way the resistance of a long pipe is greater than the resistance of a short pipe and the electrical resistance of a long wire is greater than that of a short wire. The laws of resistance are as follows:

1. **The resistance of a conductor is proportional to its length.** For example, if 50 ft. of copper wire has a resistance of 2 ohms, then 100 ft. of the same wire has a resistance of 4 ohms.

2. **The resistance of a conductor is inversely proportional to its cross sectional area.** If the wire has a circular cross section

the resistance is inversely proportional to the square of the diameter, or radius. If a copper wire having a diameter of 1 mm. has a resistance of 4 ohms, then a copper wire of the same length having a diameter of 2 mm. has a resistance of 1 ohm.

3. **The resistance of a given conductor depends on the material out of which it is made.** This resistance is affected by temperature or any other cause which changes the molecular condition of the substance. Thus if a copper wire has a resistance of 10 ohms in melting ice it will have a greater resistance when immersed in steam. In this case it is found that the wire when immersed in steam has a resistance of about 14 ohms. On the other hand the resistance of a carbon filament is only about one-half as much when cold as when it is incandescent.

342. Specific Resistance.—These laws may be brought together into a single formula from which the resistance of wires of different lengths and cross section can be calculated. Let R be the resistance of the wire in ohms, l its length in cm., a the area of its cross section in sq. cm. and k a constant depending on the material out of which the wire is made and on the temperature. Then

$$R = k\frac{l}{a}$$

This constant k is the specific resistance of the wire. **It is equal to the resistance of a wire which is** 1 cm. long and has a cross sectional area of 1 sq. cm. If the value of this constant k is known then the resistance of any other wire made out of the same material can be calculated. For example the specific resistance of silver is 0.0000016 ohm. This means that the resistance of a silver wire 1 cm. long and 1 sq. cm. in cross section is 0.0000016 ohm.

Since wires are usually circular in cross section, engineers have chosen a somewhat different method of calculating the resistance. They call a wire which is one thousandth of an inch in diameter, **1 mill in diameter** and the area of its circular cross section is one **circular mill.** As in the preceding case the resistance of any wire can be calculated by comparing it with the resistance of a wire of the same material of standard size and length. Engineers choose for their standard the resistance of a wire **one foot in length and one mill in diameter. The resistance of such a piece of wire is the resistance of a mill foot.** Aluminum has about 17.4 ohms to the mill foot.

Using this system of units the resistance of a wire is expressed as,

$$R = Kl/d^2$$

where K is the resistance of one mill foot of the wire, l is the length in feet and d is the diameter in mills.

Example.—The resistance of copper per mill foot at 20°C. is 10.4 ohms. Find the resistance of a copper wire 500 ft. long with a diameter of 0.021 in.

$$R = K\,l/d^2 = \frac{10.4 \times 500}{21 \times 21} = 11.8 \text{ ohms}$$

343. Temperature Coefficient of Resistance.—Whenever the resistance of a metal is given, it is always necessary to state the temperature at which the resistance was measured for the reason that the resistance of metals increases as the temperature of the metal is raised. The amount which the resistance increases per degree rise in temperature for each ohm in the wire is defined to be the **temperature coefficient of the resistance.** For all pure metals this coefficient is essentially the same and lies in the neighborhood of 0.004 per degree Centigrade.

Taking the resistance at zero as the standard from which to measure increases and decreases in resistance, copper wire would increase 0.0042 ohm per degree for each ohm at 0°C.

The relation between the temperature and the resistance is expressed in the form of the equation,

$$R = R_o(1 + 0.0042t)$$

where $R =$ the resistance at a temperature above or below 0°C.
$R_o =$ the resistance at 0°C.
$t =$ the temperature in degrees Centigrade.

Example.—The resistance of a copper wire is 5.25 ohms at 0°C. Find its resistance at 75°C.

$$\begin{aligned} R &= R_0(1 + 0.0042t) \\ &= 5.25(1 + 0.0042 \times 75) \\ &= 5.25 \times 1.315 = 6.90 \text{ ohms} \end{aligned}$$

344. Temperature Coefficient of Alloys.—When metals are mixed together, the resistance of the mixture may be very different from the resistance of the metals which were used to make the mixture. Alloys always have a higher resistance than the metals out of which they were made. On the other hand the rate at which the resistance changes with the change of temperature is much less in alloys than in pure metals. Manganin, an alloy consisting of copper, nickel, iron and manganese, has a resistance which is thirty or forty times that of pure copper but the rate at which its resistance changes with the temperature is practically negligible. For this reason it is an excellent substance to use for standard resistance coils.

Problems

1. Compute the resistance of 100 m. of copper wire which has a diameter of 1 mm. The specific resistance of copper = 0.000017 ohm.

2. How long must a manganin wire be in order to have a resistance of 100 ohms, if its diameter is 0.5 mm.? Specific resistance of manganin at 18°C. = 0.00042 ohm.

3. It is desired to make the two wires from a farm light plant to the barn have a resistance of 1 ohm. If the distance from the plant to the barn is 100 meters, what must be the diameter of the wire? Specific resistance of copper = .000017 ohm.

4. A piece of wire 5 m. long and 0.75 mm. in diameter has a resistance of 80 ohms. What is its specific resistance?

5. A copper wire is 500 ft. long and 0.036 in. in diameter. Find its resistance. The resistance of a mill foot of copper is 10.4 ohms at 20°C.

6. The wires from a generator to the lights which it supplies is 600 ft. long and have a diameter of 0.101 in. Find the resistance between the dynamo and the lights.

7. A piece of wire 10 m. long and 0.4 mm. in diameter offers a resistance of 3 ohms. Find the length of wire 0.5 mm. in diameter in order that it may have a resistance of 3.5 ohms.

8. Two wires of the same length and material are found to have resistances of 8 and 12 ohms respectively. If the diameter of the first wire is 0.75 mm., what is the diameter of the second wire?

9. A wire 5 ft. long and 30 mills in diameter has a resistance of 0.0321 ohm. What is the resistance of a mill foot of this material?

10. The resistance of the field coils of a dynamo is 425 ohms at 25°C. What will the resistance be at 75°C.?

11. What resistance will a coil of copper wire have at 6°C. if its resistance is 425 ohms at 30°C.?

12. A coil of copper wire has resistance of 13.5 ohms at 0°C. How high must the temperature be raised in order to make the resistance 16.5 ohms?

13. A piece of wire has a resistance of 21 ohms at 0°C. and 21.145 at 35°C. What is its temperature coefficient?

14. A piece of wire at 0°C. has a resistance of 34 ohms and at 25°C. its resistance is 44.8312 ohms. What is its temperature coefficient?

345. Electric Currents and Water Currents.—There is a definite analogy between the flow of water in pipes and the flow of electricity. Consider Fig. 241 in which two tall open vessels filled with water are connected with a pipe. A paddle wheel which is turned by a falling weight forces water from one of these vessels to the other until the back pressure becomes so great that the paddle wheel is unable to lift the water further. In this case the difference in pressure on the two sides of the paddle wheel just balances the tendency of the weight to produce rotation. This difference in level does not depend on the size or shape of the vessel but on the magnitude of the weight hanging from the wheel. This difference in level between the water in the vessels corresponds to the difference in potential or electrical pressure in the battery.

Let the vessels now be connected by a second pipe (Fig. 242). Water will be forced continuously through this pipe by the difference in level between the water in the vessels. After this flow is established, the difference in the level is less than before the flow started. The decrease in back pressure against the wheel being now reduced, the wheel is able to turn and force the water through the circuit. The flow of water thus maintained corresponds to the flow of current in the electric circuit. The falling weight represents the chemical action in the cell which produces the electrical pressure driving the current around the circuit. The rate at which the water flows depends on the difference in level which the falling weight can maintain. In like manner the rate at which the current of electricity flows depends on the electrical pressure which the battery can establish. The rate at which the water flows in the pipe depends also on the resistance or

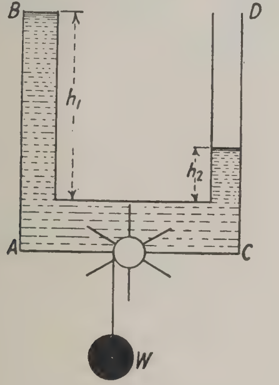


Fig. 241.—Pressure developed by a water wheel.

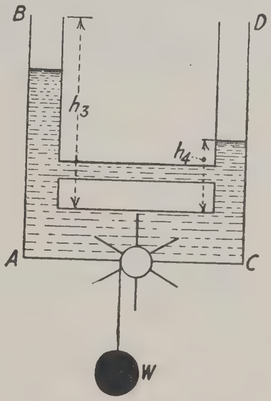


Fig. 242.—Pressure and current from a water wheel.

opposition to flow which the water experiences in the pipe. In the same way the electrical current depends on the electrical resistance or opposition to electrical flow which the electricity encounters in the wire. The work required to force the water around the circuit comes from the work done by the falling weight. The energy required to drive the electricity around the electric circuit comes from the chemical action which takes place in the cell or from the work done in driving the dynamo.

346. Ohm's Law.—The relation between current, electrical pressure driving the current and the resistance encountered by the current has been stated in what is now known as **Ohm's law.** This is one of the most fundamental laws in electricity and has a wide application in electrical machinery. **This law states that the electrical current in a conductor is equal to the electrical pressure divided by the electrical resistance.**

$$\text{Current} = \frac{\text{electrical pressure}}{\text{resistance}}$$

$$I = \frac{E}{R}$$

where I stands for the current in amperes, R the resistance in ohms; E the electrical pressure in volts. The law can be stated in two other forms. Each of these forms states the same facts.

$$\text{Voltage} = \text{current} \times \text{resistance}$$

$$E = I \times R$$

$$\text{Resistance} = \frac{\text{voltage}}{\text{current}}$$

$$R = \frac{E}{I}$$

Example.—The pressure in an electrical circuit is 50 volts; the resistance is 12.5 ohms. How many amperes are flowing in it?

$I = E/R$
$E = 50$ volts
$R = 12.5$ ohms
$I = 50/12.5 = 4$ amperes

Example.—How many volts are required to force 4 amperes through 25 ohms?

$I = 4$ amperes
$R = 25$ ohms
$E = R \times I = 4 \times 25 = 100$ volts

Example.—Through how many ohms can 100 volts force a current of 4 amperes?

$$E = 10 \text{ volts}$$
$$I = 4 \text{ amperes}$$
$$R = E/I = 100/4 = 25 \text{ ohms}$$

347. Application of Ohm's Law.—Ohm's law can be applied to the electric circuit as a whole or it may be applied to a part of the circuit. In case it is applied to part of the circuit care must be taken with respect to the form in which the law is stated. In order to avoid mistakes in this connection it is well to emphasize the facts involved by stating the law in two forms, one for the entire circuit and the other for a part of the circuit.

For the entire circuit the law is as follows: The current in the entire circuit must be equal to the voltage across the entire circuit divided by the resistance of the entire circuit.

For part of a circuit the law is as follows: The current in a part of the circuit is equal to the voltage across that part of the circuit divided by the resistance of that same part of the circuit. It is important to notice that in this case the current, the voltage and the resistance must be taken across the same part of the circuit.

Example.—A dynamo having a resistance of 2 ohms is connected in such a way that it drives a current through two resistances of 8 and 10 ohms in series. The electrical pressure generated by the dynamo is 220 volts. What is the current in the resistances?

The resistance of the entire circuit is the sum of all the resistances = $2 + 8 + 10 = 20$ ohms.

The voltage in the entire circuit = 220 volts.

Hence current in dynamo = $220/20 = 11$ amperes.

The voltage across the 8 ohm coil is found by multiplying the resistance of that coil by the current in it.

Voltage across 8 ohm coil = $8 \times 11 = 88$ volts.

In the same way the voltage across the 10 ohm coil is,

$$E = 11 \times 10 = 110 \text{ volts}$$

The voltage across the interior of the generator is,

$$E = 11 \times 2 = 22 \text{ volts}$$

The total potential around the circuit is,

$$88 + 100 + 22 = 220 \text{ volts}$$

348. Brush Potential.—In the preceding problem it was seen that only part of the potential or voltage generated by the dynamo is available for forcing the current through the external circuit. Part of the voltage is used in forcing the current through the generator itself. The voltage used in driving the current from one brush of the dynamo to the other is known as **the brush potential or the terminal voltage of the dynamo.** This is the voltage available for sending the current through an external resistance. The remainder of the voltage generated in the dynamo or battery is used in the dynamo or battery itself. This brush potential must be carefully distinguished from the total voltage made by the dynamo.

Let $R =$ the external resistance.

Let $E =$ the electromotive force of the battery or dynamo.

$r =$ the internal resistance of the battery or dynamo.

$$\text{Current} = \frac{E}{R + r}$$

$e =$ difference of potential between brushes = current × external resistance

$$e = \left(\frac{E}{r + R}\right) \times R$$

Entire drop of potential = internal drop of potential + drop of potential over external circuit

It is thus seen that the electromotive force of a dynamo or battery may be considered as made up of two parts: (1) that required to maintain the current in the external circuit; (2) that required to maintain the current in the internal resistance of the dynamo or battery. These two voltages are proportional to the resistance of the corresponding parts of the circuit. Their sum is equal to the voltage produced by the battery or dynamo.

Problems

1. An incandescent lamp uses 0.75 ampere on 115 volts. What is its resistance?

2. If the resistance in series with a battery is 11 ohms and the electromotive force of the battery is 55 volts, what is the current in the circuit?

3. What difference of potential will produce a current of 8 amperes through a resistance of 75 ohms?

4. A dynamo generates 220 volts. The resistance of the entire circuit is 20 ohms. What is the current?

5. A current of 14 amperes is flowing in a wire of 8 ohms resistance. What is the difference in potential between its ends?

6. The feed for a trolley has a resistance of 0.055 ohms. What is the fall of potential in it when it is supplying a current of 350 amperes?

7. A barn is 200 ft. from a light plant. If five lamps each taking 1

ampere are to be located in it, what size of copper wire must be used in order that the drop of potential in the line may be 2.5 volts?

8. What is the greatest distance a copper wire having a resistance of 1 ohm per 1,000 ft. will carry 6 amperes, allowing a drop of 3 volts in the wire?

9. How many lamps, each taking 0.6 ampere, can be lighted at the same time, 350 ft. away from the light plant, when copper wire having a resistance of 0.8 ohm per 1,000 ft. is used, assuming that the drop of potential in the wires must not be greater than 1.5 volts?

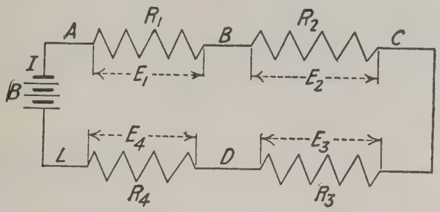


FIG. 243.—Resistance in series.

10. What brush potential must a generator make in order to supply a current of 25 amperes through a circuit which has a resistance of 0.45 ohm?

11. A motor requires 10 amperes at 55 volts. The resistance of each line wire is 0.25 ohm. What voltage must the dynamo supply?

12. A dynamo which has a resistance of 0.5 ohm makes 55 volts. If the machine is supplying 10 amperes to a motor, what is its brush potential?

13. A dynamo which is making 110 volts gives a brush potential of 100 volts when it is delivering a current of 22 amperes. Find the internal resistance of the dynamo.

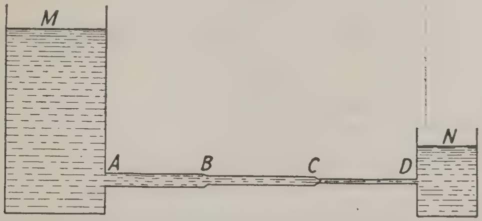


FIG. 244.—Water pipes in series.

349. Resistance in Series.—When two or more conductors are arranged as shown in Fig. 243 they are said to be connected in series. Such an arrangement is analogous to several pipes joined as in Fig. 244 so that the same current of water flows through each pipe. Whatever the area of the cross section of these pipes or their length, the flow of water in each pipe must be the same. There is no opportunity for water to accumulate in a pipe when the flow has once started. In like manner the current flowing through each of a number of resistances connected in series is always the same.

Let I = the current in each of the resistances
and R = total resistance between A and L.
The fall of potential from A to $B = E_1 = IR_1$.
The fall of potential from B to $C = E_2 = IR_2$.
The fall of potential from C to $D = E_3 = IR_3$.
The fall of potential between D and $L = E_4 = IR_4$.
Total fall of potential between A and $L = E = IR$.

The total fall of potential is the sum of the falls of potential over the individual resistances. Hence

$$IR = IR_1 + IR_2 + IR_3 + IR_4$$

or

$$R = R_1 + R_2 + R_3 + R_4$$

Thus for conductors connected in series the following laws hold:

1. **The current in each resistance is the same.**

2. **The resistance of the entire circuit is equal to the sum of the separate resistances.**

3. **The voltage across several resistances in series is equal to the sum of the voltages across each of the separate resistances.**

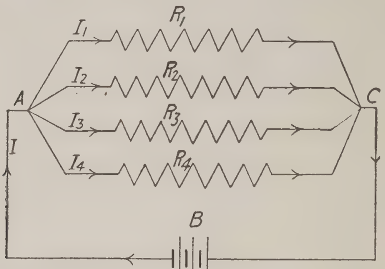


Fig. 245.—Resistances in parallel.

4. **Since the current in each resistance is the same, the fall of potential over a resistance is proportional to the resistance.** Thus over a large resistance the fall of potential is large and over a small resistance it is small.

Example.—Three resistances, one of 10 ohms, another of 5 ohms, and a third of 2 ohms are connected in series. What is the effective resistance of the circuit?

Effective resistance = sum of the individual resistances.

$$R = R_1 + R_2 + R_3$$
$$R = 10 + 5 + 2 = 17 \text{ ohms}$$

350. Resistances in Parallel.—When several conductors are joined between two points so that the current divides between them (Fig. 245) they are said to be in parallel or in multiple. This is analogous to connecting a tank *M* (Fig. 246) to another tank *N* by means of a number of pipes through each of which water may flow. The head of pressure will evidently be the same whether the water flows through one or more of the pipes at the same time. The amount which flows through each pipe will be determined by the length and diameter of that pipe, and will be independent of the amount which flows through another pipe so long as the pressure remains unchanged during the flow.

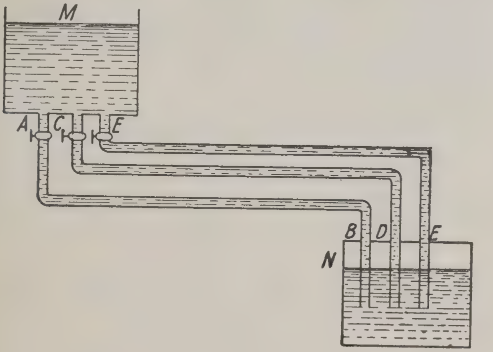


FIG. 246.—Water pipes in parallel.

The flow through all of the pipes will be equal to the sum of the flows through the individual pipes.

In the same way the current of electricity which flows through R_1 (Fig. 245) will be determined by its resistance, if the potential difference between *A* and *C* remains unchanged. The total current entering at *A* must be equal to the current leaving *C* and this total current must be equal to the sum of the currents in the separate resistances.

Let I = current entering at *A*
I_1 = current in R_1,
I_2 = current in R_2,
I_3 = current in R_3,
I_4 = current in R_4,

Total current = sum of separate currents

$I = I_1 + I_2 + I_3 + I_4.$

The fall of potential from A to C is the same by each of the paths just as the difference in pressure in the flow of water (Fig. 246) is the same between the ends of each pipe. By Ohm's law the fall of potential is given by multiplying the resistance by the current. Hence

Potential difference between A and $C = I_1 \times R_1 = I_2 R_2 = I_3 R_3 = I_4 R_4 = E$

Hence $I_1 = E/R_1,\ I_2 = E/R_2,\ I_3 = E/R_3,\ I_4 = E/R_4$

Let R denote the equivalent resistance by which these resistances could be replaced without disturbing the fall of potential between A and C or the flow of current between A and C.

Fall of potential between A and $C = IR = E$, or $I = \dfrac{E}{R}$.

Substituting these values of I, I_1, I_2, I_3, I_4 in the equation $I = I_1 + I_2 + I_3 + I_4$

we have $$E/R = E/R_1 + E/R_2 + E/R_3 + E/R_4$$

and $$1/R = 1/R_1 + 1/R_2 + 1/R_3 + 1/R_4$$

Example.—Three resistances of 2 ohms, 4 ohms and 12 ohms are connected in parallel find the equivalent resistance.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3};$$

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{4} + \frac{1}{12} = \frac{10}{12};$$

$$R = \frac{12}{10} = 1.2 \text{ ohms}.$$

351. Kirschhoff's Laws.—Two extensions of Ohm's law were made by Kirschhoff and these extensions are known as Kirschhoff's laws.

First Law.—The first law states that **at any point in a circuit there is as much current flowing away from the point as there is current flowing to that point.** This means that the sum of the currents flowing to a point is just equal to the sum of the currents flowing away from it. If this law were not true there might be more current flowing toward a point than away from it. In that case there would be an accumulation of electricity at the point. In a system of water mains no matter how many pipes may lead away from one place the amount of water coming up must equal the amount going away unless there is a reservoir to receive some of the water or unless water already stored up is discharging from some kind of a reservoir. In similar manner the electricity going away from a point must just equal that coming up to it unless there is some accumulation of electricity.

$$I = I_1 + I_2 + I_3 + I_4.$$

The Second Law.—The second law of Kirschhoff's states that **the sum of the products of the current by the resistance taken around any closed path in a network of conductors is just equal to the sum of the electromotive forces which one passes in going around the closed circuit.** In going around the circuit in the application of this law regard must be had for the direction in which the current is flowing. Currents which are flowing in the direction in which the circuit is traced out are regarded as **positive.** Currents flowing in the opposite direction are considered **negative.** Electromotive forces which , if acting alone would produce a positive current in the circuit, are considered positive and the oppositely directed electromotive forces are considered negative. If there are no electromotive forces in any of the conductors which comprise the part of the net work traced out, then the sum of the IR drops about that circuit is zero.

Problems

1. A lamp has a resistance of 100 ohms and the line wires a resistance of 5 ohms. When an electromotive force of 200 volts is applied to the circuit, what is the current in the line, the voltage across the lamp and the voltage across the line?

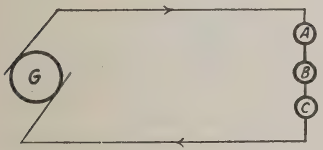


Fig. 247.—Electric lamps in series.

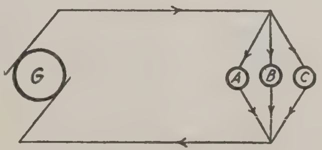


Fig. 248.—Electric lamps in parallel.

2. What electromotive force is required to send a current of 4 amperes through a circuit containing a generator having a resistance of 0.55 ohm, line wires of resistance 1.2 ohm and a lamp having a resistance of 100 ohms?

3. A circuit consists of a dynamo (Fig. 247) having a resistance of 0.75 ohm, line wires of resistance 2.5 ohms and 3 lamps connected in series. Each lamp has a resistance of 17.5 ohms. What is the current when the dynamo makes 125 volts?

4. A divided circuit consists of two branches, one of resistance 1 ohm and the other of resistance 0.35 ohm. What is the joint resistance of the circuit?

5. A circuit (Fig. 248) has three branches of 120, 40 and 60 ohms respec-

tively. When a current of 2 amperes is flowing in the branch containing 60 ohms, how much current is flowing in each of the other branches?

6. What electrical pressure is required to force 10 amperes through a branched circuit consisting of 4 ohms, 7.5 ohms and 12 ohms?

7. Three resistances of 25, 15 and 6 ohms are in series. The applied voltage is 150 volts. What is the difference of potential across each resistance? If these resistances were in parallel what would be the current in each resistance?

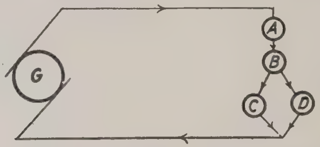


Fig. 249.—Electric lamps in series and in parallel.

8. A circuit consists of the following three parts joined in series: (1) two coils joined to each other in parallel, one of 8 ohms, the other of 12 ohms; (2) a line wire of 2.4 ohms resistance; (3) six lamps each of 120 ohms resistance joined in parallel. What is the resistance of the circuit and the voltage necessary to send 12 amperes through it?

9. Two resistances (Fig. 249) of 100 ohms and 140 ohms are joined in parallel and in series with these are joined two resistances, A and B of 140 and 100 ohms, respectively. Find the current in each resistance when a voltage of 25 volts is applied to the ends of the circuit.

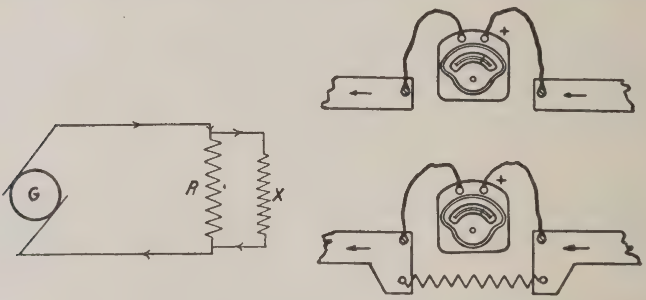


Fig. 250.—A shunt.

10. A coil having a resistance of 150 ohms is shunted with a coil having a resistance of 0.5 ohm. What part of the current flows in the coil and in the shunt?

11. A coil *R* (Fig. 250) has a resistance of 250 ohms. It is desired to provide it with a shunt so that only one-tenth of the current will flow in the coil. What is the resistance of the shunt?

352. Cells Connected in Series.—The generators or cells which supply the current may be arranged either in series or in parallel. When cells are connected (Fig. 252) so that the positive pole of one cell is connected to the negative pole of the next, they are said to be joined in series. The electromotive force of such a combination is the sum of the electromotive forces of each cell. Since the whole current passes through each cell, the internal resistance of these cells is equal to the sum of the resistances of the individual cells. If these cells are joined up so that they drive a current through an external resistance, there is only one path for the current and the current in the circuit is everywhere the same.

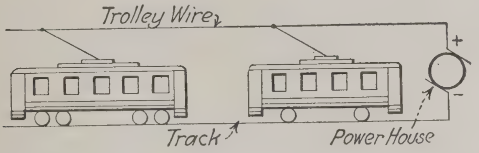


FIG. 251.—Current in a trolley wire and track.

Suppose five such cells, each having an electromotive force e and a resistance r, are connected to an external resistance R.

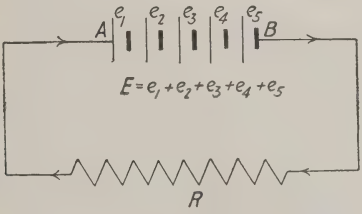


FIG. 252.—Cells connected in series.

The whole electromotive force in the circuit is $5e$ and the whole resistance of the circuit is $R + 5r$. By Ohm's law,

$$\text{Current} = I = \frac{5e}{R + 5r}$$

In general

$$I = \frac{ne}{R + nr},$$

where n = number of cells.

Example.—Five cells each having an electromotive force of 1.25 volts and an internal resistance of 0.4 ohm are connected to a resistance of 20 ohms. Find the current in this resistance.

Total electromotive force = 1.25 × 5 = 6.25 volts
Internal resistance of the cells = 0.4 × 5 = 2.0 ohms
Total resistance of the circuit = internal resistance of cells + external resistance
= 2.0 + 20 = 22.0 ohms

$$\text{Current} = \frac{\text{electromotive force}}{\text{total resistance}} = \frac{6.25}{22} = 0.28 \text{ amperes}$$

353. Cells Connected in Parallel.—If cells are joined together in such a way that all the positive poles (Fig. 253) are connected together and then all the negative poles are connected, these cells are said to be connected in parallel. The voltage of such a combination of cells is the same as the voltage of each cell. The internal resistance of the combination is equal to the internal resistance of one of the cells divided by the number of cells. The current sent through an external resistance by this combination of cells will be the sum of the currents supplied by each cell. By joining cells in this way the electromotive force is unaltered but the internal resistance of the battery is reduced. It is desirable to connect cells in this way if the internal resistance of the cells is large in comparison with the external resistance of the circuit.

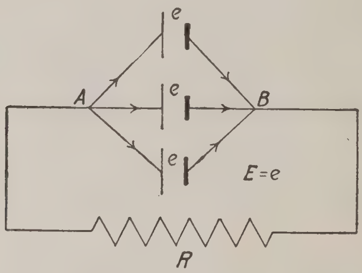


FIG. 253.—Cells connected in parallel.

The current supplied by such a battery is,

$$I = \frac{e}{R + \dfrac{r}{n}}$$

when e is the electromotive force and r the internal resistance of each cell and R the external resistance of the circuit and n the number of cells.

Example.—Three cells each having an electromotive force of e volts and an internal resistance of 4.5 ohms are connected in parallel to drive a current through an external resistance of 8 ohms. Find the current in the external resistance.

Total electromotive force = electromotive force of one cell = 2 volts

$$\text{Internal resistance} = \frac{\text{internal resistance of one cell}}{\text{number of cells}} = \frac{4.5}{3} = 1.5 \text{ ohm}$$

Total resistance of the circuit = external resistance + internal resistance
= 8 + 1.5 = 9.5 ohms

$$\text{Current} = \frac{\text{total electromotive force}}{\text{resistance of circuit}} = \frac{2}{9.5} = 0.21 \text{ amperes}$$

Problems

1. A battery of 10 cells is connected in series with an external resistance of 25 ohms. The electromotive force of each cell is 105 volts and the internal resistance of each cell is 0.85 ohm. Find the current in the circuit.

2. Ten cells each having a resistance of 2.5 ohms are arranged in parallel and produce a current of 1.2 amperes through an external resistance of 2.2 ohms. Find the electromotive force of each cell.

3. Twenty-four cells each having a resistance of 1.2 ohms and an electromotive force of 1.25 volts are connected in parallel to drive a current through an external resistance of 2.4 ohms. Find the current in the external resistance.

4. Six cells each having a resistance of 0.25 ohm and an electromotive force of 2.25 volts are connected in series to drive a current through an external resistance. If the current in the external resistance is 0.75 ampere, what is the external resistance?

5. Four cells each having an internal resistance of 2.4 ohms and an electromotive force of 1.60 volts are connected in parallel to drive a current through an external resistance. The current in the external resistance is 1.25 amperes. What is the external resistance?

6. The resistance of a certain cell is 2.4 ohms. What is the resistance of a battery made up of twenty-four such cells, when the cells are divided into four groups of six each and the cells in each group then connected in series? Three of the groups are then connected in parallel and these three parallel groups are then connected in series with the fourth group of six cells.

354. Measurement of Resistance by Ammeter and Voltmeter.—Two resistances R_1 and R_2 (Fig. 254) are connected in series so that the same current goes through both resistances. A voltmeter reads the voltage V_1 across R_1 and a second voltmeter reads the voltage V_2 across R_2. Since the current in each resistance is the same, the resistances are to each other as the voltages across them. That is, when the fall of potential across R_1 is three times as much as that across R_2, the resistance of R_1 is three times as great as that of R_2. In general,

$$\frac{R_1}{R_2} = \frac{V_1}{V_2}$$

If a standard resistance is used for R_1 and its resistance given, the magnitude of the resistance R_2 can be calculated as soon as the readings on the voltmeter V_1 and the voltmeter V_2 are observed. In this way resistances are compared by a direct comparison of the voltages across them.

Example.—A standard resistance of 10 ohms and an unknown resistance are connected as in Fig. 254. The voltmeter across the standard resistance reads 1.25 volts and that across the unknown resistance reads 3.75 volts. How large is the unknown resistance?

$$\frac{R_2}{R_1} = \frac{V_2}{V_1}$$

$$\frac{R_2}{10} = \frac{3.75}{1.25} = 3$$

$$R_2 = 30 \text{ ohms}$$

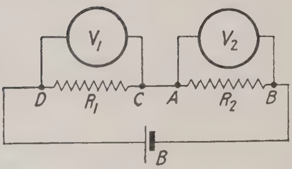


FIG. 254.—Measurement of resistance with two voltmeters.

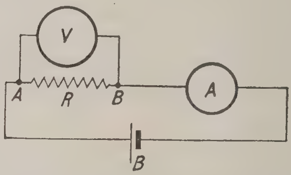


FIG. 255.—Measurement of resistance with a voltmeter and an ammeter.

Instead of using two voltmeters and a known resistance, the resistance can be determined by means of a voltmeter and an ammeter connected in the circuit as in Fig. 255. A battery B sends a current through the ammeter and the unknown resistance. The fall of potential over the unknown resistance is given by the voltmeter. If the resistance of the voltmeter is large so that very little current goes through it, the current in the ammeter is very nearly the same as the current in the unknown resistance. If the resistance of the voltmeter is not large in comparison with the resistance to be measured, correction must be made for the fact that some of the current goes through the voltmeter. In the former case where the current in the voltmeter can be neglected, the unknown resistance is given at once by means of Ohm's law from the readings of the voltmeter and the ammeter.

Let V = the reading of the voltmeter,
I = the current in the resistance R.

By Ohm's law,

$$I = V/R$$

and

$$R = V/I$$

Example.—What is the unknown resistance when a voltmeter and an ammeter connected as shown in Fig. 255 indicate 12 volts and 4 amperes respectively?

$$\begin{aligned} R &= V/I \\ &= \frac{12}{4} = 3 \text{ ohms} \end{aligned}$$

355. Resistance Boxes.—It would be inconvenient in experimental work to always have the resistances as individual coils. They are therefore joined together in boxes (Fig. 256) in such a way that any number of coils may be joined in series or any one of the coils may be used separately. In this way a great number of resistances can be obtained from a single box. Each coil is wound on a spool attached to a hard rubber sheet used as the top of the box. The wire is first doubled and then wound on the spool. The free ends are soldered to brass blocks which can be joined together by means of brass plugs inserted in holes between the brass blocks. When all of the plugs are removed the coils are all in series. Where a plug is inserted that coil is short circuited and the remainder of the coils are in series. When all the plugs are inserted the resistance is very nearly zero.

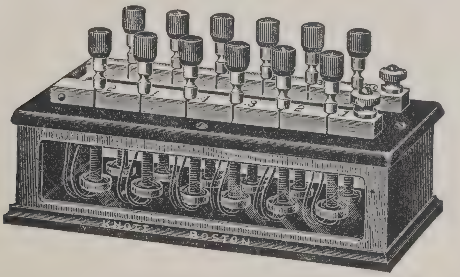


Fig. 256.—Resistance box.

356. Wheatstone Bridge.—The most accurate method of measuring resistances is by means of the Wheatstone bridge. In its simplest form it consists of two resistances R and X connected to a wire AB as in Fig. 257. To A, the junction of the wire and the resistance R, is connected one terminal of a battery. To B, the junction of the wire and the unknown resistance X, is connected the other terminal of the battery. This battery sends a current to A which divides, part of it going through R and X and the remainder through the wire AB. At D a point between R and X one terminal of a galvanometer is connected. The other terminal of this galvanometer is connected to a sliding contact which rests on the bridge wire at C. The sliding contact is moved along until there is no deflection in the galvanometer indicating that there is no difference of potential between D and C. When the bridge is thus balanced the current in R and X is the same, and the current in both parts of the bridge wire is the same.

Let $x =$ the current in X and R.
$y =$ the current in both parts of the bridge wire.
$R_a =$ the resistance of the bridge wire between A and C.
$R_b =$ the resistance of the bridge wire between C and B.

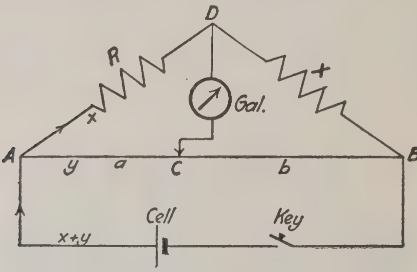


FIG. 257.—Slide wire Wheatstone bridge.

Fall of potential from A to $D =$ fall of potential from A to C.

$$Rx = R_a y$$

Fall of potential from D to $B =$ fall of potential from C to B.

$$Xx = R_b y$$

Dividing one of these equations by the other,

$$\frac{X}{R} = \frac{R_b}{R_a}$$

$$X = R\frac{R_b}{R_a}$$

When R is known and the resistances of the bridge wire between A and C and between C and B are known, X can be calculated.

If the bridge wire is made of a single homogeneous material of uniform cross section, the resistance of any part of it is proportional to the length of that part. Hence the resistance between A and C as well as the resistance between C and B is proportional to the lengths of these respective parts. In such a case instead of the preceding equation we may write,

$$X/R = b/a$$

$$X = Rb/a$$

where $b =$ the length of the bridge wire from B to C,
$a =$ the length of the bridge wire from A to C.

To determine X it is then only necessary to observe the lengths of the bridge wire when there is no current in the galvanometer and to know the value of the resistance, R.

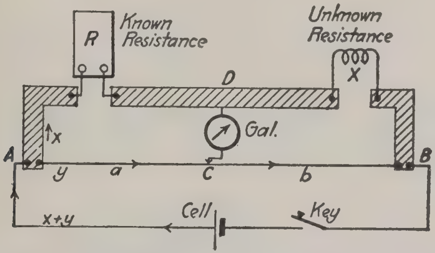


FIG. 258.—Modified slide wire Wheatstone bridge.

This slide wire bridge as it is called is modified in two ways. In one case the known resistance R is replaced by a resistance box (Fig 258) in which the resistance can be varied within certain suitable limits. In this way it is possible to make the balance point C lie near the middle of the bridge wire. The accuracy of the observations is thus increased. Fig. 259 represents the hydraulic analogue of the Wheatstone bridge. The pump replaces the battery and when there is no difference of pressure between B and D, no water flows through G.

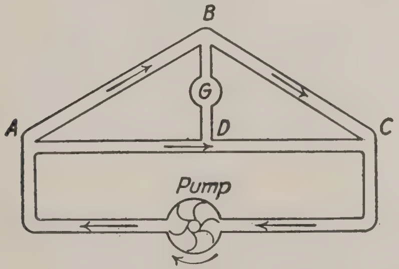


FIG. 259.—Hydraulic analogue of Wheatstone bridge.

357. The Postoffice Box Bridge.—The accuracy and convenience of the bridge may be still further increased by constructing what is known as a postoffice box bridge. In this case the bridge wire is replaced by two known resistances R_3 and R_4 of Fig. 260. These resistances can be varied within certain limits and their ratio is accurately known. The balance is reached on such a bridge by adjusting the three known resistances R_1, R_3 and R_4 until no current flows through the galvanometer. When this balance is reached,

$$\frac{R_1}{R_3} = \frac{X}{R_4}$$

$$X = R_4\frac{R_1}{R_3}$$

When the ratio of R_1 to R_3 is known and the value of R_4 for a balance has been observed the value of X can be calculated.

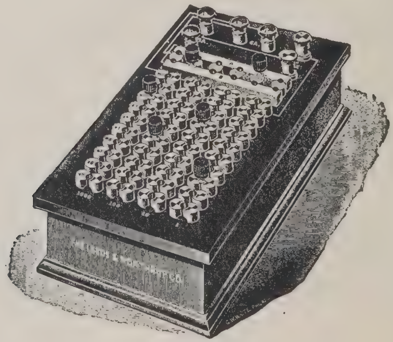


FIG. 260.—Post office box bridge.

The three known resistances are made up in a single box with provision for connecting to it the battery, the galvanometer and the unknown resistance.

Problems

1. In a slide wire bridge the balance point on the wire was found to be 60 cm. from one end and 40 cm. from the other end. The known resistance which was opposite the greater length of bridge wire had a resistance of 12 ohms. Find the unknown resistance.

2. In measuring an unknown resistance on a postoffice box bridge, the ratio arms were 1,000 to 10 ohms and the third resistance was 970 ohms. What was the unknown resistance?

CHAPTER XXVI

HEATING EFFECT OF AN ELECTRIC CURRENT

358. Heating by Electricity.—The production of heat by an electric current is of much practical importance. On account of its great convenience it is being used more and more. Electric cooking, electric soldering and electric welding are now very familiar. In an electric furnace the high temperatures necessary to melt most metals can be produced by an electric current. In an ordinary electric lamp a filament of tungsten is heated to incandescence by an electric current and becomes a source of light and heat.

359. Measurement of Heat Generated.—In order to determine the heat generated by an electric current it is necessary to measure the electric current and the resistance through which it flows or to measure the electrical current and the electrical pressure which is driving it through the resistance.

The power required to keep a current of water flowing in a pipe is found by multiplying the current in pounds of water per minute by the pressure driving the water. If the head of pressure of the water is measured in feet, the power which is the work done per minute is given by

$$\text{Pressure} \times \text{lb. per min.} = \text{ft.-lb. per min.}$$

$$\text{Horsepower} = \frac{\text{ft.-lb. per min.}}{33{,}000}$$

In exactly the same way the power or work per second required to keep a current of electricity flowing through a resistance is obtained by multiplying the current in amperes by the electrical pressure in volts.

$$P = E \times I.$$

This gives the power in watts. To reduce this to kilowatts it is necessary to divide the number of watts by 1,000 because 1,000 watts make one **kilowatt.**

$$\text{Kilowatts} = \frac{\text{volts} \times \text{amperes}}{1{,}000}.$$

1 kilowatt = 1.34 horsepower

1 horsepower = 746 watts.

Example.—A given pump raises water 100 ft. at the rate of 2,000 gal. per min. What power is supplied to the pump?

$$\begin{aligned}\text{Power} &= \text{pressure} \times \text{current}\\ &= \text{ft.} \times \text{pounds per min.}\\ &= 100 \times 2{,}000 \times 8.3\\ &= \frac{1{,}660{,}000}{33{,}000} = 46 \text{ horsepower}\end{aligned}$$

Example.—What power is required to drive a dynamo which delivers 15 amperes at a pressure of 150 volts?

$$\begin{aligned}\text{Power} &= \text{pressure} \times \text{current}\\ &= \text{volts} \times \text{amperes}\\ &= 150 \times 15 = 2{,}250 \text{ watts}\\ &= \frac{2{,}250}{746} = 3.02 \text{ horsepower.}\end{aligned}$$

360. To Compute the Power.—When it is desired to compute the power which is being consumed in an electric resistance, an ammeter is inserted and the current in the resistance is measured. At the same time a voltmeter is attached to the terminals of the resistance and the electrical pressure or voltage across the resistance is measured. This gives sufficient data from which to compute the power from the equation,

$$P = E \times I$$

This important equation may be written in a variety of forms. One form of the equation will be more convenient than the other depending on the way in which the characteristics of the circuit are stated.

$$P = EI$$

But by Ohm's law $E = RI$

Hence $P = I(IR) = I^2R$

Again $I = E/R$

Hence $P = E/R \times E = E^2/R$

Example.—Suppose that a dynamo having a voltage of 100 volts sends a current of 2 amperes through a lamp having a resistance of 55 ohms. Find the power expended in the lamp.

$$\text{Power in lamp} = E \times I = 110 \times 2 = 220 \text{ watts}$$

$$\text{Power in lamp} = I^2R = 2 \times 2 \times 55 = 220 \text{ watts}$$

$$\text{Power in lamp} = E^2/R = \frac{110 \times 100}{55} = 220 \text{ watts}$$

361. Electrical Energy in Heat Units.—Where the heat generated by allowing a current of electricity to flow through a wire is measured by noting the amount it raises the temperature of a given mass of water (Fig. 261), it is convenient to express the electrical energy in calories instead of in watts. By multiplying the current in amperes by the electrical pressure in volts the electrical energy is expressed in watts. But a watt is defined as joule per second. Hence the number of joules generated in the circuit in one second is found by multiplying the current in amperes by the electrical pressure in volts. In the study of the mechanical equivalent of heat it was found that 4.2 joules generate one calorie of heat. **Thus one joule is equal to 0.24 calories.** This is another way of saying that a current of one ampere flowing through a resistance of one ohm or under an electrical pressure of one volt generates 0.24 calories each second.

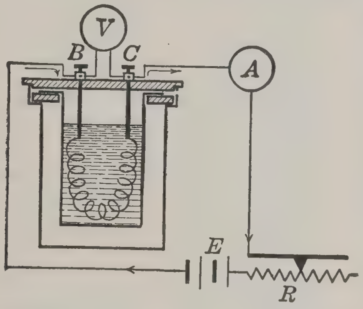


FIG. 261.—An electric calorimeter.

It would therefore raise the temperature of one gram of water 0.24°C each second. Therefore the heat generated in an electric circuit may be expressed as

H in calories

$$=0.24 \times I^2R \times t$$
$$=0.24 \times E \times I \times t$$
$$=0.24 \times \frac{E^2}{R} \times t$$

where H = heat in calories
I = current in amperes
R = resistance in ohms
t = time in seconds.

Example.—How much heat is generated per hour in an electric iron using 3.5 amperes at 100 volts?

$$\text{Heat generated} = \frac{\text{volts} \times \text{amperes} \times \text{seconds}}{4.2}$$

$$\text{Heat in calories} = .24 \quad E \times I \times t = .24 \times 100 \times 3.5 \times 3{,}600$$
$$= 302{,}400 \text{ calories}$$

362. Electric Fuses.—It is necessary to have some sort of device to protect electric machines and appliances from excessive currents. One method of furnishing this protection is by means of fuses. These fuses consist essentially of a wire which has a low melting point. Such wires are made by preparing an alloy of tin or lead with some other metals so that the melting point is lower than that of either tin or lead. When an excessive current passes through this fuse wire, the heat generated in accordance with the law just discussed becomes sufficient to melt the wire, the circuit in which the wire was inserted is opened without harm to the machinery or other electrical appliances. The size of the fuse is so chosen that it melts when the current becomes greater than a certain amount. The fuse is enclosed in some material like asbestos or porcelain so that there is no danger from fire when the fuse is being melted. The manner in which such fuses are inserted in the lighting circuits of a house is shown in Fig. 262.

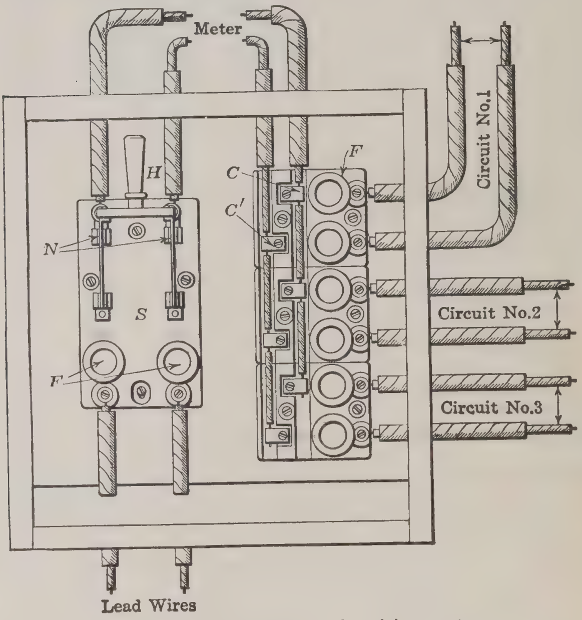


FIG. 262.—Electric fuses in wiring system.

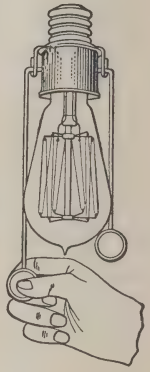


FIG. 263.—Electric dimmers.

363. Electric Light Dimmers.—Where it is necessary to use an electric light of small intensity throughout the night or to dim the headlights of an automobile, provision is made for inserting a resistance (Fig. 263) in series with the light. If the voltage applied to the light remains constant, the insertion of a resistance in series with the lamp reduces the current through it and the energy consumed by it. If R is the resistance of the lamp, I the current in it and E the electromotive force supplied, then the energy consumed by the lamp is, $P = EI = RI^2 = \frac{E^2}{R}$. If now a resistance R_1 is connected in series with the lamp, the new current is $I_1 = \frac{E}{R+R_1}$. The energy supplied to the light is $P_1 = RI_1^2$, and that supplied to the resistance is, $P_2 = R_1I_1^2$. The total energy in this case is,

$$P_1 + P_2 = RI_1^2 + R_1I_1^2 = I_1^2(R + R_1) = \frac{E^2}{(R+R_1)^2}(R + R_1) = \frac{E^2}{R+R_1}$$

The total power supplied before the insertion of the resistance was $P = \frac{E^2}{R}$. Since $R+R_1$ is greater than R the energy used after the insertion of the resistance is less than before its insertion.

Problems

1. How much heat will be given off by an electric heater which carries a current of 6 amperes at a voltage of 110 volts? What will it cost to run this heater if electricity costs 7 cents per kilowatt hour?

2. How long must a current of 7.5 amperes flow in a wire having a resistance of 20 ohms in order to melt 1 kilogram of ice?

3. How much does it cost per hour to heat an electric stove which has a resistance of 10 ohms and is used on a circuit giving 115 volts? Assume that the electricity costs at the rate of 7 cents per kilowatt hour.

4. The temperature of 250 g. of water is raised from 20°C. to 40°C. in 10 min. If the resistance of the wire carrying the current is 5 ohms, find the current in it.

5. A piece of copper weighing 750 g. is heated 10 degrees C. in one hour by a current of 1 ampere. Find the voltage applied to the coil of wire surrounding the copper.

6. If current costs at the rate of 5 cents per kilowatt hour, what will it cost to operate a 32-volt motor giving ¼ horsepower for 4 hours?

7. How many kilowatts are used in burning 25 twenty-watt lamps for 4 hr.?

8. How many twenty-watt 32-volt lamps can be lighted at the same time by a current of 10 amperes?

CHAPTER XXVII

MAGNETIC EFFECT OF AN ELECTRIC CURRENT

364. Magnetic Field about a Current.—If a copper wire is placed over a magnetic needle (Fig. 264) so that the wire is parallel to the axis of the needle, the needle will be deflected and tend to set itself at right angles to the wire. If the direction of the current is reversed, the direction of the deflection is also reversed. This experiment shows that a current of electricity produces a magnetic field about the wire and that this magnetic field has a definite direction depending on the direction in which the current flows in the conductor.

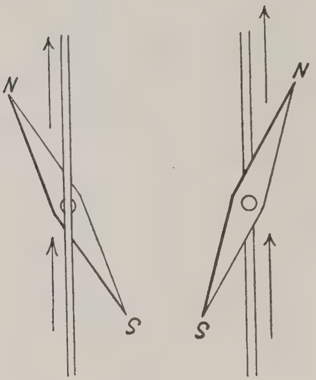


Fig. 264.—Deflection of magnetic needle by an electric current.

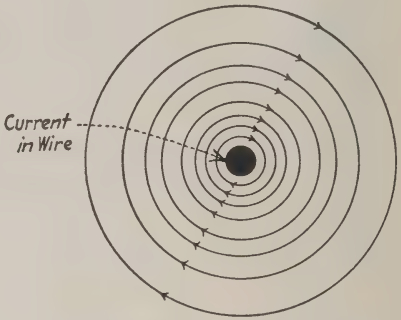


Fig. 265.—Magnetic lines of force around a conductor.

The existence of a magnetic field about a wire carrying a current (Fig. 265) may be clearly shown by passing the wire through a sheet of cardboard with its plane perpendicular to the current. If iron filings are now sprinkled on the cardboard they will arrange themselves in concentric circles around the wire. The wire is the center of all these circles.

To help in fixing the direction of the lines of force due to a current, a number of rules have been given. The following are the most useful. (1) **Grasp the wire carrying the current with the right hand and then let the thumb point in the direction of the current. The fingers will then point in the direction of the lines of magnetic force.** (2) Another rule states that if the current is flowing away from the observer the lines of force go around the current in the direction in which the hands of a clock move around the face of the clock.

365. Magnetic Field around a Coil.—If a wire is bent in a spiral so that the successive turns have the same diameter, this spiral or helix is called a **solenoid** (Fig. 266). The easiest way to make a solenoid is to wind the wire spirally around a cylinder. If a current is sent through this coil all the lines of force enter the coil at one end and leave it at the other. Consequently, nearly all the lines of force will thread the entire coil and then return outside the coil to the other end of the solenoid. If a compass needle is brought near one end of such a closely wound solenoid when it is carrying a current, the coil behaves like it were a bar magnet. When the current in the solenoid is reversed, the poles of the magnet are reversed and the direction in which the magnetic needle is deflected is reversed.

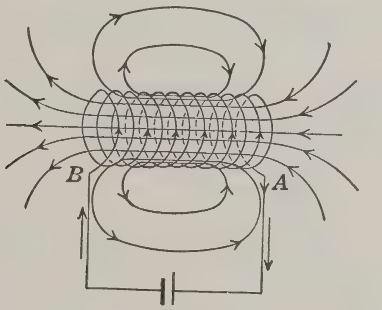


Fig. 266.—Magnetic lines of force in a solenoid.

The magnetic intensity in the interior of a solenoid whose length is great in comparison with its cross section can be found from the following equation,

$$H = \frac{4\pi NI}{10} \text{ gauss.}$$

where N = the number of turns per unit length of the solenoid.
I = the current in amperes.

This equation also gives the magnetic field inside of a solenoid bent in the form of a ring. The magnetic field in the case of a ring solenoid (Fig. 267) is entirely confined to the closed space inside the turns of wire forming the ring.

Example.—A solenoid which is 50 cm. long has 1,500 turns of wire on it. What is the magnetic field in the inside of it, when it is carrying a current of 5 amperes?

$$H = \frac{4\pi NI}{10}$$

$$N = \frac{1{,}500}{50} = 30 \text{ turns per cm.}$$

$$H = \frac{4\pi \times 30 \times 5}{10} = 188 + \text{ gauss}$$

Problems

1. What is the magnetizing force in a coil of wire wound in the form of a ring solenoid having 300 turns to the cm., when a current of 1.5 amperes is flowing in it?

2. How many amperes are necessary to produce a magnetic field of 25 gausses in solenoid having a wooden core? The solenoid which is 75 cm. long is wound with 1,200 turns of wire.

3. A cylindrical coil of 400 turns and 40 cm. in length lies with its axis parallel to the magnetic meridian. The horizontal component of the earth's magnetic field is 0.18 gauss. What current must flow through the solenoid to make the magnetic field at its center zero?

4. In two concentric solenoids the currents flow in opposite directions. The inner solenoid has 200 turns to the cm. and carries 6 amperes. The outer solenoid has 350 turns to the cm. How much current must it carry in order that the magnetic field inside the inner solenoid be zero?

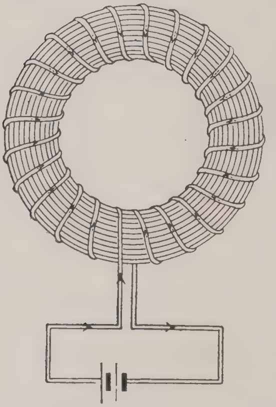


FIG. 267.—Ring solenoid.

366. Electromagnet.—When a piece of soft iron is placed inside of a solenoid it becomes magnetized and is then called an **electromagnet.** Such magnets are much more powerful than permanent magnets. The iron core loses much of its magnetism as soon as the current is removed from the coil. Under the action of the current the minute molecular magnets are nearly all oriented in one direction. When the current ceases to flow this directive force is removed and these molecular magnets again point in any direction whatsoever.

There are many applications of electromagnets. In the horseshoe type the windings are carried around the two legs of the magnet so as to make the windings continuous if the bar were straightened out. In this form of magnet the opposite poles are brought closer together and both poles are thus made available for lifting or holding. An illustration of this kind of magnet is found in the ordinary door bell. A hammer is pressed against a metallic point *F* by means of a spring *A* (Figs. 268 and 269). It thus closes the electric circuit containing the battery *B*. A soft iron armature attached to the hammer is pulled over to the electromagnet when the circuit is closed through a push button. This breaks the electric circuit and allows the hammer to fly back and again close the circuit. The circuit is thus alternately opened and closed.

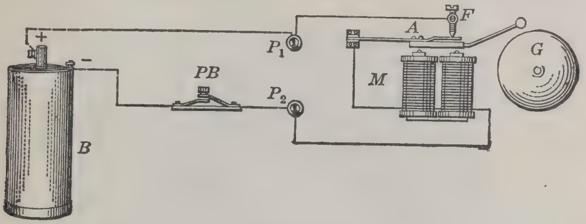


FIG. 268.—Electromagnet in a door bell.

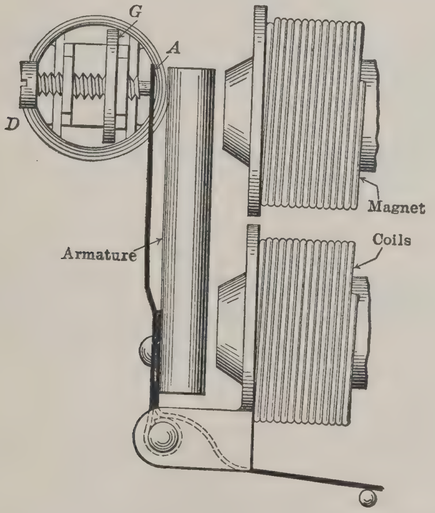


FIG. 269.—Vibrating mechanism in door bells.

Large electromagnets have great lifting power. They are now frequently used for lifting heavy loads in foundries. They hold the heavy load in the air as it is carried from one part of the building to another They have also found a use in medical work especially in extracting foreign magnetic materials from the eye. A magnet used for this purpose is represented in Fig. 271.

367. Magnetic Brake.—The magnetic brake is an application of the forces to be obtained from the magnetic field inside a solenoid. It consists of a solenoid (Fig. 272) by means of which the weight *W* on the lever arm of the brake can be lifted. When current is supplied to the solenoid, this weight is lifted and the brake released. If the current is shut off, the weight *W* is released, the brake is applied to the brakewheel and the machinery is stopped.

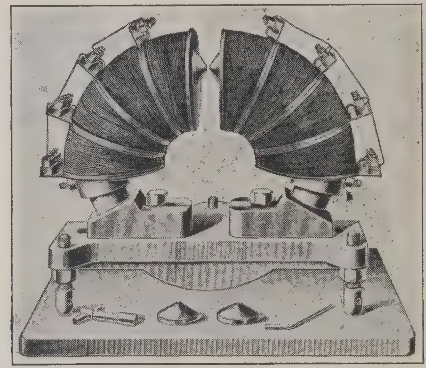


Fig. 270.—Large electromagnet.

368. Magnetic Separator.—A magnetic separator (Fig. 273) consists of two pulleys over which passes a belt which carries from the hopper the material to be separated. Some of this material is magnetic and some non-magnetic. This belt with its load passes over the pulley *C* which is magnetized. The magnetic material is carried around further than the non-magnetic material so that magnetic material separates from the non-magnetic. The magnetic material is caught in the hopper *D* and the non-magnetic in the hopper *E*.

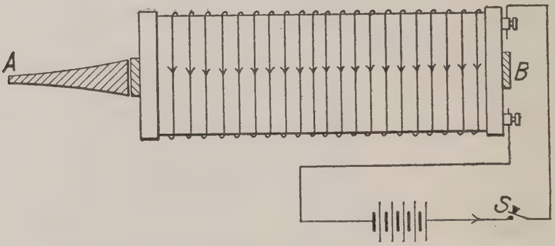


Fig. 271.—Electromagnet for medical work.

369. Electric Hammer.—The electric hammer shown diagramatically in Fig. 274 illustrates the action of a solenoid on a soft iron plunger. A current passing through the coil *C* makes the plunger a magnet with the polarity indicated in the figure. If now a current is sent through the coils *A* and *B* in the direction indicated by the arrows, the plunger *P* will be attracted by *A* and repelled by *B* and will therefore move toward the left If the current in *A* and *B* is now reversed, the plunger will move in the opposite direction. By reversing the current flowing in *A* and *B* the plunger *P* is made to move back and forth and act as a hammer.

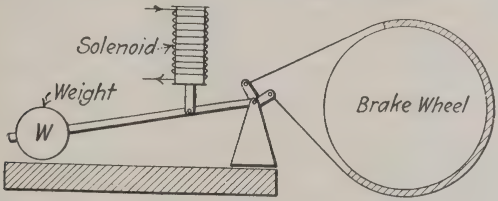

Solenoid
Weight
W
Brake Wheel

Fig. 272.—Magnetic brake.

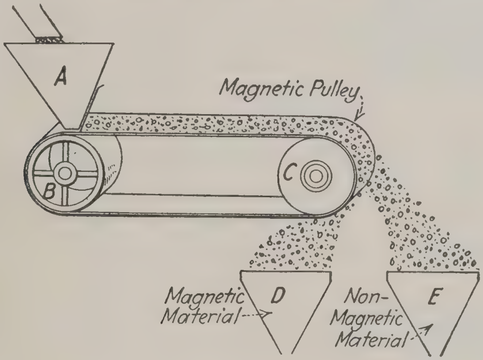

A
Magnetic Pulley
B
C
Magnetic Material
D
Non-Magnetic Material
E

Fig. 273.—Magnetic separator.

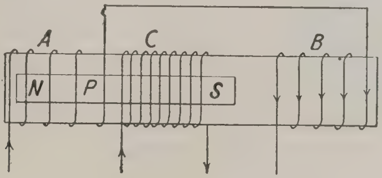


Fig. 274.—Electric hammer.

370. Moving Coil Galvanometer.—In the preceding sections it was seen that when a current flows through a coil of wire, the coil is made into a small magnet with most of the properties of a magnetic needle. If such a coil is suspended between the poles of a permanent magnet, it tends to turn so that the lines of force due to the coil and those due the permanent magnet will run in the same direction. This gives a means of measuring the current in the coil by observing the deflection which is produced when it is flowing in the coil of wire between the poles of a magnet. The magnet *NS* (Fig. 275) is usually made in the shape of a horseshoe so that it will be as strong as possible. The coil is wound on a very light frame and is suspended by means of a very fine phosphorbronze wire between the poles of the magnet. The bottom of the coil is connected to a binding post by means of a fine wire or ribbon wound in the form of a spiral. The current to be measured enters the coil through the wire by which it is suspended and leaves through the wire or ribbon spiral at the bottom of the coil. Of course the direction of the current may be reversed. When there is no current flowing in the coil of the galvanometer, the plane of the coil lies in the plane of the poles *N* and *S* of the magnet. When a current is flowing in the coil it acts like a small magnet with its axis perpendicular to the plane of the figure. The coil, which is equivalent to a little magnet, tries to turn itself so that its north pole points in the direction of the south pole of the permanent magnet and its south pole in the direction of the north pole of the permanent magnet. The coil turns until the restoring torque due to the torsion in the suspension is as great as the torque produced by the current in the coil. Then the coil comes to rest and the amount which it has turned is a measure of the current in the coil. A mirror is fastened to the coil so that its deflection can be read with accuracy. The deflection is nearly proportional to the current in the coil.

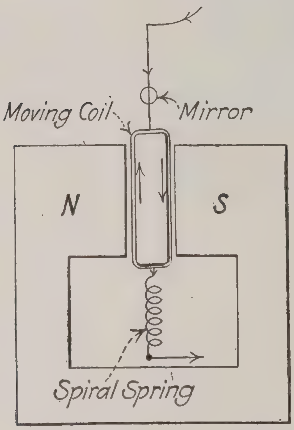


Fig. 275.—Moving coil galvanometer.

371. Ammeter.—The galvanometer just described is used to measure small currents. An important modification of it is used to measure large currents. This modification is called an **ammeter.** The form in most common use (Fig. 277) consists of a coil of fine copper wire wound on a light frame. This coil is like the coil of a moving coil galvanometer. Instead of being suspended from a fine wire it is mounted on jeweled bearings between the poles of a strong horseshoe magnet. When a current flows through the coil it rotates between the poles of the magnet. Two spiral springs, one at the top and the other at the bottom, carry the current into and out of the coil. They serve also to keep the coil in position between the poles of the magnet. To the coil is fastened a light pointer which moves over a scale and indicates the current in the ammeter. Since it is desired to measure large currents with such an instrument it is not possible to allow all the current to flow through the coil of fine wire.

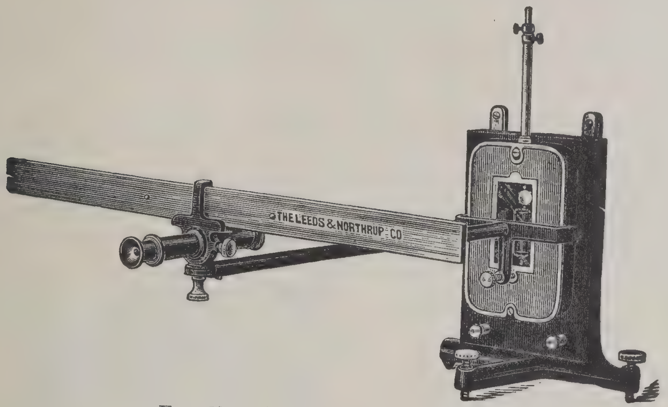


Fig. 276.—D'Arsonval galvanometer.

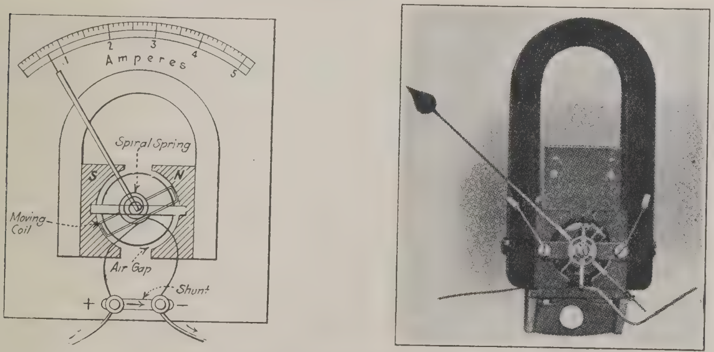


Fig. 277.—Ammeter.

For this reason there is placed between the binding posts a short thick piece of wire through which much of this current flows without flowing through the movable coil. This heavy wire or strip of metal is called a **shunt.** Since the current flowing through the movable coil is always a constant fraction of the whole current coming up to the binding post, the pointer which is fastened to the movable coil can be made to indicate the entire current coming up to the instrument.

372. Voltmeter.—The voltmeter is very similar in construction to the ammeter. Like the ammeter it consists of a moving coil which is mounted between the poles of a horseshoe magnet (Fig. 278). Under the action of the current flowing in the coil, the coil turns as in the ammeter and this rotation is indicated by the movement of a pointer. The voltmeter differs from the ammeter in one important particular, **the ammeter has a low resistance and the voltmeter a high resistance.** The ammeter is placed in series with the resistance in which the current is to be determined. Whatever current goes through the resistance also goes through the ammeter. On the other hand the voltmeter is placed in parallel with the resistance to find the voltage driving the current through the resistance. Only a small part of the whole current goes through the voltmeter. It is desired that the current which the voltmeter takes be as small as possible, so that the voltage indicated by the voltmeter be the true voltage over the resistance when the voltmeter is not in the circuit. For this reason the voltmeter has a high resistance. To make its resistance large, a coil of high resistance is placed in series with the moving coil.

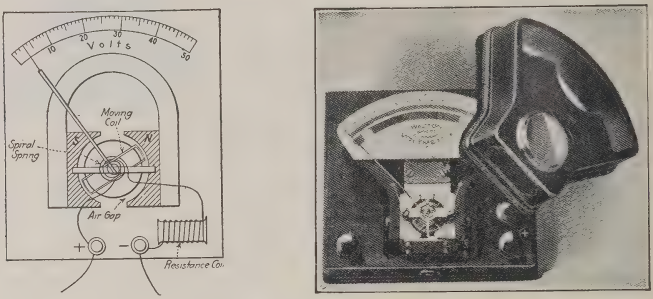


Fig. 278.—Voltmeter.

CHAPTER XXVIII

THE MAGNETIC CIRCUIT

373. Flux and Flux Density.—The introduction of an iron core into a solenoid increases very much the number of lines of force in the solenoid. The number of lines of force passing through one square centimeter of the solenoid before the iron is introduced is a measure of the field intensity in the solenoid or the **magnetizing force. It is equal to the number of dynes acting on unit magnetic pole placed in the solenoid.** The number of lines of force passing through each square centimeter after iron or some other substance is introduced into the solenoid is called the **flux density** and will be denoted by B. The total flux is the flux density times the area of the cross section. If ϕ denotes the total flux and A the area, then,

$$\phi = BA$$

374. Permeability.—Since the number of lines of force in the iron core is always much greater than the number in air for the same current in the solenoid, it is convenient to take the ratio of the number in the iron to the number in the air as a measure of the magnetic properties of the iron. This ratio of the flux density B set up in the iron or other magnetic substance to the magnetizing force is called the **permeability.** It is equal to the number of lines of force per square centimeter in the iron divided by the number of lines per square centimeter when the solenoid is filled with air or some other gas.

$$\text{Permeability} = \frac{\text{flux density}}{\text{magnetizing force}} = B/H$$

Example.—The magnetizing force in a specimen of iron is 3 dynes and the corresponding flux density is 600 lines per sq. cm. What is the permeability of the iron?

$$\text{Permeability} = \frac{\text{flux density}}{\text{magnetizing force}} = \frac{600}{3} = 200$$

375. Magnetomotive Force.—Just as hydraulic pressure is necessary to force water through a pipe and an electric pressure to force a current of electricity through a wire so also a magnetic pressure is necessary to produce magnetic flux in a magnetic circuit. This magnetic pressure is called **the magnetomotive force** and bears the same relation to the magnetic circuit that the electric pressure bears to the electric circuit. **The magnetomotive force can be defined as the magnetic pressure necessary to force the magnetic flux through the circuit.** It is found by multiplying the magnetic field by the length of the magnetic circuit.

$$\text{M.m.f.} = H \times l$$

$$\text{For a solenoid } H = \frac{4\pi NI}{10\,l} = \frac{1.26\,NI}{l}$$

where N is the total number of turns in the solenoid.

$$\text{M.m.f.} = \frac{1.26\,NIl}{l} = 1.26\,NI.$$

The product NI is known as the **number of ampere turns** on the circuit.

The distinction between the magnetic field inside the solenoid and the magnetomotive force must be kept clearly in mind. The magnetic field is the force on unit pole inside the solenoid. It is determined by the product of the number of turns per cm. and the electric current.

$$H = \frac{4\pi NI}{10l}$$

The magnetomotive force is the work to carry unit pole around the circuit. The unit of M.m.f. in the c.g.s. system is the erg per unit pole and is called the gilbert. It is determined by the product of the current and the total number of turns on the circuit.

$$\text{M.m.f.} = \frac{4\pi NI}{10.}$$

Example.—Find the magnetomotive force of a solenoid consisting of 1,500 turns, when the current is 20 amperes.

$$\text{M.m.f.} = \frac{4\pi NI}{10} = 1.26NI = 1.26 \times 1{,}500 \times 20 = 3{,}780 \textit{ gilberts}$$

376. Reluctance.—In a magnetic circuit there is a magnetic resistance which is analogous to the electric resistance in an electric circuit. **The opposition or resistance which must be overcome when the magnetic flux is established is the reluctance**

of the circuit. The reluctance increases with the length of the magnetic circuit and decreases as the area or the cross section of the circuit is increased. The reluctance is inversely proportional to the permeability of the material out of which the circuit is made. To find the resistance of a conductor the following equation is used,

$$R = kl/a$$

where k = the resistance per unit length of the wire of unit cross section.
l = the length of the wire.
a = the area of the cross section.

The reluctance of a magnetic circuit is given by the equation,

$$R = \frac{1}{\mu}\frac{L}{A}$$

where L = the length of the magnetic circuit in cm.
A = the area of the cross section in sq. cm.
μ = the permeability of the material of the circuit.

These equations are similar in form except that the reciprocal of the permeability is used in the equation for the reluctance in place of the specific resistance in the equation for the resistance. **The unit of reluctance is the oersted.** (See Appendix C.)

Example.—A piece of iron which is 50 cm. long has cross section of 10 sq. cm. and a permeability of 500. What is the reluctance?

$$\text{Reluctance} = \frac{\text{length}}{\text{area} \times \text{permeability}}$$
$$= \frac{50}{10 \times 500} = \frac{1}{100} = 0.01 \text{ oersteds}$$

377. The Law of the Magnetic Circuit.—There is a magnetic pressure called the magnetomotive force driving the flux through a magnetic circuit and there is a magnetic resistance which must be overcome in establishing the flux in the circuit. The relation between the magnetomotive force, the reluctance and the flux in the circuit is the same as the relation between the electromotive force, the electric resistance and the current in an electric circuit. Ohm's law states that

$$\text{Current} = \frac{\text{electromotive force}}{\text{resistance}}.$$
$$I = E/R$$

The **law of the magnetic circuit** states that,

$$\text{Flux} = \frac{\text{magnetomotive force}}{\text{reluctance}} = \frac{\text{M.m.f.}}{\text{Rel.}}$$

$$\phi = \frac{\text{M.m.f}}{\text{Rel.}}$$

where I = current in amperes.
ϕ = magnetic flux lines of force.
E.m.f. = electromotive force in volts.
M.m.f. = magnetomotive force in gilberts.
R = electrical resistance in ohms.
Rel. = reluctance in oersteds.

Example.—Find the number of lines of force in magnetic circuit in which the reluctance is 0.05 oersteds and the magnetomotive force is 45 gilberts.

$$\text{Flux} = \frac{\text{magnetomotive force}}{\text{reluctance}} = \frac{45}{.05} = 900 \text{ lines of force}$$

378. Magnetization of Iron.—To study the magnetic properties of iron it is best to have the iron in the form of a ring surrounded by a solenoid. The iron fills the whole space in which there are lines of magnetic force and no lines pass out into the air. As the magnetizing force is slowly increased from zero to larger values the flux density or induction increases slowly at first but soon rises rapidly. The rate of increase of the induction falls off again at higher values of the magnetizing force and for large values of the magnetizing force increases very slowly. If the induction is plotted against the magnetizing force the resulting curve is called the **magnetization curve.** (Fig. 279). From the inspection of such a curve it is seen that sometimes a small increase in the magnetizing force causes a large increase in the flux density and that at other places on the curve a large increase in the magnetizing force is necessary to produce even a small change in the flux density. This means that the permeability of iron or steel depends on the number of lines of force which it already contains. Hence the permeability is not a constant but varies with the flux density. The dotted curve of Fig. 279 shows the way in which the permeability of a specimen of iron changes with the number of lines of force in it.

The fact that after a time the flux density increases very slowly with the increase of the magnetizing force makes it impracticable to magnetize a piece of iron beyond a certain flux density. By this is meant that a point has been reached where a large increase in the magnetizing force is necessary to produce a small increase in the flux density. This point is marked by the knee of the curve (Fig. 279). When iron is used in a motor or generator it is not profitable to use a flux density which is greater than the flux density corresponding to knee of the magnetization curve. It requires too many ampere-turns to produce the added flux.

379. Hysteresis.—If a specimen of iron has been magnetized by subjecting it to constantly increasing field intensities and if then the field intensity is decreased, the flux does not decrease along the same curve by which it had increased but it follows a curve which lies above the magnetization curve. The flux lags behind the magnetizing force. When the magnetizing force has become zero there still remains a considerable amount of flux in the iron. The flux which remains when the magnetizing force has been reduced to zero is called the **residual magnetism.** If the direction of the magnetizing force be now reversed the flux quickly becomes zero. That magnetizing force which is necessary to reduce the flux to zero is termed the **coercive force.** It is represented in Fig. 280 by the line *OE*. When the magnetizing force is still further increased, the iron becomes magnetized in the opposite direction. If the magnetizing force is again gradually decreased, the flux again gradually lags behind the magnetizing force, giving a curve which in this case lies below the original curve. When the magnetizing force has been made

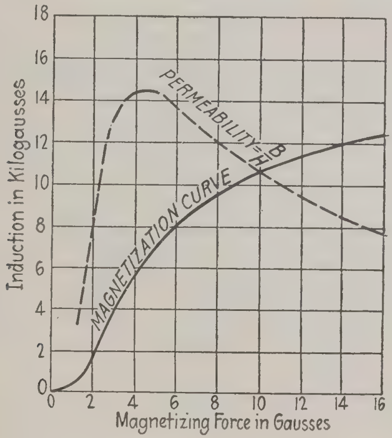


FIG. 279.—Magnetization curve for iron.

zero again, there is a residual magnetism in the iron in the opposite direction to that in the former case. This residual magnetism is represented by the line *OG*. The reversal of the magnetizing force again causes the residual magnetism to disappear. The coercive force necessary to reduce the residual magnetism to zero is in this case represented by the line *OF*. By increasing the magnetizing force sufficiently the cycle closes and the iron is in the condition in which it was at the beginning of the cycle.

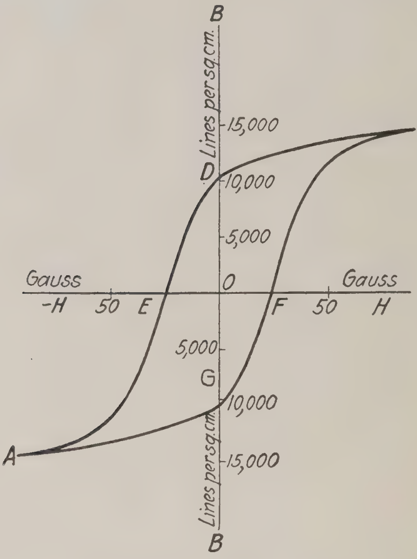


Fig. 280.—Hysteresis cycle.

During the complete cycle of the field intensities the induction has described a loop called a **hysteresis cycle.** This arises out of the fact that the induction lags behind the magnetizing force. The area included in this loop is a measure of the loss of energy in the iron during the cycle. This energy appears as heat and is lost for all practical purposes. It therefore cuts down the efficiency of the machine. It is consequently of importance to use soft iron in an electric machine, for soft iron has a smaller loss due to hysteresis than is found in cast iron.

Problems

1. An iron ring is wound with a coil of wire containing 1,500 turns. The ring has a mean circumference of 35 cm. and the wire carries a current

of 0.6 ampere, the permeability of the iron being 850. What is the flux density within the iron?

2. How many ampere-turns per centimeter are necessary to produce a flux density of 16,000 lines per sq. cm. in an iron ring which has a permeability of 500?

3. An electromagnet has in it a total flux of 150,000 lines. It is wound with a coil containing 750 turns and this coil carries a current of 0.8 ampere. What is the reluctance of the magnetic circuit?

4. A magnetic circuit consists of (1) 20 cm. of iron cross section 8 sq. cm. and permeability 200; (2) 60 cm. of wrought iron, cross section 9 sq. cm. and permeability 1,600; (3) 1.2 cm. of air with a cross section of 12.5 sq. cm. Calculate the reluctance of this circuit.

5. A magnetic circuit has a length of 85 cm. How many turns of wire must be wound on it to produce a flux density of 8,000 lines when the current in the wire is 1.5 amperes?

6. In order to magnetize an iron rod a magnetomotive force of 600 gilberts is required. The magnetizing coil has 300 turns and 75 ohms resistance. What voltage must be applied to it?

7. An iron ring has a mean diameter of 60 cm. and a cross section of 42 sq. cm. Find the number of ampere-turns which will produce a flux of 750,000 lines. Permeability of iron is 850. What is the reluctance of the circuit?

8. Find the number of ampere-turns necessary to establish a magnetic flux of 12,000 lines in the air gap of a magnetic circuit which consists of an iron path which is 150 cm. long and 25 sq. cm. in cross section and has a permeability of 400. The area of the air gap is 30 sq. cm.

CHAPTER XXIX

INDUCED CURRENTS

380. Currents Induced by Moving Magnets.—It was discovered by Faraday that if the ends of a coil of wire of many turns are connected to the terminals of a galvanometer (Fig. 281) and if the pole of a magnet is pushed into the coil, the coil of the galvanometer moves, showing the presence of a current in the circuit while the magnet is in motion. As soon as the magnet ceases to move the current no longer flows in the galvanometer. If the opposite pole is now thrust into the coil, the galvanometer deflects in the opposite direction, showing that the current is flowing in the opposite direction to that in which it flowed in the former case. When the pole of the magnet is removed from the coil the current flows in the opposite direction to that in which it flowed when the magnetic pole was inserted into the coil.

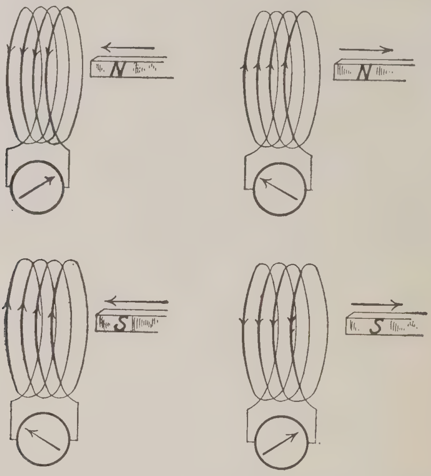


FIG. 281.—Currents induced by a moving magnet.

It is immaterial whether the coil is moved with respect to the magnet or the magnet is moved with respect to the coil. The experiment may then be performed in the following way instead of in the way already described. Over one pole of a magnet is thrust a coil containing many turns of wire. As the coil is moved over the pole a deflection is noted in the galvanometer indicating an induced current. When the coil is removed the deflection is in the opposite direction. If the coil is thrust over the north pole instead of the south pole the deflections are reversed.

These experiments show that it is possible to produce a momentary current by moving a magnet with respect to a coil of wire or by moving a coil with respect to a magnet. Currents produced in this way are known as **induced currents.** This is the most important method of producing electric currents for it is in this way that electricity is generated in a dynamo.

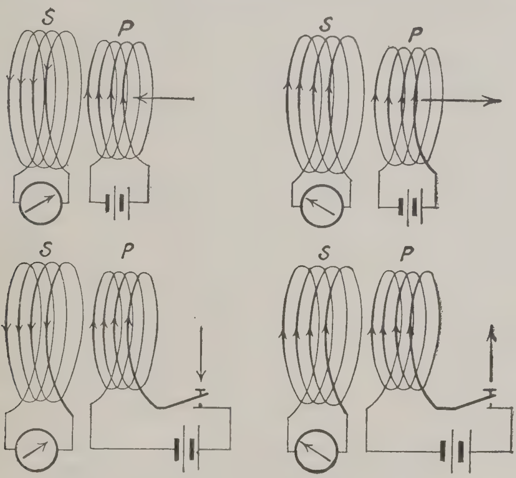


FIG. 282.—Currents induced by currents.

381. Currents Induced by Currents.—It has already been seen that a coil of wire in which is flowing a current of electricity behaves in every way like a magnet. If therefore a coil of wire carrying a current of electricity is brought near another coil of wire connected to a galvanometer we will expect to observe effects very similar to those observed when a magnet is moved with respect to such a coil. In Fig. 282 a coil of wire S is connected to the terminals of a galvanometer and a second coil P is connected to the terminals of a battery. If the coil P which is carrying a current of electricity, is brought near the coil *S*, a deflection is noted in the galvanometer while *P* is in motion. When *P* is removed there is again a deflection in the galvanometer which is in the opposite direction to that noted while the coil *P* was being brought up. If the coils are wound in the form of solenoids so that one can be inserted into the other, the induced current is larger in each case. The effect observed in the galvanometer is the same whether *P* is brought near to the coil *S* or the coil *S* is brought near to *P*. The greater the current in the coil *P* the greater is the deflection in the galvanometer caused by the induced current. The larger the current in *P* the greater the magnetic field surrounding it. By inserting an iron core in the coil *P* the magnetic field about it may be very greatly increased and when the coil *P* containing such an iron core is introduced into the coil *S* the induced current in *S* is much increased over that which was obtained when the coil *P* did not contain an iron core.

382. Making and Breaking the Circuit.—When the coil *P* is placed inside the coil *S* with the iron core in position and the current through *P* is suddenly broken, a large induced current is set up in the coil *S*. By breaking the circuit of the coil *P* the current in this coil disappears and with it also disappears the magnetic field which surrounded it. The removal of these lines of force from the coil *P* and the coil *S* at the same time causes a current in the coil *S*. This current in the coil *S* continues only so long as the current in *P* is changing. If now the circuit of *P* is again closed allowing the current in *P* to again flow, there is an induced current in *S* while the current in *P* is building up to its final value. This current, however, is in the opposite direction to the current observed when the circuit was broken.

By comparing the currents obtained in these cases with those obtained by bringing up or taking away the coil *P* it is found that the effect of bringing the coil *P* up to *S* is the same as closing the circuit in the coil *P* and the effect of taking *P* away from the coil *S* is the same as opening the circuit of the coil *P*. The coil *P* through which the current flows from the battery is known as the **primary** and the coil *S* in which the electromotive force is induced is called the **secondary.**

383. Law of Induced Electromotive Forces.—These observations show that there is an induced current set up in the secondary (1) when the current in the primary is started or stopped;

(2) when the strength of the current in the primary is either increased or decreased; (3) when the primary coil carrying a current of electricity is moved either closer to or farther from the secondary; (4) when a permanent magnet is moved with respect to a coil of wire. All of these facts may be included in the two following laws:

1. **There is an induced electromotive force in any coil of wire in which the number of lines of force are changing and the magnitude of this electromotive force is equal to the rate at which the number of lines of magnetic force through the coil is changing.**
2. **There is an induced electromotive force in any conductor which is moving across lines of force and the magnitude of this electromotive force is equal to the rate at which the lines of force are being cut by the conductor.**

384. Lenz's Law.—A consideration of the direction of the induced currents in these circuits shows that **the induced current always flows in such a direction that its magnetic field opposes the change of conditions giving rise to the induced current.** This is known as Lenz's law.

To illustrate Lenz's law, consider a north-seeking pole which is being pushed into a coil of wire. A current will flow in the coil and this current will flow in such a direction that its magnetic action will oppose the motion of the magnet. Now the force exerted by a north-seeking pole is the only force which can oppose the approach of a north pole. Hence the current in the coil of wire will flow in such a direction that it will produce a north pole at the end of the coil lying nearest the approaching magnet. Consider now that the north pole of the magnet is taken away from the coil. The current in the coil will flow in such a direction that it will oppose the taking away of this north pole. In order to oppose the taking away of the pole an attraction must be produced. It is therefore necessary to make the end of the coil nearest the receding magnetic pole a south pole. Consequently the current in the coil flows in such a direction that a south pole is developed at the end nearest the receding north pole, thus producing an attraction which opposes the motion of north magnetic pole.

385. Direction of Induced E.m.f.—It is of much convenience to have a rule by which to remember the direction in which the induced current flows when the direction of the motion and the direction of the magnetic field are known. Suppose that a straight wire is moved across a magnetic field (Fig. 283). If the conductor is carried upward the current is found to flow from *A* to *B*. When the conductor is carried downward the current flows in the opposite direction. If the direction in which the magnetic field is acting is reversed without changing the direction of motion of the conductor, the direction of the current is reversed. In order then to know the direction of the current it is necessary to know the direction of the magnetic field and the direction of motion of the conductor. The following rule has been suggested by Fleming.

Extend the thumb, forefinger and middle finger of the right hand so they are at right angles to each other. Let the thumb point in the direction in which the conductor is moving and the forefinger in the direction in which the magnetic field is acting. Then the middle finger will point in the direction in which the induced current is flowing.

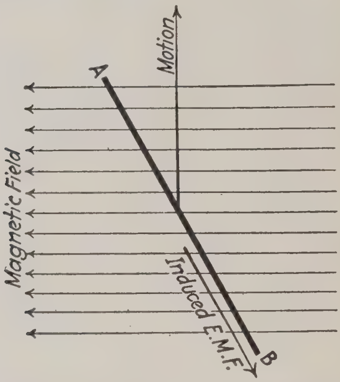


FIG. 283.—Direction of induced E.m.f.

386. Magnitude of Induced E.m.f.—The magnitude of the electromotive force induced in a moving conductor depends on three factors.

(1) The speed with which the conductor moves, (2) the number of lines of force per unit area in the magnetic field, (3) the number of turns of wire in the coil which is being moved. In Fig. 284

$$E = \frac{H \times v \times l}{10^8} \text{ volts,}$$

where E is the induced electro motive force in volts; v the velocity in cm. per sec. and l the length of the wire in cm. The greater the speed with which the conductor moves the greater is the induced electromotive force. The stronger the magnetic field, the larger is the induced electromotive force and the induced electromotive force is also increased by increasing the number of turns of wire on the moving coil. These relations may all be expressed by saying that the induced electromotive force is proportional to the number of lines of force cut each second by the moving conductor. If a conductor cuts 100,000,000 lines of force per second, there is a difference of potential of 1 volt between its ends.

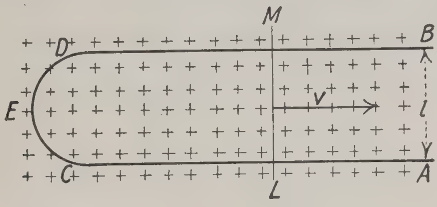


FIG. 284.—Magnitude of induced E.m.f.

Example.—A wire which is 50 cm. long is carried through a magnetic field of 15,000 lines per sq. cm. at the rate of 100 cm. per sec. What is the electromotive force between its ends?

The number of lines cut = 15,000 × 50 × 100 = 75,000,000 per sec.

$$\text{E.m.f.} = \frac{\text{number lines cut}}{10^8} = \frac{75{,}000{,}000}{10^8} = 0.75 \text{ volt.}$$

Problems

1. How many volts are induced across the ends of a wire if the wire cuts 5×10^8 lines of force in 5 sec.?

2. A wire cuts a magnetic field of 3,500,000 lines 750 times per min. What is the voltage in the wire?

3. A wire which is 200 cm. long moves across a magnetic field with a velocity of 1,500 cm. per sec. Find the voltage induced in the wire, when the magnetic field is 5,000 lines per sq. cm.

4. If a wire which is moving across a magnetic field at the rate of 300 cm. per sec. has 5 volts generated in it, how rapidly must it move to have 7.5 volts generated in it?

387. The Induction Coil.—An important illustration of induced electromotive forces is found in the induction coil. This coil consists essentially of a core made of fine soft iron wire (Fig. 285). Around this iron core is wound the primary coil made of a few turns of heavy copper wire. Insulated carefully from this primary coil is the secondary which is wound on the outside of the primary. The secondary contains a large number of turns of fine wire which is silk-covered for better insulation. By making or breaking the current rapidly in the primary an induced electromotive force is set up in the secondary. In order to break the current rapidly in the primary a hammer interrupter is connected in the primary circuit. This interrupter consists of a heavy spring which is fastened to a piece of iron. This piece of iron is near the iron core of the primary. As this iron core is magnetized by the primary current it pulls over the springs with this piece of iron. The primary current flows through the spring to a point *D* in contact with the spring and then out to the battery *B*. When the spring is pulled away from the contact

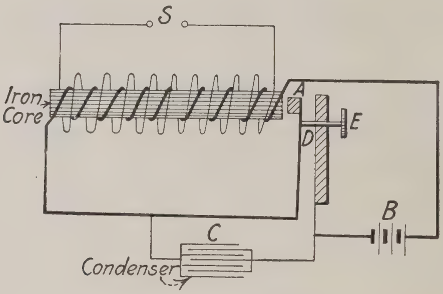


FIG. 285.—Induction coil.

D, the primary current is broken. The current in the primary dies away and the magnetism of the iron core disappears. The spring with the piece of iron flies back into position and the primary circuit is again closed. This is the same action which takes place in the electric door bell (Fig. 268).

When the primary circuit is broken an electromotive force is induced in the secondary. This electromotive force is large since the number of turns in the secondary is much larger than the number of turns on the primary. If the terminals of the wire forming the secondary are not joined together a spark tends to jump from one end of the wire to the other.

When the current in the primary is broken the electric current tends to keep on going just as a current of water tends to keep on flowing when a faucet is closed. In the case of water when a faucet is suddenly closed the water strikes against the pipes with considerable force, until it has lost all its velocity. The tendency of the current to keep on going increases the time required to stop the current. To get the greatest induced electromotive force the primary current must be annulled as quickly as possible. To make the break more abrupt, a condenser C is connected across the the gap in the primary. This condenser is made of sheets of tinfoil which are separated by paraffin paper. It acts as a storage place into which the current can surge when the circuit is broken. This increases the rapidity with which the current is eliminated from the primary and increases also the magnitude of the induced electromotive force in the secondary. The condenser also helps to prevent sparking at the contact when the primary current is broken.

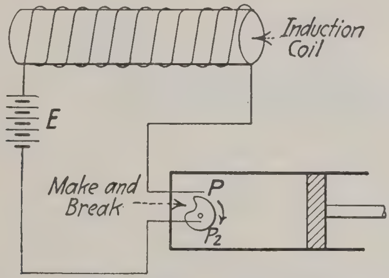


Fig. 286.—Make and break ignition.

388. Make and Break Ignition.—The principle of the induction coil is made use of in the make and break system of ignition. (Fig. 286.) A coil of wire consisting of many turns is wound on a soft iron core. This coil is connected through a battery E to two points inside the cylinder of the gas engine. One of these points is stationary and the other moves. When the points separate the current is broken but the tendency of the current to keep on going is so great that a spark occurs at the time at which the points are separated. This spark which jumps across the gap between the points ignites the gases in the cylinder of the engine.

389. High Tension Ignition.—The electrical connections of a high tension ignition system are represented in Fig. 287. Such a system consists essentially of an induction coil with terminals of secondary connected to the spark plug. The terminals of the primary are connected through the battery E to the stationary and revolving arms of the contact-maker. In order to insure quick cessation of the primary current and prevent destructive arcing at the vibrator-contacts, a condenser is connected across the battery and the vibrator contacts. When the piston of the engine is near the end of its forward stroke, the revolving of the contact-maker opens the primary circuit, and causes a spark in the secondary at the spark-plug. This produces an explosion of the gases in the cylinder.

390. The Transformer.—If an iron ring (Fig. 288) of any form be wound with two separate coils which are insulated from each other and an alternating current be sent through one of them, an alternating current of the same frequency will flow through the other. Let one of these coils be the primary and the other the secondary. Energy by this means has been handed from the primary to the secondary through the medium of the iron core. If the small losses in the iron core can be neglected, the energy in the primary circuit must be equal to the energy in the secondary.

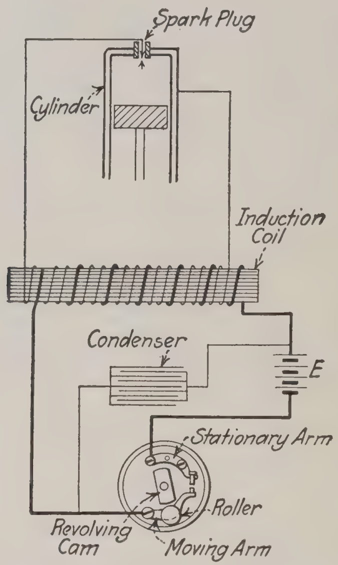


Fig. 287.—High tension ignition.

Let E_p = the voltage in the primary,
I_p = current in the primary,
E_s = voltage in the secondary,
I_s = current in the secondary.
Then, $E_pI_p = E_sI_s$.

The ratio of the number of turns in the primary to the number of turns in the secondary determines the ratio of E_p to E_s. If there are a large number of turns on the secondary and few turns on the primary, the electromotive force of the secondary will be large in comparison with that of the primary. It can be shown that

$$\frac{E_p}{E_s} = \frac{N_p}{N_s} = \frac{I_s}{I_p}$$

It is seen from this equation that where the electromotive force is large, the current is small and that where the electromotive force is small the current is large. Hence by means of such a transformer a small electromotive force and a large current may be transformed into a large electromotive force and a small current. There are then two types of transformers, **step-up transformers,** which increase the voltage and decrease the current and **step-down transformers,** which decrease the voltage and increase the current. In the former the secondary has a large number of turns in comparison with the primary. In the latter the reverse is the case. In practice the current is generated at high voltage, then transmitted to the place where

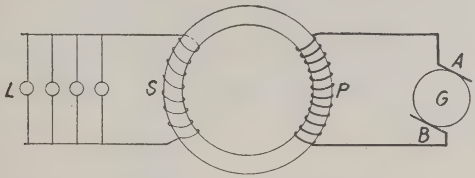


Fig. 288.—A transformer.

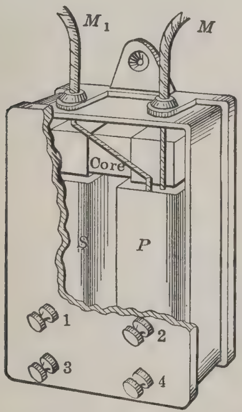


Fig. 289.—Doorbell transformer.

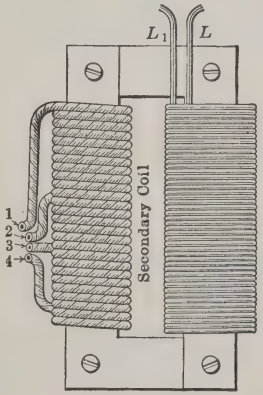


Fig. 290.—Details of doorbell transformer.

it is to be used and there transformed into a lower voltage. There is less loss in transmitting small currents at high voltages than there is in transmitting large currents at low voltage.

A transformer for ringing door bells has been represented in Figs. 289 and 290. It takes a small current at 110 volts and transforms it into a larger current at a potential of a few volts.

391. The Autotransformer.—For small transformers where the ratio of transformation is not large, economy of construction and efficiency of operation are obtained by using the same coil for both primary and secondary. Such a transformer is known as an **autotransformer.** The arrangement and connections of the coils are shown in Fig. 291. The entire coil AC is the primary of the transformer and the part between B and C is the secondary. If N_1 is the total number of turns on the primary AC; N_2, the number of turns on the secondary between B and C, E_1 the electromotive force applied to the primary, E_2 the electromotive force set up in the secondary, I_1 the current in the primary and I_2 the current in the secondary, then

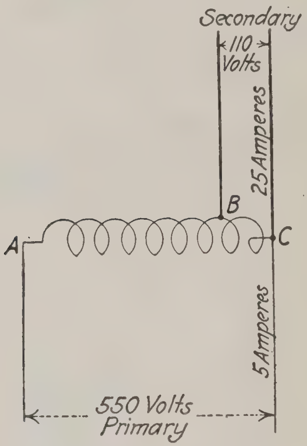


Fig. 291.—Autotransformer.

$$\frac{E_1}{E_2} = \frac{I_2}{I_1} = \frac{N_1}{N_2}$$

Since N_2 is less than N_1, E_2 is also less than E_1 and the transformer steps down the voltage.

Such step-down autotransformers are used for reducing the electromotive force applied to alternating current motors while the motor is being started. In such cases they take the place of a starting box.

392. Telephone.—The receiver of a telephone (Fig. 292) consists of a hard-rubber handle containing a hard-steel cylindrical magnet, around one end of which is wound a coil *AB* of many turns of fine wire. The ends of this wire are brought to binding screws on the end of the receiver. Close to the end of the magnet *MN* which carries the coil of wire there is placed a disc *D* of thin sheet iron. This disc is supported at its edges so that it is free to vibrate in the middle which is placed near the end of the magnet. It is clamped at the edges to the receiver by means of a hard-rubber cap with a hole in its center.

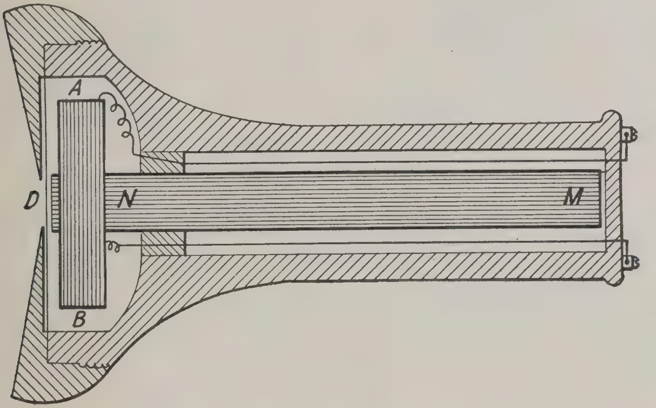


Fig. 292.—Telephone receiver.

The transmitter (Fig. 293) consists of a cell *C* which contains carbon granules between two plates of polished carbon. In front of this cell is mounted a metal diaphragm *D*. The terminals of the battery *B* are connected to metal plates at the front and back of this carbon cell so that the current from the battery goes through the carbon resistance. The battery *B* is also connected in series with the primary *P* of a small induction coil. The secondary *S* of this induction coil is connected to the line wires leading to the receiving station. As the diaphragm *D* vibrates in response to sound waves, it changes the pressure on the carbon granules in the cell *C*. This causes a change in the resistance of the cell, since the resistance of loose carbon contacts varies rapidly with the pressure. This variation in resistance produces a fluctuation in the current from the battery B. This fluctuation of current in the primary produces induced currents in the secondary. These induced currents in the secondary pass along the line wires to the receiver at the other end of the line. There they change the strength of the magnet which is acting on the diaphragm in the receiver. Vibrations are thus set up in the diaphragm of the receiver. These vibrations correspond to the sounds which produced the vibrations of the diaphragm in the transmitter. Hence the sounds imparted to the diaphragm of the transmitter are reproduced by the diaphragm of the receiver.

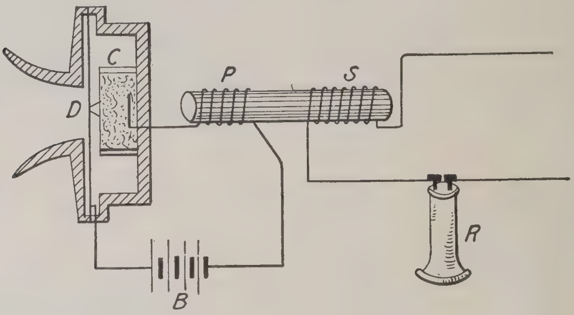


FIG. 293.—Telephone transmitter.

CHAPTER XXX

THE DYNAMO

393. The Current in a Revolving Loop.—If we rotate a rectangular coil of wire between the poles of a large electromagnet, we have a picture of what goes on in a dynamo in the simplest case. Let the loop of wire be revolving as shown in Fig. 294. Suppose we start with the coil in the vertical position and turn it clockwise. The side *AB* of the loop will be moving down across the magnetic field. Applying the right hand rule to the wire *AB* the induced e.m.f. is found to be in the direction *B* to *A*. Applying the same rule to *DC* the e.m.f. is found to be from *D* to *C*. Hence a current flows around the loop in the direction indicated.

When the loop has moved one-half revolution from its vertical position, the side *AB* begins to move up and the side *DC* to move down. Applying the right-hand rule to this case as before, the current in the wire will be reversed since the direction of motion has been reversed. During this half of the revolution the current will go around the loop in the opposite direction to that in which it flowed in the first half of the revolution. When such a loop is revolving in a magnetic field, it has in it an alternating current which reverses its direction once during each revolution. This current will have its least value when the coil is in the vertical position. In this position the wires in the coil are moving parallel to the magnetic field and are therefore not cutting any lines of magnetic force. When the coil is in the horizontal position each of the wires are cutting the lines of force most rapidly because they are moving at right angles to these lines of force. At the instant the coil is in the horizontal position, the current has its maximum value.

394. Sine Curve of E.m.f.—If we measure the angle through which the coil has rotated from its vertical position, the horizontal position would be 90 degrees from this position. When it is again vertical it would be 180 degrees from this position and when it is again horizontal it is 270 degrees from its initial position.

Now plot a curve between the position of the coil and the electromotive force generated in it. The position of the coil will be measured from its first vertical position and will be plotted on the horizontal axis. The electromotive force generated in the coil will be plotted on the vertical axis. Electromotive forces in one direction are arbitrarily called positive and those in the opposite direction are called negative. By this method a curve like that shown in Fig. 295 is obtained. This curve shows that the voltage generated in the coil rises rapidly to a maximum value when the coil has turned through 90 degrees and then decreases to zero again when the coil is vertical again.

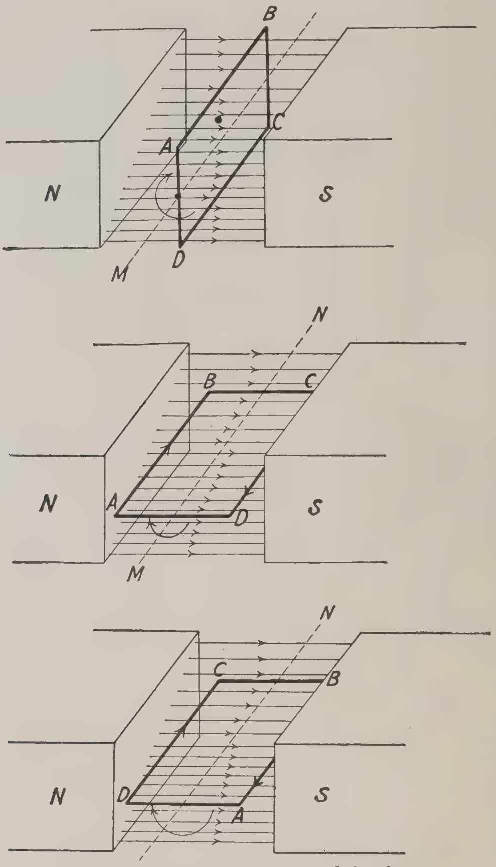


FIG. 294.—Current in a revolving loop.

Here its direction is reversed and it rises to a maximum value in the opposite direction. From this position it decreases to zero a second time when the coil has made one complete revolution. This cycle of events takes place every complete revolution of the coil.

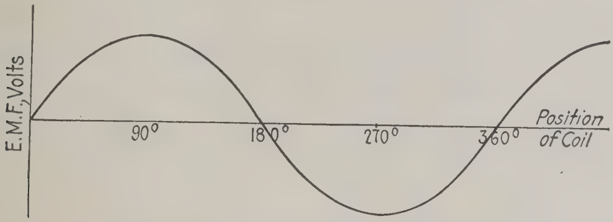


Fig. 295.—Sine curve of electromotive forces.

395. Collecting Rings.—If an outside circuit is connected continuously to the two ends of the revolving coil the electromotive force in the outside circuit behaves just like the voltage in the revolving coil. This is an electromotive force which reverses its direction once each revolution of the coil. In order to make continuous connection between the revolving coil and the outside circuit, the ends of the wire forming the coil are fastened to two rings (Fig. 296) which are mounted on the axis of the revolving coil. On each of these rings presses a spring called a **brush** and to these brushes are connected the terminals

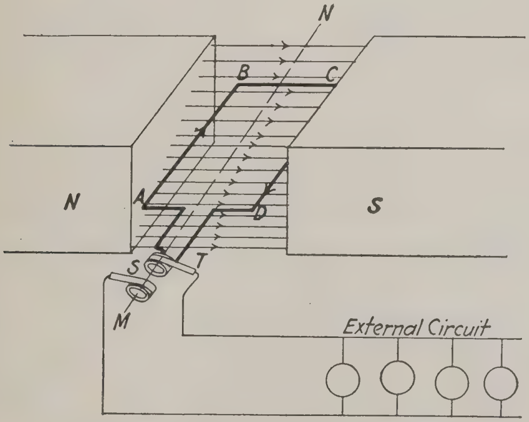


Fig. 296.—Collecting rings on dynamo.

of the outside circuit. As the coil revolves these rings revolve with it, moving under the brushes and always making contact with the wires forming the external circuit. In Fig. 298 the external circuit is represented as a number of lamps. These lamps form a continuous circuit with the revolving coil and the electromotive force generated in the coil drives an alternating current through these lamps.

396. The Commutator.—To get a direct current, that is, one which always flows in the same direction through the circuit from a loop of wire revolving between the poles of a magnet, the terminals of the wire instead of being joined to two rings are joined to a divided ring called a **commutator.** Such a divided ring is shown in Fig. 297. On this divided ring rest two brushes which are connected to the external circuit and so placed that they slip from one segment to the other at the time at which the electromotive force in the revolving coil is zero. To understand better how this commutator works consider a simple case. If the revolving loop is rotating clockwise, the wire *CD* which is moving downward will give a current in the direction of the arrow and the current in the wire *AB* will be in the direction of the arrow on that wire. Hence the current will leave the brush *T* and circulate through the external circuit in the direction of the arrow from *T* to *S*. When the loop has made one-half revolution, the segment of the commutator which was in contact with the brush *T* makes contact with the brush *S* and the other

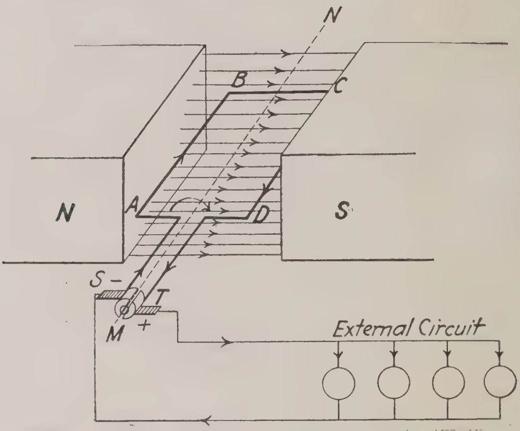


Fig. 297.—Direct current dynamo.

segment makes contact with the brush *T*. The wire *CD* is now moving up and the current in it is reversed. The wire *AB* is moving down and the current in it is also reversed. The segments of the commutator have, however, reversed their positions. Hence the current will still leave by the brush *T* and flow through the external circuit in the same direction in which it flowed in the former case. By this means the current in the external circuit is always kept flowing in the same direction. In the revolving coil the current flows first in one direction and then in the other.

The voltage generated at the brushes may now be represented by a curve of the form shown in Fig. 298. When the coil is in the vertical position between the poles, it will be moving parallel to the lines of force and no electromotive force will be generated in it. When it has revolved 90 degrees from this position, it will be cutting lines of force most rapidly and the electromotive force will have its greatest value. When the coil has again turned into the vertical position, the electromotive force at the brushes is again zero and one-quarter of a turn later the electromotive force will be again a maximum. Since the position of the commutator has been reversed with respect to the brushes this electromotive force will have the same direction at the brushes as in the former case. A machine provided with a commutator for making the current always flow in the same direction in the external circuit is a direct current dynamo.

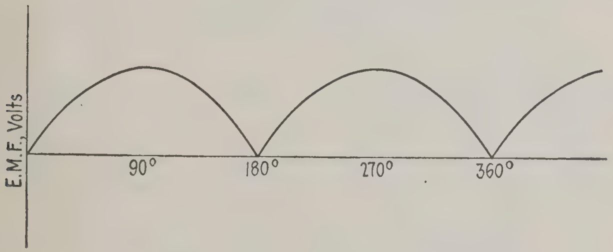


Fig. 298.—Reversal of current by a commutator.

397. Ring and Drum Armatures.—There are two general ways in which armatures are wound (1) the drum type (2) the ring type.

In the drum type a cylinder of iron is built up out of thin sheets of iron. The use of such sheets of iron for the armature decreases the losses which would occur if the cylinder were made of a solid piece of iron. Around the outside of this cylinder there are wound the wires in which the electric currents are induced. The surface of such a core is usually slotted so that the loops of wire lie between projections as in Fig. 300.

In the ring type of winding the cylinder is built up as in the drum type except that it is hollow and the wires are wound in and out in such a way that they do not pass continuously around the the outside of the cylinder (Fig. 301). In the ring armature only one side of the wires cut the lines of force. The other wires are on the inside of the ring where there are few lines of force. On the other hand in the drum armature all the wires are on the outside and each loop cuts all the lines of force. The drum type is the one now most commonly used because it is stronger and simpler.

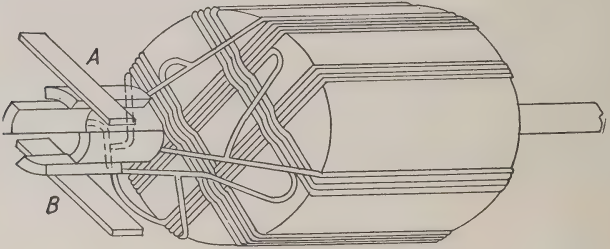


Fig. 299.—Drum armature.

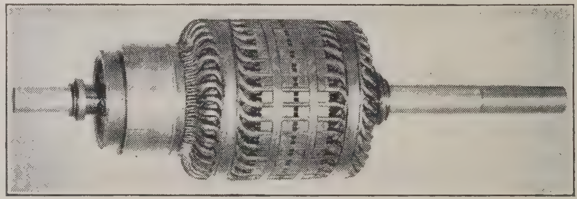


Fig. 300.—Drum armature and commutator.

398. Excitation of the Fields of Generators.—Generators and motors are often classified with regard to the manner in which the electromagnets which produce their field are excited. From this point of view generators may be classified as **separately excited** or **self excited.** The self excited generators may again be divided into **series, shunt** and **compound wound** generators according to the way in which the field windings are connected to the external circuit. In a separately excited generator or motor the current for the field coils is taken from an outside source of current. In the self excited generators the current for the field coils is supplied by the generator itself. Where it is desirable to keep the field strength in the machine independent of the current which the machine is taking or delivering separately excited generators or motors are used. Self excited generators may change their polarity in starting and stopping but this is not true of separately excited generators.

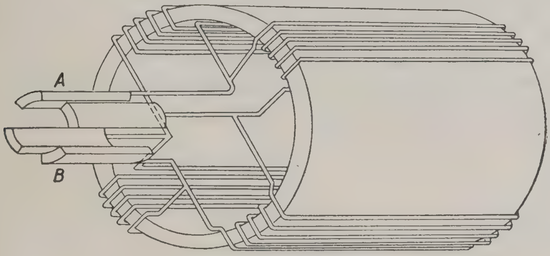


Fig. 301.—Ring armature.

399. Series-Wound Dynamo.—In a series-wound dynamo the current which goes through the main line also goes through the wires wound around the field coils. The coils producing the magnetic field are thus in series with the external circuit (Fig. 303). The field coils consist of a few turns of coarse wire and have therefore a low resistance. When the generator is not delivering current there is no magnetic field except that due to residual magnetism.

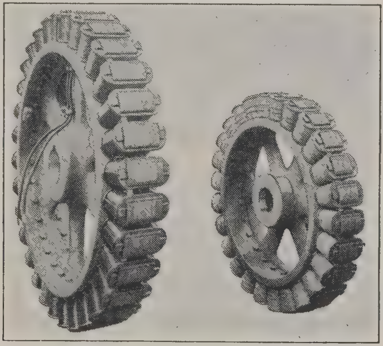


Fig. 302.—Revolving magnetic fields of a generator.

The greater the current taken by the line, the greater is the current in the field coils and the greater is the magnetic field in which the armature rotates and consequently the greater the electromotive force generated in the armature. In order to keep the magnetic field constant and therefore the electromotive force constant it is necessary to always use the same current in the external circuit. Such a dynamo is satisfactory for operating a number of arc lights in series for in that case the current is always the same. It is not satisfactory for operating a number of incandescent lamps in parallel for every time one or more lamps is turned off or on the voltage of the dynamo changes.

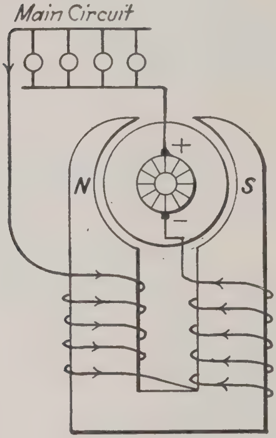


Fig. 303.—Series wound dynamo.

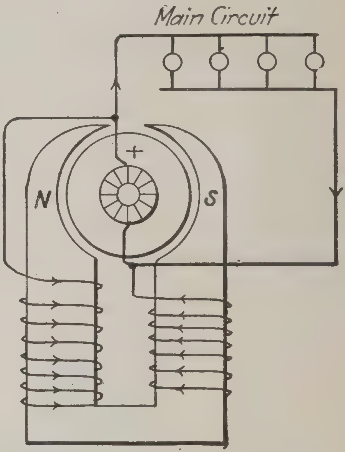


Fig. 304.—Shunt wound dynamo.

400. Shunt-Wound Dynamo.—In this type of winding the field coils are excited by a current shunted around the main circuit as in Fig. 304. Many turns of fine copper wire are wound on the field coils giving them a high resistance. It is not desirable to have a large current in the field coils because all the current that goes through the field coils is lost to the main circuit and decreases the efficiency of the machine.

When the current in the external circuit is increased, the voltage at the terminals of this kind of generator decreases somewhat. This drop in voltage tends to still further decrease the current in the field coils and cut down the magnetic field and the voltage of the machine. A shunt-wound generator does not therefore supply constant voltage for all leads.

401. Compound-Wound Dynamo.—On increasing the current supplied by a series wound dynamo the voltage also increases. On the other hand increasing the current supplied by a shunt wound generator causes a drop in the voltage. It should then be possible to combine these two types of windings and produce a type of winding which gives constant voltages for all loads. This is what is done in the compound wound dynamo (Fig. 305). The field coils are wound first with a large number of turns of fine wire just as in the shunt wound type. On top of these windings is then wound a series coil through which passes all the current which goes to the main circuit. When the current in the external circuit is increased these series windings tend to increase the voltage which the machine is making. At the same time the effect of this increase of current in the external circuit is to decrease the magnetic field produced by the shunt windings. These two effects may be made to neutralize each other and the voltage thus kept constant for different loads.

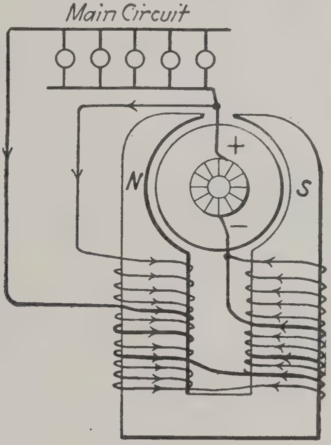


FIG. 305.—Compound wound dynamo.

402. Losses in a Generator.—The losses which occur in a generator may be divided into three classes (1) mechanical losses such as friction (2) the copper losses in the field and armature (3) the iron losses due to eddy currents. To find the mechanical losses it is necessary to determine the mechanical forces necessary to overcome such forces as friction. The copper losses can be computed by multiplying the square of the current in the armature by the resistance of the armature and adding to this the product obtained by multiplying the square of the current in the field coils by the resistance of the field coils. Thus

$$W_a = i_a{}^2R_a$$
$$W_f = i_f{}^2R_f$$
$$\text{Copper loss} = W_a + W_f.$$

Whenever a conductor cuts lines of force there is set up in it induced electromotive forces which tend to send electric currents through it. The copper wires of an armature are wound on an iron core and this iron core is an electrical conductor which is revolving in a magnetic field and cutting lines of force. There will be set up in this iron core induced electromotive forces which will tend to send currents of electricity through it. These **eddy currents** as they are called will heat the iron of the armature. Their presence also makes it more difficult to rotate the armature in the magnetic field and thus requires that more energy be supplied for the operation of the generator. In order to reduce these eddy currents and the losses to which they give rise to a minimum, the armature is built up of thin sheets of iron which are insulated from each other. These sheets of iron are so placed that their planes are perpendicular to the direction in which the currents tend to flow. This cuts down the induced electric currents without changing the magnetic flux. The thinner these sheets of iron, the less the eddy current losses.

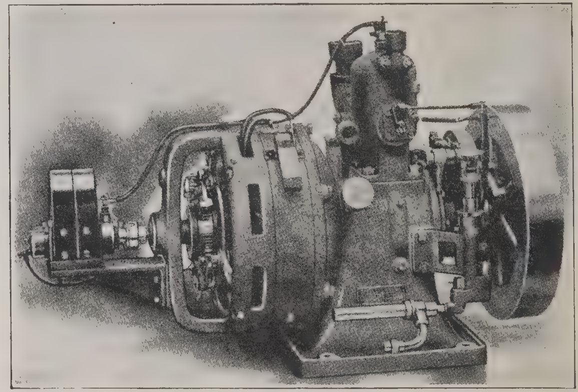


FIG. 306.—1 K-w gasoline electric generating set (General Electric Co.)

Problems

1. The armature of a dynamo has resistance of 0.14 ohms. The dynamo gives 112 volts on open circuit and 110 volts on full load. What is the current in the dynamo when it is running on full load?

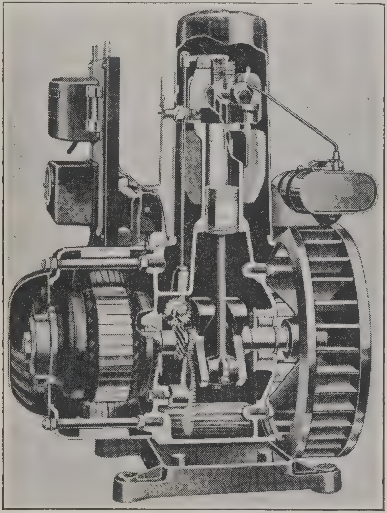


FIG. 307a.—A farm light plant.

2. In a generator the potential drops from 125 volts on no load to 112 volts on full load. The current in the armature is 32 amperes. Find the resistance of the armature.

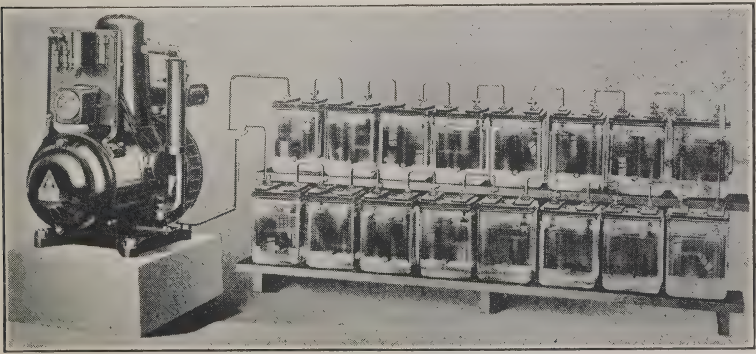


FIG. 307b.—A farm light plant.

3. The resistance of the field coils of 3/4 k.w. generator is 125 ohms. The brush potential at full load is 110 volts. Find the current in the field coils and the power consumed in them.

4. The resistance of a series generator is 125 ohms. The brush potential is 105 volts when the machine is delivering 12 amperes. Assuming that the field strength and the speed do not change, find the brush potential when the machine delivers 30 amperes.

5. The resistance of the armature of a bipolar shunt generator is 0.60 ohm and the resistance of the field coils is 85 ohms. The brush potential is 102 volts when the machine is delivering 18 amperes. What e.m.f. is it generating?

6. The resistance of the armature of a series generator is 0.50 ohm. The field coil has a resistance of 0.35 ohm. The generator supplies 15 incandescent lamps connected in series, each lamp taking a current of 8 amperes at 55 volts. What e.m.f. is generated by the machine and what is the drop of potential inside of it?

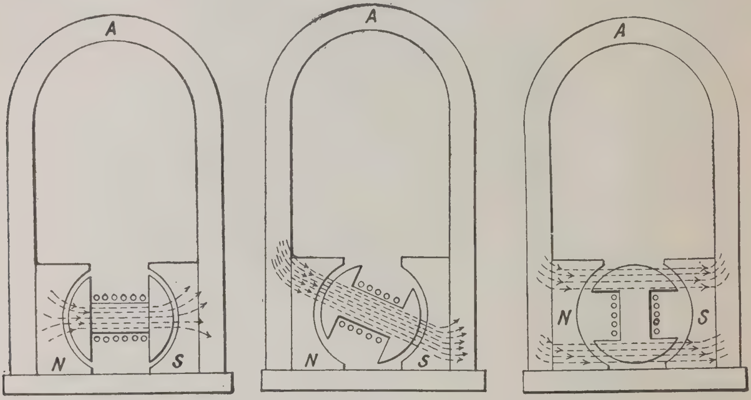


Fig. 308.—Distribution of flux in a magneto.

403. Magneto.—The operation and construction of a magneto are the same as those of the simple dynamo except that the magnetic field in which the armature rotates is produced by permanent magnets in the magneto and by electromagnets in the dynamo. The cycle of operation through which the magneto goes will be exactly like that described for the dynamo. The distribution of flux in a magneto in different positions is shown in Fig. 308. The construction of a magneto is shown in Fig. 309.

If the external circuit from such a magneto is carried to the spark plug of an engine, the armature must be driven so that it is making its maximum electromotive force when the spark is needed in the engine. This is provided for in practice by properly timing the spark and controlling the rate at which the armature rotates. The armature is driven from the flywheel of the engine through a reducing gear so that the armature makes one revolution to two revolutions of the flywheel. It would be possible to reduce the rate of revolution of the armature still further but this is rarely done.

404. Efficiency of Generators and Motors.—The efficiency of an engine has been defined as the output divided by the input.

$$\text{Efficiency} = \frac{\text{output}}{\text{input}}$$

In a generator it is easy to measure the output by measuring the current and the voltage. The input is obtained by adding the losses to the output. In this case the efficiency may be written as,

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}}$$

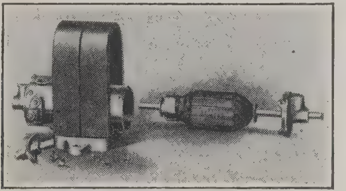


FIG. 309.—Disassembled magneto.

Example.—In a shunt dynamo the stray power loss is 415 watts and the copper loss is 220 watts. What is the efficiency of the dynamo when it is delivering 65 amperes at 110 volts?

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}}$$

$$\text{Total loss} = \text{stray power loss} + \text{copper loss}$$
$$= 415 + 220 = 635$$

$$\text{Output} = 65 \times 110 = 71{,}500 \text{ watts}$$

$$\text{Input} = 71{,}500 + 635 = 72{,}135 \text{ watts}$$

$$\text{Efficiency} = \frac{71{,}500}{72{,}135} = 99.3 \text{ per cent}$$

In motors the losses are subtracted from the input to get the output. The input is calculated from the current and the voltage supplied to the motor. For a motor the efficiency becomes,

$$\text{Efficiency} = \frac{\text{output}}{\text{input}} = \frac{\text{input} - \text{losses}}{\text{input}}$$

Example.—A motor running at full load takes 75 amperes at 110 volts. The stray power loss is 800 watts, and the copper loss is 450 watts. What is the efficiency of the motor?

$$\text{Efficiency} = \frac{\text{input} - \text{losses}}{\text{input}}$$

Input = 75 × 110 = 82,500 watts

Losses = Stray power loss + copper loss
800 + 450 = 1,250 watts

Output = input − losses
= 82,500 − 1,250 = 81,250

$$\text{Efficiency} = \frac{81{,}250}{82{,}500} = 98.7 \text{ per cent}$$

Example.—The armature of a shunt wound generator has a resistance of 0.5 ohm and the field coils a resistance of 50 ohms. What is the copper loss when the generator is delivering 25 amperes at 100 volts?

$$\text{Current in field coils} = \frac{100}{50} = 2 \text{ amperes}$$

Joules generated in field coils = $2^2 \times 50 = 200$

Joules generated in armature = $(25)^2 \times 0.5 = 312$

Copper loss = 512 joules.

Problems

1. A 40 k.w. generator at full load has a stray power loss of 1,250 watts and a copper loss of 2,500 watts. What is its efficiency?

2. What is the efficiency of a motor which takes 90 amperes at an electrical pressure of 115 volts, when the copper loss is 3,500 watts and the stray power loss is 1,800 watts?

3. A shunt motor requires 60 amperes at 110 volts. The stray power loss is 500 watts. The field has a resistance of 125 ohms and the armature a resistance of 0.15 ohm. What is the efficiency of the motor?

4. A shunt generator delivers 60 amperes at a brush potential of 110 volts. If the stray power loss is 750 watts, what is the efficiency of the generator? The resistance of the field coils is 55 ohms and the resistance of the armature is 0.25 ohm.

CHAPTER XXXI

MOTORS

405. The Dynamo Reversed.—When a dynamo is driven by some source of power as by a steam engine, it can generate electricity. If now this same dynamo is supplied with current from some outside source as from a storage battery it is capable of doing work. For example it may be now used to operate almost any kind of machinery. This dynamo being driven backward by a current is now called a **motor**. It is exactly like a dynamo consisting of an armature, a commutator and field magnets. In order to understand its operation, it is necessary to consider the effect which is produced by placing a wire carrying a current of electricity in a magnetic field.

406. Force on a Wire in a Magnetic Field.—When a wire carrying a current of electricity is brought into a magnetic field so that the lines of force are perpendicular to the wire carrying the current, the conductor carrying the current experiences a force driving it out of the magnetic field. To understand this fact recall that every current generates about itself a magnetic field of its own. This magnetic field is represented by concentric circles with the wire carrying the current as center. If a wire carrying a current is brought into a magnetic field which is uniform and runs from left to right, the lines of force from the magnetic field of the wire will add to those in the uniform field above the wire and subtract from them for points below the wire. In consequence of this fact there will be a crowding of the magnetic lines of force above the wire (Fig. 310). This crowding of the magnetic lines of force above the wire will cause it to move from the position where the magnetic field is large to where it is smaller. In this case the wire will move downward. If the current in the wire had flowed in the opposite direction, the magnetic lines of force would have been crowded below the wire and the wire would have moved upward.

Fig. 311 shows the cross-section of a single loop of wire which, carrying a current of electricity, was introduced into a uniform magnetic field. It is assumed that the current is sent in at A and out at B. The flux is weakened below A and strengthened below B. There results a force tending to push A downward and B upward. These two forces cause the loop to rotate. The stronger the current in the wire the greater the force tending to rotate it. The greater the length of the wire and the greater the magnetic field in which it is placed, the greater the force tending to rotate the loop. The force acting on one of the wires is

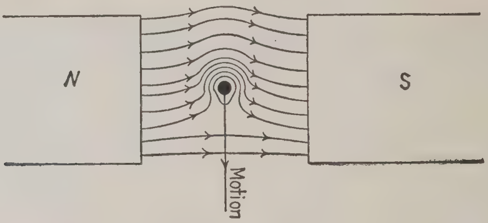


FIG. 310.—Force on a conductor in a magnetic field.

$$F = \frac{IHl}{10 \times 980} \text{ grams.}$$

where F = force in grams
I = current in amperes;
H = magnetic field in gausses;
l = length of wire in centimeters.

The torque acting on the loop is

$$Fd$$

where d is the perpendicular distance between the conductors.

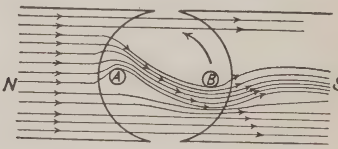


FIG. 311.—Torque on a coil in a magnetic field.

Example.—A wire which is 40 cm. long is lying in a magnetic field of 40,000 gausses. If the current in the wire is 50 amperes what is the force acting on it?

$$F = \frac{I \times H \times l}{10 \times 980}$$

$$= \frac{50 \times 40{,}000 \times 40}{10 \times 980} = 8{,}163 \text{ g.}$$

$$= 8.16 \text{ kg.}$$

Problems

1. A wire 30 cm. long lies at right angles to the line of magnetic force. The force on the wire is 2 kg. when the current through it is 20 amperes. Find the strength of the magnetic field in which the wire lies.

2. A wire which is 35 cm. long lies at right angles to the magnetic field and is acted on by a force of 1.5 kg. when the magnetic field is equal to 250 g. Find number of amperes flowing in the wire.

3. A wire lies in a magnetic field which has a strength of 3,000 gausses. The current flowing in the wire is 15 amperes and the force required to hold the wire in position in the magnetic field is equal to 250 g. Find the length of the wire.

4. A rectangular loop of wire lies in a magnetic field (Fig. 311). The length of the side of the loop is 25 cm. and the width of the loop is 15 cm. The strength of the magnetic field in which the loop lies is 9,000 gausses and the current in the wire is 40 amperes. Find the torque acting on the loop.

5. The distance between the sides of a rectangular loop lying in a magnetic field of 15,000 gausses is 25 cm. The length of each side lying perpendicular to the magnetic field is 35 cm. and the current in the wire is 8 amperes. Find the torque on the loop.

407. Hand Rule for Motor.—For finding easily the direction in which a conductor carrying a current in a magnetic field will move, there is a hand rule which is very similar to that described for telling the direction in which a current is induced in a moving conductor. In this case the left hand is used instead of the right hand as in the other case. **Take the left hand and place the thumb, the forefinger and the middle finger so that they are at right angles to each other. Point the forefinger in the direction in which the magnetic field is pointing and the middle finger in the direction which the current is flowing in the wire. Then the thumb will point in the direction in which the conductor will move under the action of these magnetic fields.**

408. Types of Motors and Back E.m.f. in a Motor.—Dynamos and motors for direct currents are often made in exactly the same way. There will therefore be three types of motors for direct currents just as there were three types of generators. These three types of motors are series, shunt and compound. The type of motor chosen will depend on the uses for which it is intended.

In the study of the dynamo it was seen that as the armature consisting of a large number of turns of wire revolves in a magnetic field, there is generated in it an electromotive force. Now since in the motor the construction is precisely the same as in the dynamo, we again have an armature revolving in a magnetic field. There should therefore be set up in this armature an electromotive force. No matter what the machine, when a coil of wire revolves in a magnetic field, it will have an electromotive force produced in it. This means that every motor is at the same time a dynamo. There is only one difference. In the case of the dynamo the armature is made to revolve by the action of some external force whereas in the case of the motor the current flowing into the armature from some outside source causes the armature to revolve. In other words the cause for the rotation of the armature is different in the two cases.

By applying the right hand rule to the armature revolving in the magnetic field it is found that the direction of the electromotive force generated in the armature is such that it opposes the current flowing in the armature. For this reason this electromotive force is called *a back electromotive force or a counter electromotive force.*

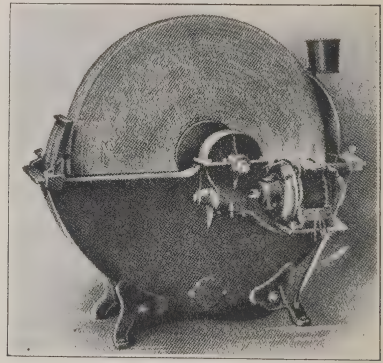


Fig. 312.—Motor driven grindstone (General Electric Co.)

When the armature revolves more rapidly the back electromotive force is increased. Hence the opposition to the flow of the current from the outside is increased and the current in the armature is decreased. That is, the faster the armature revolves the less current will flow through it.

If an incandescent lamp is connected in series with a small motor while the armature is held stationary, the lamp glows almost as if the motor were absent. When, however, the armature is allowed to revolve the lamp becomes dim. This is because the electromotive force generated in the armature opposes the impressed electromotive force and thereby reduces the current in the armature.

409. Starting Box.—Since the current in the armature depends on the difference between the impressed electromotive force and the back electromotive force, this current will be largest when the armature is at rest for in that case the back electromotive force is zero. In order to keep the current from being excessive at the start it is customary to introduce in series with the motor a resistance which reduces the current flowing in the circuit. As the speed of the motor increases and the back electromotive force becomes appreciable, this resistance is reduced. Such a resistance is called a starting resistance or a starting box (Fig. 314).

FIG. 313.—Motor driven threshing machine (General Electric Co.)

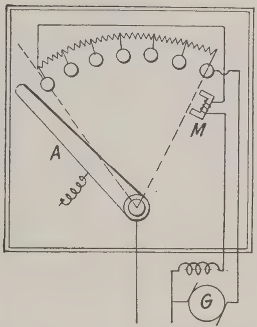


FIG. 314.—Starting box.

Example.—The armature of a shunt motor contains 0.3 ohm resistance. A difference of potential of 110 volts is applied to it. What is the current before the motor starts?

$$I = 110/0.3 = 366 \text{ amperes}$$

If the back electromotive force is 108 volts when the motor has speeded up, the current becomes,

$$I = \frac{110 - 108}{0.3} = \frac{2}{.3} = 6.66 \text{ amps.}$$

Problems

1. A motor is running at 115 volts and taking 5.5 amperes in the armature. The resistance of the armature of the motor is 0.25 ohms. What is the counter e.m.f. in the armature?

2. In a motor it was found that the impressed e.m.f. was 220 volts, and the counter e.m.f. was 210 volts when the current in the armature was 15 amperes. What is the resistance of the motor?

3. A 110-volt shunt motor has an armature resistance of 0.8 ohm. The resistance of the field coils is 220 ohms. If the motor takes 10 amperes when running under full load, what is the counter e.m.f. in the armature?

4. The armature of a shunt motor has a resistance of 2 ohms. The field has a resistance of 250 ohms. Find the current in the field coils and in the armature when the machine is at rest if the impressed voltage is 115 volts.

5. When the motor in the preceding problem is running fast enough to make 105 volts counter e.m.f. what is the current in the armature and the current in the field coils?

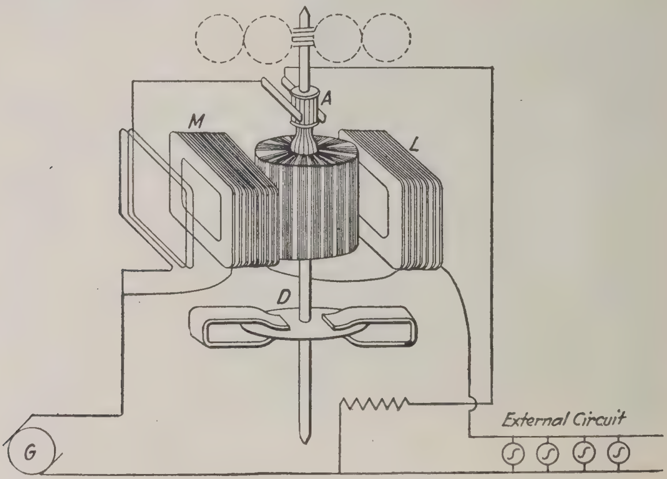


Fig. 315.—Recording wattmeter.

6. A shunt motor takes a total current of 50 amperes from 110 volt mains. The resistance of the armature is 0.10 ohm and the resistance of the field coils is 75 ohms. Find current in the field coils and in the armature and the power used in heating the armature and the field coils.

7. A 5-k.w. 110 volt shunt motor has a field resistance of 75 ohms and an armature resistance of 0.10 ohm. Find the current in the armature and in the field coils at full load.

8. The impressed voltage across a motor armature is 32 volts. The counter e.m.f. is 30 volts and the current in the armature is 5 amperes. What is the resistance of the armature?

9. A motor armature has a resistance of 2 ohms. An impressed voltage of 35 volts gives a current of 5 amperes in the armature. Find the counter e.m.f. of the motor.

410. Recording Wattmeter.—The Thomson form of recording **wattmeter** (Fig. 315) consists of a little shunt motor whose armature turns at a speed which is proportional to the energy which is supplied to it. The armature is geared to dials which record the energy in kilowatt-hours. The stationary field coils L and M are connected in series with the generator supplying the current to the external circuit. The field strength of the motor is therefore proportional to the current supplied to the lamps. The armature is connected across the line, like a voltmeter would be connected. The voltage across the armature is thus the voltage supplied to the lamps and the current in the armature is proportional to the voltage. The torque which turns the motor is proportional to the product of the current in the armature and the magnetic field in which the armature turns. Hence the torque which turns the armature is proportional to the product of current supplied to the lamps and the voltage at which it is supplied, that is to the watts in the line.

In order that the motor may not run too fast and in order also that it may stop as soon as the current ceases to flow an electromagnetic brake D is attached to it. This brake consists of an aluminum or copper disc rotating between the poles of permanent magnets. The eddy currents induced in the disc by its rotation between the poles of the magnet retard its motion and cause it to stop as soon as the current ceases to flow.

This type of wattmeter can be used with either direct or alternating currents.

CHAPTER XXXII

APPLICATIONS OF DISCHARGE OF ELECTRONS FROM HOT WIRES

411. Discharge from Hot Wires.—If a platinum wire *DE* (Fig. 316) is placed inside of a metal cylinder *C* which is inserted in a very perfectly evacuated vessel, it is found that when the wire is heated to incandescence there is an evaporation of electrons from its surface. If now the cylinder *C* is connected to the earth through a galvanometer *G*, and a battery *B*, there will be a flow of electrons from the wire to the cylinder, through the galvanometer and then to the earth. When the wire is at room temperature, this flow of electrons does not take place. As the temperature of the wire is increased, the number of electrons escaping from the surface of the wire increases rapidly and the current through the galvanometer *G* also increases. The relation between the current through the galvanometer and the temperature is shown in Fig. 317. Since the current in the galvanometer is proportional to the number of electrons evaporating each second, this curve also shows how the rate of evaporation of electrons increases with the temperature.

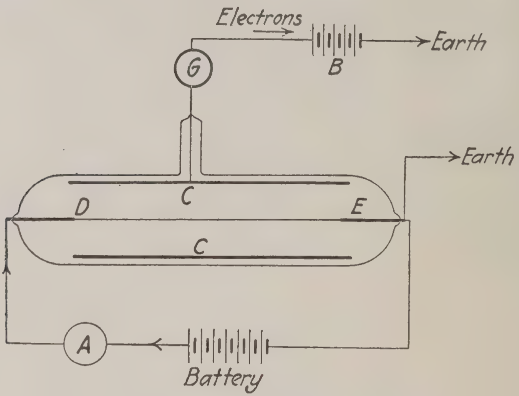


FIG. 316.—Discharge from hot wire.

This escape of electrons from the surface of a hot wire is very analogous to the evaporation of the molecules from the surface of a liquid. The curve in Fig. 317 showing how the number of electrons increases with rise of temperature is similar to the vapor pressure curve of a liquid (Fig. 163). There is, however, this important difference. In the evaporation of a liquid the molecules are uncharged but in the discharge of electricity from hot wires each escaping electron carries the elementary charge of negative electricity. This discharge of electricity from hot wires has found many very important practical applications.

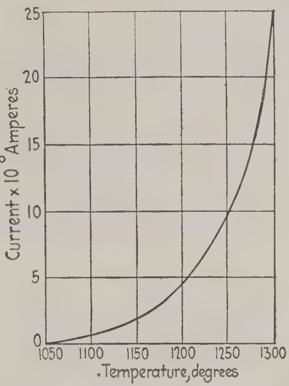


FIG. 317.—Increase of discharge of electrons with the temperature.

412. Coolidge X-ray Tube.—When electrons moving with very great velocity strike a target *T* (Fig. 318) and are stopped they set up a series of very short electric and magnetic waves. These waves travel in the **ether** with the velocity of light and do not differ in their nature from light except in the fact that their wave lengths are extremely short in comparison with the wave length of light. These waves which are known as Roentgen rays or X-rays can pass through many substances which are opaque to light and produce many important physical, chemical and physiological effects. They have thus come to occupy an important place in physics, chemistry, medicine and engineering.

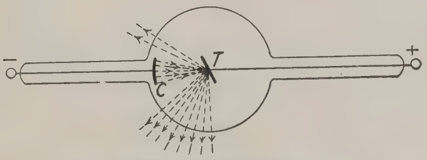


FIG. 318.—X-ray tube.

To produce X-rays it is necessary to have a source of electrons and a means of imparting a high velocity to them so that when they are suddenly stopped by a target electromagnetic waves of short wave length are produced. The most successful way to generate X-rays is by means of a Coolidge X-ray tube. It consists of a highly evacuated tube (Fig. 319) in one end of which is a small tungsten filament which is heated to incandescence by means of an electric current. This filament behaves essentially like a miniature electric lamp. A large number of electrons are emitted from the filament and by the application of a high potential, 100,000 to 200,000 volts, between the filament and the cold target at the other end of the tube, a high velocity is imparted to these electrons. When they strike the tungsten target very penetrating X-rays are generated. The intensity of these rays is greater than that obtained from any other tube and the penetrating power of these rays is also very great. The tube is capable of very excellent control so that it produces rays which are well suited for most purposes.

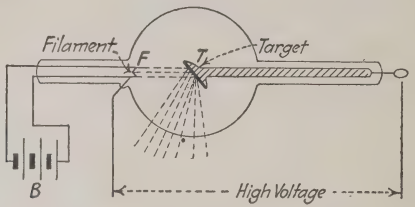


FIG. 319.—Coolidge X-ray tube.

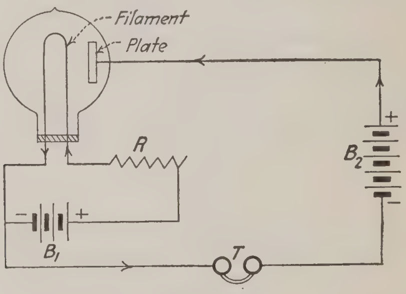


FIG. 320.—Thermionic valve.

413. The Thermionic Valve.—The thermionic valve (Fig. 320) consists of a bulb from which air or other gases have been completely exhausted. On the inside of the bulb is a tungsten filament much like the tungsten filament in a small electric light bulb. This filament is heated to incandescence by means of an external battery B_1 as an electric light filament is heated. As the filament is heated, electrons escape from it and fly out into the surrounding space. In the bulb there is mounted a plate which is connected to a wire passing through the side of the bulb. In some cases this plate takes the form of a cylinder surrounding the filament. There is no electrical connection between the plate and the filament inside of the bulb. If the filament is now connected to the negative terminal of a battery and the plate to the positive terminal, the electrons will be repelled from the filament and attracted to the plate. There will thus be a current of electrons from the filament to the plate through the circuit containing the battery B_2. If now the terminals of the battery B_2 are reversed so that the negative terminal is connected to the plate and the positive terminal to the cell, the electrons will remain in the filament and no current will pass from the plate to the filament. The bulb has, therefore, the power to let a current pass in one direction through it but prevents the current from flowing in the opposite direction. The bulb when placed in an electric circuit, behaves like an electric valve, permitting the current to flow only in one direction. The electronic current can pass only from the filament to the plate. Thus when the coil L_2 induces alternating e.m.fs. in the coil L_1 (Fig. 321), the current in the coil L_1 flows in only one direction. This is equivalent to a positive current on the basis of the ordinary conventions with respect to the direction of the current, flowing from the plate to the filament. The power of such a bulb to rectify a current has important applications.

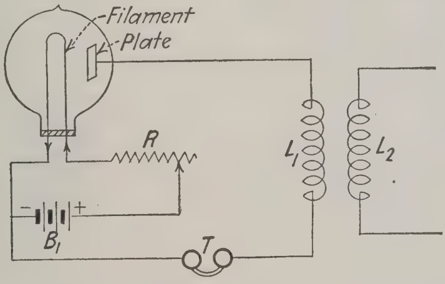


FIG. 321.—Retification of signals by a thermionic valve.

414. Tungar Rectifier.—In the Tungar rectifier (Fig. 322) the generator G supplying the voltage to be rectified is connected to the terminals of the primary P of a transformer. The secondary terminals S of the transformer are connected to the filament of the vacuum tube. The voltage is transformed until it is sufficiently low to be suitable for heating the filament. If it is desired to charge a battery from the alternating current generator, the positive terminal of the battery is connected to the primary of the transformer and the negative terminal through a regulating resistance R to the anode of the vacuum tube. The other terminal of the primary of the transformer is connected to the filament at A. The electrons from the hot filament are able to pass only from the filament to the anode. The positive current can only flow from the anode to the filament in the bulb. Hence the current in the circuit containing the battery flows through the battery in the direction indicated by the arrows.

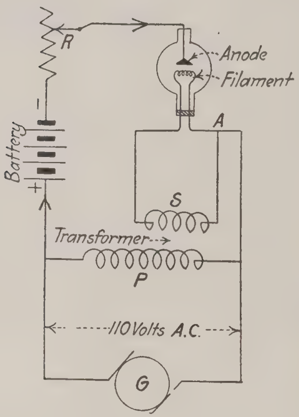


FIG. 322.—Diagram of tungar rectifier.

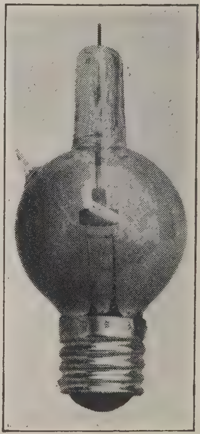
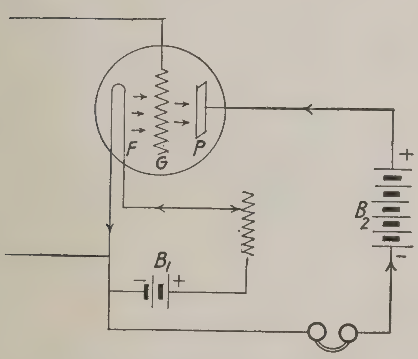


FIG. 322a.—Tube of tungar rectifier.

This direction is opposite to the direction in which the current would flow if the battery were delivering current. Because of the rectifying power of the tube the current only flows in one direction through the battery although the generator G supplies an alternating voltage. By means of such a rectifier it is possible to charge a storage battery from an alternating generator. In other words it is possible to obtain direct current from an alternating generator. Rectifiers of this kind are now made sufficiently large to carry 12 amperes at an electrical pressure of 75 volts.

415. The Audion.—The audion, now extensively used in receiving and sending wireless messages, differs from the thermionic valve (Fig. 323) in that a wire gauze or grid *G* is inserted between the plate and the filament. A wire passing through the side of the bulb is connected to this wire gauze or grid. But the grid is not in any way connected electrically to either the plate or the filament inside of the tube. The electrons emitted by the filament will charge the gauze or grid negatively. The passage of electrons from the filament to the grid will then nearly cease because of the negative charge on the grid. The space around the filament is then nearly a non-conductor for positive currents going from the grid to the filament. This effect is increased if the grid is given a negative potential with respect to the filament. The battery B_2 is then unable to send current from the plate to the grid.



ig. 323.—The action of the audion.

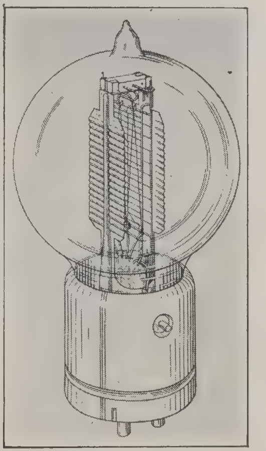


Fig. 324.—The audion (Western Electric Co.)

If, on the other hand, the potential of the grid be made positive with respect to the filament, electrons pass freely toward the grid and reach the plate. Hence the battery B_2 can now produce considerable current. A small negative potential of grid over filament nearly stops the current from the battery B_2 and a small positive potential of grid over filament allows a large current to be supplied by the battery B_2. Thus small changes in potential when applied to the grid are able to produce large changes in the current supplied by the battery. By this means an amplification of currents too feeble to be otherwise detected is produced.

Such an amplifier is used in long distance telephones. A telephone current usually varies in accordance with the speech of the user. It is an alternating current of varying frequency and wave form. When the current is made to impress a similar varying voltage on the grid of an amplifying tube the plate current varies in just the way the voltage impressed on the grid varies, except that the variation of the plate current is much greater than the variation of the grid current. A weak current may thus be amplified indefinitely and the distance to which telephone messages can be heard much increased.

416. Wireless Telegraphy.—When an electric field is changing it produces a magnetic field at right angles to it and when a magnetic field is changing there is produced an electric field. Thus a varying electric force has associated with it a magnetic force and a varying magnetic force has associated with it an electric force. These forces are always at right angles to each other. If now the potential difference between two neighboring conductors *A* and *B* (Fig. 325) becomes sufficiently large that a discharge occurs, there is set up between the conductors a series of electric oscillations which are accompanied by magnetic oscillations. At first *A* is positive and *B* negative. Then both are neutral and a little later *B* is positive and *A* negative. A series of electromagnetic pulses are thus sent out and these pulses travel through empty space with the velocity of light. These pulses have a frequency and a wave length which are determined by the characteristics of the discharging conductors. These waves are transmitted with the velocity of light and have the same characteristics as light waves except they have much longer wave lengths.

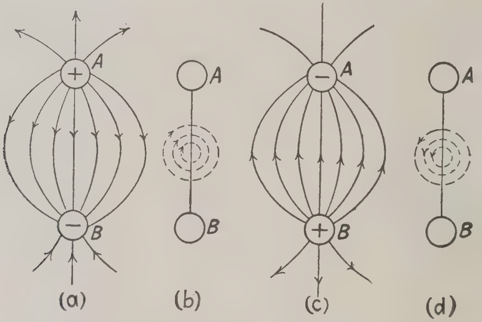


FIG. 325.—Electric oscillations.

When these electromagnetic disturbances or waves encounter an electric circuit, there is set up in the electric circuit an induced electromotive force. This electromotive force is oscillatory and if the electric circuit contains a narrow air gap each side of the air gap will rapidly alternate between high and low potential. When the potential difference is sufficiently great, there will be a discharge across the gap. The frequency of the currents produced in the circuit is so high that they can not be heard in a telephone receiver because the ear can not detect sounds having a frequency of more than about 15,000 per sec. The oscillations are also so rapid that the diaphragm of the telephone can not follow them. Before these currents can be detected some means of rectifying them must be provided. This is done by means of a crystal **rectifier or an audion.**

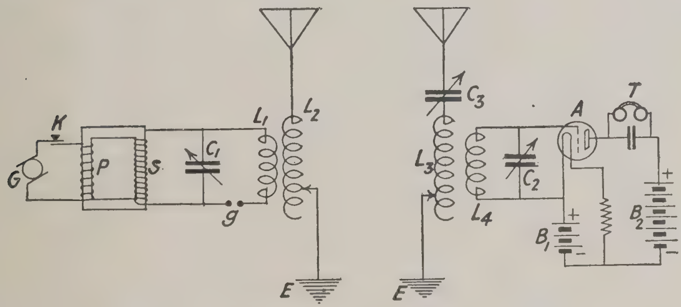


FIG. 326.—Diagram of system of wireless telegraphy.

In wireless telegraphy there are then two important parts (1) a means of sending electromagnetic waves (2) a means of receiving such waves. In Fig. 326 a common method of sending and receiving electric waves is represented diagrammatically. The sending system consists of a high frequency generator G which is connected through a key K to the primary P of a transformer. The secondary S of the transformer is connected to the variable capacity C_1. The terminals of this capacity C_1 are also connected to the primary L_1 of an air core transformer. One terminal of the secondary L_2 of this transformer is connected to the earth and the other to a number of parallel wires which form the **antenna.**

When the key K is closed an alternating electromotive force is produced in the secondary S of the transformer. When by this means the condenser C_1 is charged up to a sufficiently high potential to cause a spark to jump across the gap g, there is set up a series of electric oscillations in the coil L_1. These electric oscillations set up in turn electric oscillations in the secondary L_2 of the air core transformer and in the antenna to which it is attached. The electromagnetic waves thus produced are radiated out into space from the antenna. Whatever their wave length they travel with the velocity of light, 186,000 miles per second.

The equipment at the receiving stations consists of another antenna connected through a variable condenser C_3 to one end of the primary L_3 of an air core transformer. The other terminal of this primary is connected to the ground. The terminals of the secondary L_4 of this air core transformer are connected to the terminals of the variable capacity C_2. One of the terminals of this capacity is then connected to the grid of the audion A and the other terminal to the filament. A battery B_1, supplies the necessary energy to heat the filament. A telephone T is inserted in series with the battery B_2 which is connected from the plate of the audion to the filament.

The electromagnetic waves radiating out from the antenna of the sending station reach the antenna of the receiving station and produce induced currents in the secondary L_3 of the air core transformer. By means of the condenser C_3, the frequency of the receiving circuit, consisting of the antenna and the coil L_3, can be made to have the same frequency as the corresponding part of the sending circuit. When the circuits are thus tuned, the receiving circuit responds much more forcibly to the waves sent out from the sending antenna. These induced currents in the coil L_3 produce electric oscillations of high frequency in the coil L_4 and thus set up variations of potential between the grid and the filament of the audion. Because of the amplifying and rectifying action of the audion, these variations of electric potential between the grid and the filament set up amplified currents in the circuit containing the telephone. In this way the electric waves sent out from the sending station are reproduced at the receiving station and after amplification produce audible sounds in the telephone.

417. Wireless Telephony.—When undamped waves are sent out continuously by the transmitting circuit, there is in the receiving circuit after rectification an unvarying direct current. This current simply deflects the diaphragm of the telephone receiver without setting it into vibration. If now the energy radiated by the transmitting antenna is varied, a corresponding variation takes place in the undirectional current in the receiving circuit and in the telephone. If these variations of current in the transmitting antenna are of audible frequency, a sound having the same frequency and characteristics will be produced in the telephone receivers of the receiving circuit. Thus under suitable conditions sounds or speech which have produced variations in the current of the transmitting circuit can be reproduced in the receiving circuit.

A radio telephone circuit must then consist essentially of a radiating circuit in which oscillations are set up by some generator of undamped vibrations and a receiving circuit which does not differ essentially from the receiving circuit used in the wireless telegraphy. There must be in connection with the transmitting circuit an arrangement of circuits by which the amplitude of the oscillations in the transmitting circuit can be varied or modulated by the vibrations of the human voice. There are a number of methods of producing these modulations of the vibrations in the transmitting circuit. The most successful of them is by means of an audion tube used as an oscillator.

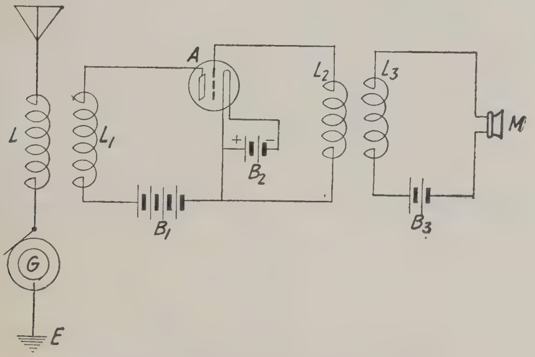


FIG. 327.—Wireless telephone.

One of these methods is shown diagrammatically in Fig. 327. Continuous undamped oscillations are set up in the antenna, by the generator G or by means of a vacuum tube used as an oscillator. Coupled electromagnetically to the antenna circuit is the plate circuit of the modulator tube A, the coupling being made through the coils L and L_1. The grid circuit of the vacuum tube is connected to a telephone transmitter circuit. When no speech is impressed on the telephone transmitter M, the grid potential of the modulator A has an unvarying value. The current in the plate circuit is the resultant of the current due to the battery B_1 and the high frequency alternating electromotive force of constant maximum value induced in the coil L_1 by the antenna current supplied by the high frequency generator. The plate current is then a pulsating current in which the direct current component and the alternating current component each has an unvarying amplitude.

When now an operator talks into the microphone M, variations in the resistance of the carbon contacts thus produced, causes changes in the current from the battery B_3 which in turn produce variations in the grid potential of the modulator A. These variations in the grid potential will cause amplified changes in the plate current which is flowing in the coil L_1. There will thus be produced by induction a very great modulation of the currents in the antenna. These fluctuations will be radiated from the antenna of the transmitting circuit and received by the antenna of the receiving circuit where they will reproduce in the receiver of the telephone the sounds which were originally imparted to the transmitter M.

PART IV.—WAVE MOTION, SOUND AND LIGHT

CHAPTER XXXIII

WAVE MOTION AND SOUND

418. Wave Motion.—When a stone is dropped into a pool of still water, the surface of the water is covered with circular wavelets which widen out from the central point where the stone fell. The water does not really move outward from the central point but it rises and then falls again. That such is the case is seen by observing a floating leaf or piece of wood. It does not move forward but returns again and again to its former position. Hence the water (Fig. 328) on which the leaf rests must have this same kind of upward and downward motion rather than a forward motion.

The motion of the heads of wheat in a field on which the wind is blowing gives the impression that there is a forward motion.

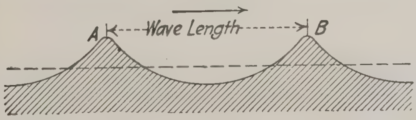


Fig. 328.—Water waves.

There is, however, only an upward and downward motion of each head of wheat. The steady onward motion of the waves in the wheat is not a forward motion of the wheat. It is a forward motion of a state of things, a shape, or a wave form.

In like manner when one end of a rope is fastened to a rigid wall and the free end moved up and down rapidly, each jerk travels along the rope, each portion of the rope communicating the jerk to the next portion. Each particle of the rope imparts its upward or downward motion to its neighbors. The motion moves forward but the particles of the rope only move up and down. Motions of this kind are **wave motions.**

419. Medium Necessary for Waves.—In all cases of wave motions it is necessary to have a medium in which the waves can travel. This is illustrated by considering an audience in which those in the front row begin to cheer. If each row in turn takes up the cheering, a wave of cheering passes over the room. The presence of the people was necessary for the passage of the wave of cheering across the room. In the illustrations of the preceding section we had the water, the rope or the field of wheat acting as the medium in which the disturbance travels. Sound which is an example of a wave motion is handed on from place to place by means of some medium like air, water or wood. Where no medium is provided the sound can no longer move forward. Light which is also a wave motion requires some kind of a medium in which to travel. In this case the medium need not be a material substance since light travels through empty space. That medium through which light travels has been called the **ether.** It is a hypothetical substance whose existence has not been definitely proved.

420. Characteristics of Wave Motion.—Wave motion is characterized by the handing on from place to place in an elastic medium of disturbances emanating from some source. These disturbances follow each other at definite intervals and the wave motion thus maintained transfers energy from one point to another. The individual particles in the medium move to and fro about their normal positions of rest and the wave form only moves forward. A medium is necessary for the transmission of the disturbance and a definite time is required for the disturbance to travel from one place to another.

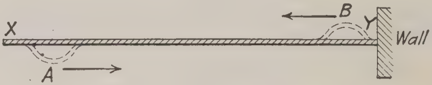


FIG. 329.—Waves in a string.

421. Transverse Vibrations.—If one end of a rope is fastened to the wall and the other end is displaced it carries with it the portions of the rope next to it. Each portion affects the neighboring portion and after a while the whole rope is affected by the jerking of one end of it. This does not occur all at once for it requires time for the disturbance to travel down the rope. If the rope is uniform the action will be handed on at a uniform rate. When the hump *A* (Fig. 329) reaches the fixed end of the rope it is reflected as the hump *B*. In this case the wave is transmitted along the rope while each particle vibrates in a direction at right angles to the rope. This type of motion is called a **transverse wave** because the direction of motion of the particles is perpendicular to the direction in which the wave is traveling.

422. Longitudinal Vibrations.—If a number of steel balls (Fig. 330) are suspended so that they are in contact and if now *K* is drawn aside and allowed to fall against *J*, the shock is transmitted through the other balls and the ball *A* on the further end of the row moves away from *B* into the position *A′*. When the ball *K* strikes the ball *J*, the ball *J* is momentarily compressed and this compression is handed on to the other balls. In this way a wave of compression travels along the balls and forces *A* away from its neighbor *B*. This wave of compression is not instantaneous. Each ball takes a little time to be compressed and then to recover its former shape. The velocity with which the compression is transferred is, however, great.

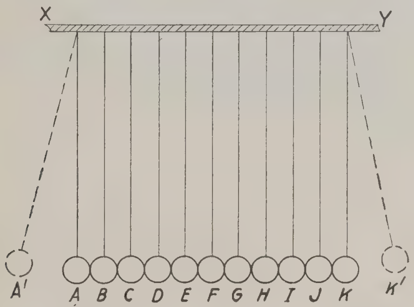


Fig. 330.—Elastic waves.

Another kind of longitudinal wave is found in a coiled spring. In this case the wire spring is the medium through which the wave passes. If the first few turns of the spring are compressed and then released the turns will move forward a little and compress those ahead. These will compress the next in turn and the compression will be handed on from turn to turn. When a displaced turn of the spring is released it moves through its position of equilibrium but on account of its inertia it does not stop in this position but moves forward still further causing the turns of the spring to separate more than the normal amount. This separation moves along the spring following the compression. Since the turns of the wire move back and forth in the direction which the wave is traveling waves of this kind are called **longitudinal waves.**

423. Amplitude and Frequency.—A vibrating body such as the prong of a tuning fork passes back and forth through its position of equilibrium. The extreme displacement on either side of the position of equilibrium is called the **amplitude**. Except for the fact that the vibration dies away slowly this displacement is the same on the two sides of the position of equilibrium.

The number of complete vibrations made by the body in one second is called the **frequency of vibration.**

424. Relation between Wave-length, Frequency and Velocity.—The distance between successive crests or between successive troughs in a water wave is called the **wave length**. It is in general the distance between successive points in like phase of vibration. Let n be the number of vibrations made by the body in one second; v the distance traveled by the wave in one second, and λ the length of each wave. Then

$$\lambda = \frac{v}{n}$$

Velocity of the wave = number of waves × wave-length

$$v = n\lambda$$

425. Nature of Sound.—The source of sound is always a state of vibration. As the vibration dies down the intensity of the sound diminishes. If a ringing bell is touched with the fingers the sound ceases because the vibrations are stopped by the fingers. If the bell is held firmly in the hands vibrations can not be set up by striking the bell with the hammer and no sound is produced. When a weight falls to the floor the weight as well as that part of the floor which is struck is set in vibration and sound waves are produced. If a stretched guitar string is plucked, it gives a musical note owing to the vibrations set up in it. These vibrations take place too fast for the eye to follow them and the string seems to be drawn out into a ribbon in the middle. In a sounding tuning fork the prongs alternately approach and recede from each other. These movements of the prongs (Fig. 331) can be felt by touching the prongs with the fingers. They produce compression and rarefactions in the surrounding air. These compressions travel forward and are the sound waves.

426. Velocity of Sound.—It is a common observation that sound takes an appreciable time to travel. Thunder is not heard until some time after the lightning is seen. When a distant but visible gun is fired, the flash reaches the eye before the report is heard. The escaping steam from the whistle of a locomotive can be seen before the sound of the whistle is heard. All of these observations show that a sound requires an appreciable time to travel from one place to another. On the other hand light travels almost instantaneously from one place to another.

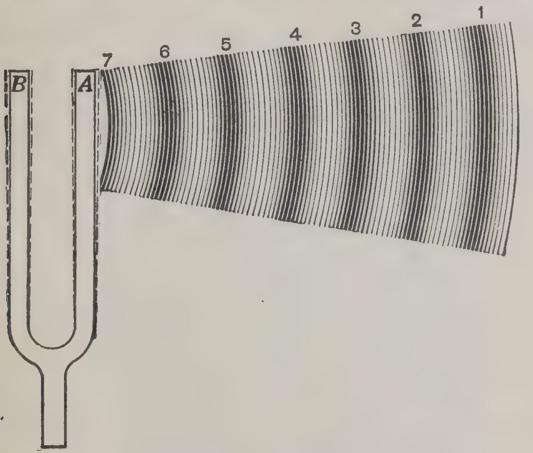


FIG. 331.—Sound waves from a tuning fork.

The velocity of sound in different media is not the same. It depends on the density and elasticity of the medium. The greater the elasticity and the less the density, the more rapidly the sound will travel. The law for velocity is,

$$v = \sqrt{\frac{e}{\rho}}$$

where $e =$ the coefficient of elasticity
$\rho =$ the density of the medium.

Where the coefficient of elasticity divided by the density is large, the velocity is great. For this reason the velocity of sound in steel is greater than in air. Although the density of the steel is greater than that of the air, its elasticity is so much greater than that of the air that the velocity in steel much exceeds that in air.

427. Intensity of Sound.—The intensity of sound diminishes rapidly as the distance from the sounding body is increased.

If a bell is ringing at S (Fig. 332) so that the sound can travel in all directions without interference, the intensity of the sound decreases inversely proportional to the square of the distance from the bell.

Let $I_1 =$ the energy per c.c. in a spherical shell of radius R_1.
$I_2 =$ the energy per c.c. in a spherical shell of radius R_2.

The energy in a spherical shell of radius R_1 and of unit thickness is

$$= \text{volume of shell} \times \text{energy per c.c.} = 4\pi R_1^2 I_1$$

The energy in a shell of radius R_2 and of unit thickness is

$$= \text{volume of shell} \times \text{energy per c.c.} = 4\pi R_2^2 I_2$$

If the energy sent out by the sounding body is constant, the amount of energy in shells of equal thickness is constant,

$$\therefore 4\pi R_1^2 \times I_1 = 4\pi R_2^2 \times I_2$$

$$\frac{I_1}{I_2} = \frac{R_2^2}{R^2}$$

$$\frac{I_1}{I_2} = \frac{\text{intensity of sound at distance } R_1}{\text{intensity of sound at distance } R_2} = \frac{R_2^2}{R_1^2}$$

Hence the intensity of the sound varies inversely as the square of the distance from the source of sound.

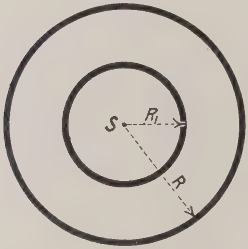


FIG. 332.—Intensity of sound waves.

When the sound can not spread out freely in all directions, the intensity will not vary inversely as the square of the distance. If the sound travels down a speaking tube, the energy in the sound wave does not spread out over a larger area as the sound moves along but it remains concentrated in the tube. In cases of this sort the intensity of the sound does not decrease with the distance very much but remains nearly constant. It is for this reason speaking tubes are used. They direct the sound and prevent it from spreading out and decreasing in intensity.

428. Reflection of Sound.—If an observer stands at some distance in front of a cliff or large building and produces a sound, the sound is returned to him. The sound travels to the wall or cliff and then back without having its characteristics changed. If the observer were about 1,100 ft. from the cliff, it would take the sound about 2 seconds to return to him. The rolling of thunder is due to the reflection of the original sound by clouds at different distances from the observer. These reflections reach the observer at different times and produce the rolling or continuation of the sound. Sounding boards are often placed behind pulpits in order to reflect the voice of the speaker toward the audience. This prevents the sound from traveling upward and backward and thus being lost. In large halls and churches these reflections often make it difficult to hear the words of the speaker. In such cases precautions must be taken to reduce the reflections and the resultant echoes to a minimum. Whispering galleries are so constructed that they reflect the sound to a particular point without much loss in intensity.

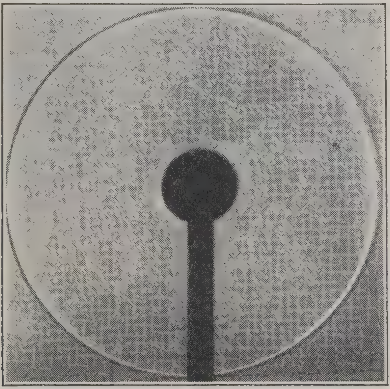


FIG. 333.—Spherical sound wave.

Fig. 333 shows the way in which an undisturbed spherical sound wave spreads out from its source. In Fig. 334 a spherical wave has been reflected by a plane surface. When the source of the sound is at the focus of a parabolic mirror (Fig. 335) the reflected wave front is a plane. Fig. 336 shows the simultaneous reflection and transmission of sound waves. The transmitted system of waves has come through four rectangular apertures and the reflected system has come from three rectangular bars and the surface above and below them. These sound photographs were made by Professor Foley of the University of Indiana.

429. Pitch and Frequency.—If a sound recurs at regular intervals, it is heard as a series of separate impulses. If the

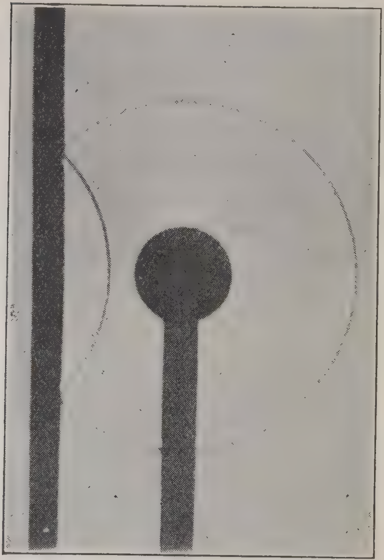


FIG. 334.—Reflection of spherical sound wave by a plane surface.

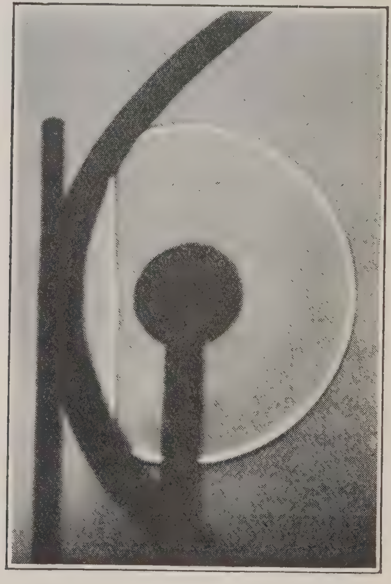


FIG. 335.—Reflection of spherical sound wave by parabolic mirror.

number of these impulses per second is increased, the separate impressions disappear and a continuous note is heard when the number of impulses is about 30 per second. As the frequency of the impulses are increased beyond this number the pitch of the sound is increased. That there is a quantitative relation between the pitch and the frequency can be shown by aid of the siren (Fig. 337) which consists of a disc having in it a number of holes distributed on concentric circles. If a draught of air from a tube A is directed against the holes while the disc is in rotation, a puff of air passes through each hole and comes out on the other side. When the number of these puffs is increased either by increasing the speed of the disc or by directing the stream of air

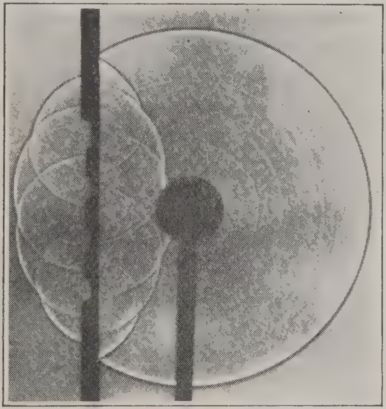


Fig. 336.—Reflection and transmission of spherical sound waves by rectangular apertures.

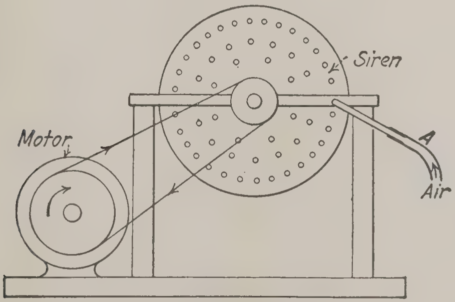


Fig. 337.—The siren.

against the outer holes, the frequency of the sound is increased and the pitch becomes higher. If the number of holes in one ring is twice as great as in another, the pitch of the sound is made twice as high by moving the stream of air from the ring where the number of holes is less to the ring where it is larger. The pitch of a sound is determined completely by the frequency. **The greater the frequency, the higher the pitch.**

430. Loudness.—The loudness of a sound depends on the energy of the vibrating source and the energy in turn depends on the amplitude of the vibrating body. The distance through which the vibrating body moves determines the displacement of the medium through which the sound travels and the amount of this displacement determines the energy of the vibrating body and the medium surrounding it. The vibrating body is continually under the action of a force which is directed toward the center from which the body was displaced. The greater the displacement the greater this force and the greater the work to produce the displacement. Since the force is proportional to the displacement and the work to produce the displacement is equal to the force times the displacement, the work to produce the displacement is proportional to the square of the displacement, that is, the energy of the vibration is proportional to the square of the amplitude. This is true for the sound waves as well as for the vibrating body. Consequently the loudness of a sound is **proportional to the square of the amplitude of the vibration of the medium in the waves.**

431. Limits of Audibility.—If the frequency of a sound rises very much, the sound becomes very shrill. When the frequency exceeds about 15,000 per second, the sound ceases to be heard. The ear of man can not respond to frequencies above this number. Some people have a higher limit of audibility than others and are able to hear sounds which have a frequency of 20,000 or 25,000 vibrations per second. Young children have a higher audibility than older people. Animals and especially insects are known to be able to hear sounds not audible to man.

432. Quality.—Sounds which have the same loudness and pitch may differ in the complexity of the vibrations and may therefore produce a very different effect on the ear. It very rarely happens that a sounding body produces a simple harmonic vibration. A tuning fork on a resonance box produces such a simple tone but in most cases the tone is not pure. Besides the fundamental vibration which determines the pitch of the note there is present one or more superposed notes which are called **overtones.** They are very simply related to the frequency of the fundamental, the frequency of the overtones being 2, 3, 4, etc., times that of the fundamental. The number of overtones present and the complexity of the sound will depend on the way in which the body is set into vibration. Thus the number of overtones in a stretched string is not the same when it is bowed in the middle as it is when it is bowed near one end. In an open organ pipe all the overtones may be present but in the closed pipe only the even numbered overtones can occur. Hence the character of the sound in an open pipe and in a closed pipe of equal frequency may not be the same. The ear resolves unconsciously these complex sounds and thus distinguishes differences which are not due either to pitch or loudness. These differences due to the complexity of the sound are differences in **quality.**

CHAPTER XXXIV

SOUNDING BODIES

433. Tuning Forks.—When the free end of a rod is pulled aside and released, a wave travels to the fixed end where it is reflected and travels back again to the free end. There is thus set up in the rod a set of stationary waves. The modes of vibration of such a rod are indicated in Fig. 338.

A tuning fork is essentially a rod fixed at its middle point. When the prongs vibrate, the stem moves up and down with the same period. There are two nodal points, one on either side of the middle point. The closer the prongs are together, the more nearly do these nodal points coincide with the central point of the fork. By giving a tuning fork the form indicated in Fig. 339 and making the prongs close together, the secondary or harmonic vibrations present in vibrating rods are nearly excluded and the tuning fork gives a pure musical tone.

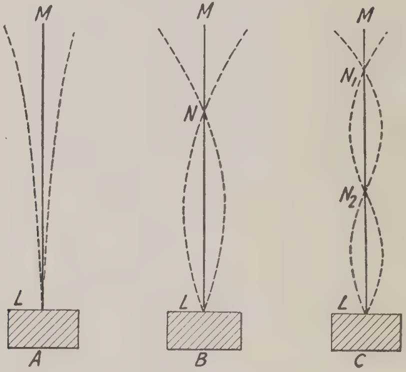


FIG. 338.—Vibrations in a rod.

The prongs set so little air into vibration that the loudness of the sound due to their vibrations is not sufficiently great. By resting the stem on a board or on the top of a table, the air in contact with the board is set in vibration and the loudness of the sound very much increased. For this reason tuning forks are ordinarily mounted on the top of a box which is open at one end. If the natural period of vibration of the box is made equal to the period of vibration of the fork, the box and the fork will vibrate in unison and the box will become a resonator and will greatly increase the intensity of the sound. Since the resonator reinforces only the fundamental of the fork, the sound emitted by the fork on the resonator box will be free from overtones.

434. Vibrations of Wires and Strings.—When the center of a stretched string is displaced and then released the disturbance is handed on from one element of the string to the next and a transverse wave is produced. These waves are reflected at the fixed ends of the string, return to the center of the string where they pass each other and go on to the ends where they are again reflected. These waves combine to form standing waves in the string. When the string is plucked in the middle (Fig. 340) the whole string vibrates as one segment and the length of the string is equal to one-half of the wave length of the sound emitted by the string. When the string is held fast at the middle point and plucked at a point midway between the middle and one end, the string vibrates in two segments and the length of the string is equal to the wave length of the sound. In a similar way the string may be made to vibrate in three, four, etc., segments. The

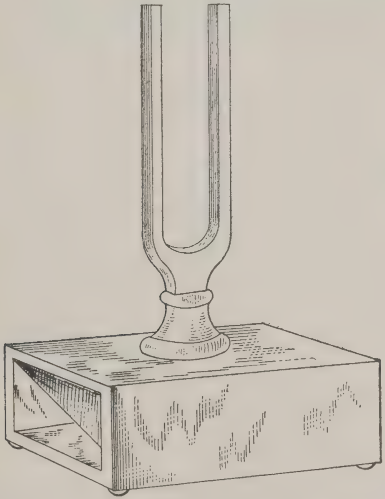


Fig. 339.—A tuning fork mounted on a resonator.

frequency of the sound is thus increased. The string may be vibrating as a whole and at the same time be vibrating in segments. The note from the string then consists of the fundamental together with one or more overtones. In this case the stationary wave in the string is the resultant of the fundamental vibration and all the overtones.

Let L = the length of the string, n = the number of vibrations per second, v = the velocity of the disturbance in the string and λ = the wave length of the disturbance. Then,

$$v = n\lambda$$

In the case of the fundamental the disturbance travels twice the length of the string for each vibration, therefore $\lambda = 2L$ and

$$v = 2nL$$

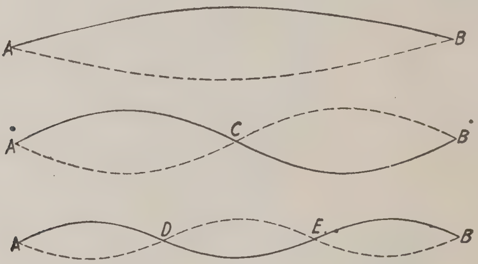


FIG. 340.—Vibrations in strings.

It can be proved by experiment that the velocity of a disturbance in a string is equal to the square root of the tension in the string divided by the mass of the string per unit length.

$$v = \sqrt{T/M}$$

where T = the tension in dynes and M = the mass in grams and v = the velocity in cm. per sec. From these two expressions it follows that

$$n = \frac{1}{2L}\sqrt{T/M}$$

By means of this relation the pitch or frequency of a sound emitted by a string can be calculated. An illustration of this law is found in all stringed musical instruments like the violin or guitar. The strings are of the same length and the pitches are determined by the masses of the strings per unit length and the tensions to which they are subjected. The strings of lowest pitch have large masses per unit of length and are under small tensions. The strings of high pitch are light in weight and are under greater tensions.

These laws may be verified by a sonomoter (Fig. 341). A weight W is fastened to one end of a string passing over a pulley P. By adding different weights the tension in the string may be varied and by moving the supports A and B the length of the part of the string in vibration can be changed. Finally strings of different mass per unit of length can be used. In this way the law for the frequency of vibration of strings can be verified.

435. Vibration of Closed Pipes.—In an organ pipe in addition to the box containing the column of air there is a device at one end by which the column of air can be set in vibration. A compression thus set up travels to the end of the column where in the case of the closed pipe (Fig. 342) it is reflected as a compression and returns to the open end of the pipe. The column of air is thus set in vibration and imparts its motions to the surrounding air. The frequency of the wave emitted is determined by the lengths of the column.

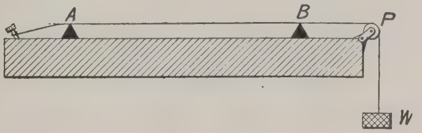


FIG. 341.—Sonometer.

When a steady state of vibration has been reached, the length of the pipe is equal to one-quarter of a wave length for the lowest pitch of the pipe. The closed end of the pipe is a node where the movement of the air is zero and the open end is an antinode where the movement of the air is a maximum.

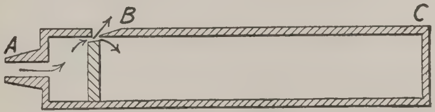


FIG. 342.—Closed organ pipe.

There are other possible modes of vibration of this column of air which will make the closed end of the pipe a node and the open end an antinode. Whenever the column of air has such a frequency that the closed end is a node and the open end an antinode, it is possible for it to vibrate. Some of the different modes of vibration in which this is possible are shown in Fig. 343. In case A the pipe is emitting its fundamental. For this case the frequency is

$$n = \frac{v}{4l}$$

In case B the second overtone is present. For it the frequency is,

$$n_1 = \frac{3v}{4l}$$

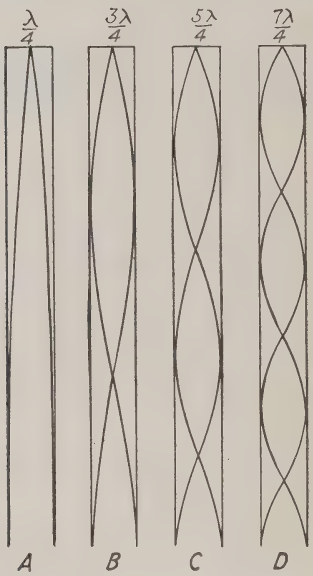


Fig. 343.—Vibrations in closed pipes.

In case C the fourth overtone is present, its frequency is,

$$n_2 = \frac{5v}{4l}$$

The odd numbered overtones can not be emitted by a closed organ pipe.

It therefore follows that the possible frequencies of vibration in a closed pipe are in the ratio of the odd numbers

1, 3, 5, 7, etc.

436. Vibrations of Open Pipes.—In an open pipe (Fig. 344) the displacement of the air must be a maximum at the open ends so that the open end must be an antinode or a place of maximum vibration. In such a pipe the simplest possible mode of vibration is the case in which there is a node at the middle and an antinode at each end (Fig. 345). It follows that the length of the pipe, which in this case is the distance between two antinodes, is equal to one-half the wave length of the emitted sound. Then since $v = n\lambda$ the frequency of the fundamental is

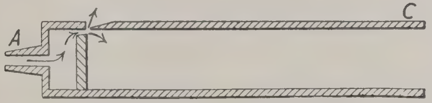


Fig. 344.—Open organ pipe.

$$n = \frac{v}{2l}.$$

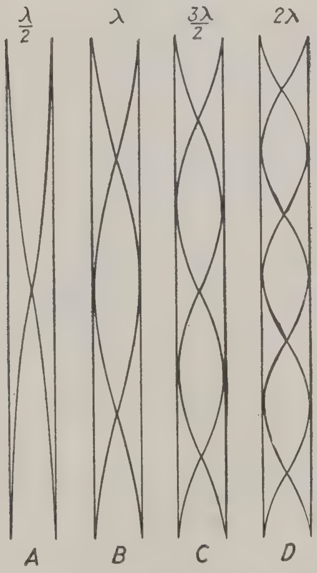


Fig. 345.—Vibrations in open pipes.

The next possible mode of vibration is the case where the pipe is one wave length and there are two nodes in it. This is the first overtone and for it

$$n_1 = \frac{v}{l}$$

It has twice the frequency of the fundamental. In the second overtone

$$n_2 = \frac{3v}{2l}$$

and this overtone has three times the frequency of the fundamental. The frequencies of the overtones in the open pipe are therefore in the ratio of the natural numbers 1, 2, 3, 4, etc. In the open pipe it is possible to have all the overtones but in the closed pipe only the even numbered overtones are possible.

437. Beats.—Let two organ pipes which have exactly the same frequency be sounded together. At any given place the state of the disturbance emitted by the pipes remains constant. If the upper end of one of the pipes is now partially closed by sliding a brass plate over it, the pitch of this pipe is slightly lowered. When the two pipes are now sounded together there is no longer unison but bursts of sound with comparative silence between them are now noticed. These bursts of sound where the disturbance from one pipe reinforces the disturbance from the other pipe are called **beats.** The reason for these beats is that at a certain instant the compressions from both sounds arrive at a given point simultaneously. A short time later (Fig. 346) the more rapidly vibrating body is one-half vibration ahead of the other and the compressions from one sound arrive at the same time as the rarefaction from the other body. The two disturbances neutralize each other and produce a minimum of sound. One train of sound waves gradually falls still further behind the other until there is now a difference of one period and the compressions from each sounding body arrive at the point at the same time. The disturbances are thus added together and a maximum of sound is produced. There results a pulsating effect in which soft and loud pulses alternate. These pulses are **beats.**

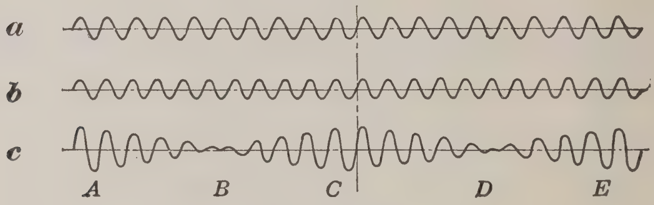


Fig. 346.—Beats.

When the two sounding bodies are almost in unison the number of beats per second is small. As the difference in the frequencies of the sounds is increased the number of beats is increased. If the sounds differ in frequency by unity, they will reinforce once each second and there will be one beat each second. In general the difference in the frequency of the two sounding bodies is equal to the number of beats per second.

Number of beats = difference in frequencies

$$N = n_1 - n_2$$

Beats are of great service in tuning stringed instruments. As two strings are brought more and more nearly into unison the number of beats per second become less and when no beats can be observed, the strings have the same pitch. In organs two pipes having nearly the same frequency are sometimes used to produce a beating of the sounds giving a tremulous effect imitating the human voice.

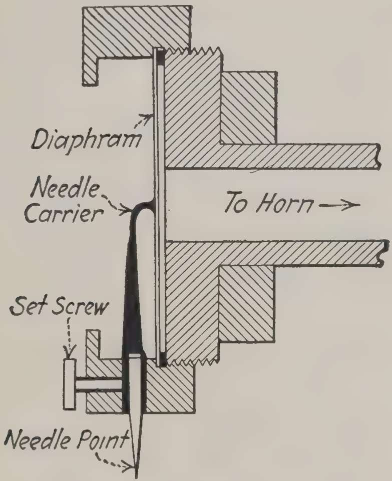


Fig. 347.—The phonograph.

438. Phonograph.—In a phonograph (Fig. 347) a wax cylinder or drum is rotated by clock work and the sounds to be recorded are spoken into a horn. These sound waves travel down the horn and fall on a thin disc which is somewhat like the disc used in a telephone transmitter. The sound waves cause the disc to vibrate back and forth in unison with them. These vibrations are recorded on a drum or disc of hard wax. A short needle fastened to the vibrating disc presses on the wax and ploughs a fine furrow in it.

The furrow is the record of the vibration of the disc. To reproduce the sound the needle is allowed to travel again along the furrow. The needle moving up and down reproduces the motion by which the furrow was made. This causes the disk attached to the needle to vibrate and repeat the original sound waves which were sent down the horn. In practice it is found better to use different needles and discs for recording and for reproducing the sounds.

In the gramophone a disc shaped record is used instead of a cylindrical record. The grooves vary in width instead of in depth as in the cylindrical records. The needle moves from side to side instead of up and down. The construction of the part of the gramophone which reproduces the sound is represented in Fig. 347. The needle carrier is attached to the center of the diaphragm which in turn is immediately in front of the end of the horn. The vibration of the diaphragm causes the needle to move to and fro and conversely the motion of the needle to and fro causes the diaphragm to vibrate and reproduce the sounds by which the record was formed.

Problems

1. What is the frequency of a sound, the wave length of which is 75 cm. in air?

2. A tower is 750 ft. high. How long does it take a sound to travel from the bottom to the top of it?

3. An open organ pipe is 85 cm. long. What is the frequency of its fundamental note?

4. What is the length of a closed organ pipe which yields a fundamental note having a frequency of 350 per sec.?

5. A man standing between two large parallel vertical cliffs, produces a sound. He hears one echo after 0.5 sec. and a second echo after 1.25 sec. Find the distance between the cliffs.

6. Two tuning forks *A* and *B* originally had the same frequency. To the fork *B* is attached a bit of wax and the forks are found to make 4 beats per sec. If the frequency of *A* is 256 vibrations per second, what is the frequency of *B*?

7. A wire has a length of 125 cm. and a mass of 48 g. It is stretched by means of a weight of 32 kg. What is the frequency of the fundamental and the first overtone?

8. If the fundamental of a stretched wire 125 cm. long has a frequency of 150 vibrations per sec. when the wire is stretched by a load of 14 kg., what will be its frequency when the wire is stretched with a load of 6 kg.?

9. A string is vibrating 140 times per sec. The length of the string is doubled and the tension made threefold its original value. What is the frequency of the string?

10. Two organ pipes are 3 ft. 6 in. and 3 ft. 5 in. in length. These pipes are sounded together and produce five beats per second. What is the velocity of sound in the medium in which the organ pipes are located?

CHAPTER XXXV

NATURE AND PROPAGATION OF LIGHT

439. Nature of Light.—A hot body gives off radiations of different wave lengths. Those which produce heating effects are called heat waves and those which are sufficiently short to affect the optic nerve are called light waves. These two kinds of waves are identical except with respect to their wave lengths. The heat waves have long wave lengths in comparison with the light waves. Both types of waves travel in the same medium, **the ether,** with the same velocity. They obey the same laws of reflection, refraction, interference, etc. Light, with wave length longer than the wave length of red light, is called infra-red radiation and light with a wave length shorter than the wave length of the violet is called ultra-violet light. Ultra-violet light produces chemical effects like those which are found when a photographic plate has been exposed to the light.

That light consists of a wave motion in a medium known as the ether has been proved in many ways. If an ordinary incandescent lamp bulb is evacuated of air and other gases, the incandescent filament illuminates its surroundings in spite of the fact that the light from the filament travels through a vacuum. The light from the sun could not reach the earth if a material medium were necessary for its transmission. That light is a transverse wave motion will be more evident from a discussion of the facts of polarization which is given in a subsequent chapter.

440. Velocity of Light.—The velocity of light which is 186,000 miles per second, is so great that in one second it would travel more than seven times around the earth at the equator. Light travels from the sun to the earth in a little over 8 min. but it requires 4 years for light to travel from the nearest star to the earth. If the north star were obliterated the earth would continue to receive light from it for about 44 years. Several methods have been devised for determining the velocity of light. It is sufficient for our purposes to describe one of them.

The method of Foucault which has been modified and improved by Michelson, has given very accurate results for the velocity of light. An understanding of this method can be obtained by examining Fig. 348. A source of light S sends a beam of light to a mirror AB which can be rotated about an axis through its center O, perpendicular to the plane of the diagram. The mirror AB reflects the light through the lens LN to the plane mirror M which sends it back to the mirror AB and thence to S where an image is formed. When the mirror AB is not rotating, this image coincides with the source of light S. When, however, the mirror AB rotates through an angle x while the light travels from the mirror AB to the mirror M and back again to the mirror AB, the beam after reflection at the mirror AB will not return to S but to some other point S'. As the mirror AB turns through an angle x its normal OP also turns through this same angle. The angle between the ray incident on the mirror AB after returning from the lens LN is thus decreased by x and since the angle of incidence is always equal to the angle of reflection, the angle of reflection will also be decreased by x. The ray after reflection at the mirror will therefore have been deflected through an angle of $2x$ from its position when the mirror AB was not in rotation.

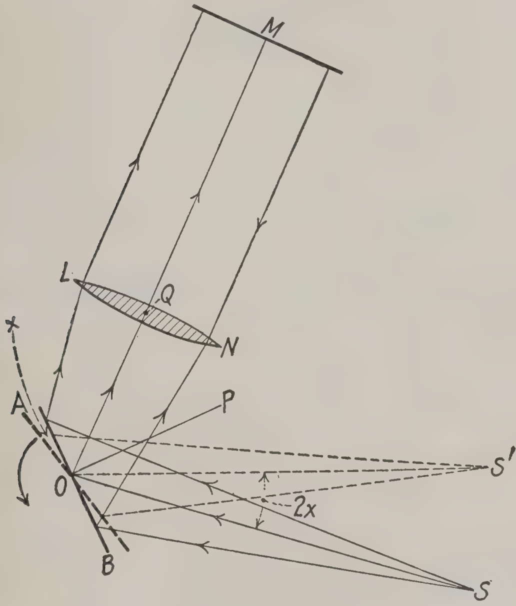


FIG. 348.—Measurement of velocity of light.

By measuring the distance SS' and the distance d from the mirror AB to S, the angle $2x$ can be found, for $2x = \frac{SS'}{d}$ radians. If the number of revolutions which is made per second by the mirror AB is observed and denoted by n, then $x = 2\pi nt$, where t is the time for the light to pass from the mirror AB to the mirror M and back again to the mirror AB. If V denotes the velocity of light and L the distance from the mirror AB to the mirror M,

$$2L = Vt$$

$$t = \frac{x}{2\pi n}$$

$$2L = V\frac{x}{2\pi n}$$

$$V = \frac{4\pi nL}{x}.$$

Example.—In determining the velocity of light by Foucault's method it was found that the rotating mirror made 100 revolutions per sec. The distance from the mirror to the source of light was 603 meters and the angle through which the ray of light was deflected was 0.0025 radian. Find the velocity of light.

$$\begin{aligned} V &= \frac{4\pi nL}{x} \\ &= \frac{4 \times 3\tfrac{1}{7} \times 100 \times 60300}{0.0025} \\ &= 3.02 \times 10^{10} \text{ cm. per sec.} \end{aligned}$$

441. Frequency and Wave-length.—The relation between frequency, velocity and wave-length is the same for light waves as it is for sound waves. Hence in this case as in the other we have

$$v = n\lambda$$

where v is the velocity, n the frequency and λ the wave-length. Waves of yellow light have been found to have a wave-length equal to about 0.000059 cm. Taking the velocity of light to be 3×10^{10} cm. per second the frequency n of yellow light is,

$$n = v/\lambda = \frac{3 \times 10^{10}}{0.59 \times 10^{-4}} = 5.01 \times 10^{14} \text{ per sec.}$$

442. Sources of Light.—The sun is the chief source of light and heat but there are many artificial sources. Any body when heated to a sufficiently high temperature becomes a source of light. The ultimate source of light and heat waves is the vibration of the electrons in the atom. The motions of these electrons set up waves in the ether and thus produce both light and heat waves.

As the temperature of a body is raised the body emits invisible radiations until the body is red hot. At this temperature visible radiations begin to be emitted. The greater the temperature, the greater is the amount of both heat and light waves which are emitted but the percentage of visible radiations becomes larger and larger as the temperature of the source of radiations is increased. For this reason the modern tungsten lamp is much more efficient than the old carbon incandescent lamp. Tungsten has a very high melting point and when it is surrounded by nitrogen or when it is in a vacuum, it can be heated to a high temperature and its efficiency thus made large.

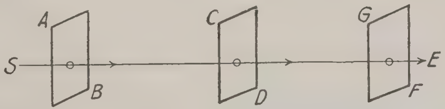


FIG. 349.—Rectilinear propagation of light.

443. Rectilinear Propagation of Light.—Under ordinary circumstances light travels in straight lines and does not appreciably bend around objects. That light travels in straight lines may be shown by placing a candle or other source of light behind a screen having in it a small hole (Fig. 349). In front of this screen *AB* are placed two screens *CD* and *GF* each with a small hole at the center. When these screens are so adjusted that the eye *E* can see the source of light *S* distinctly, it will be found that the straight line joining *S* and *E* passes through the holes in the screens. This shows that light from *S* to *E* comes in a straight line.

The rectilinear propagation of light is also shown by the fact that a small source of light will cast a sharp shadow of an object. Thus a small source of light *S* (Fig. 350) will illuminate all points on a screen above and below *A* and *B*. No light arrives at the screen between *A* and *B* because it is stopped by the opaque body *M*. The fact that the boundary of the shadow is sharp shows that light from *S* does not bend appreciably into the shadow of the opaque body.

When the source of light is not small, the boundary of the shadow will not be sharp (Fig. 351). Points on the screen above *A* and below *D* will be fully illuminated by *S*. The part of the screen between *B* and *C* will receive no light and will therefore be a complete shadow. The screen between *A* and *B* and between *C* and *D* will be partly illuminated and will be a less dense shadow than the part between *B* and *C*. This outer shadow will gradually shade off from complete shadow at *B* to complete illumination at *A*. This gradual shading off gives the shadow that blurred appearance which characterizes most shadows.

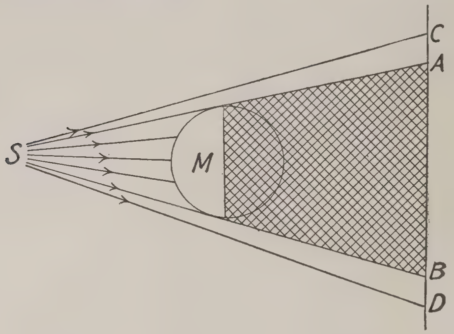


Fig. 350.—Shadows by opaque bodies.

When the source of illumination is larger than the object casting the shadow (Fig. 352) the only region of complete shadow is the cone *FEN* having the object as a base. If the screen is placed near the object there will be cast on it a small complete central shadow surrounded by a large partial shadow. When the screen is far from the object there will be only a diffuse shadow with no complete shadow at the center.

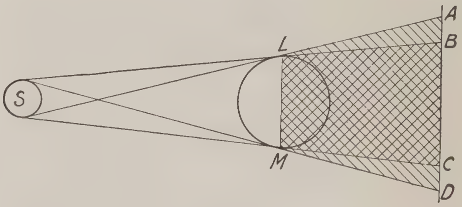


Fig. 351.—Shadows from finite sources.

444. Total, Annular and Partial Eclipses.—The best illustration of shadows on a large scale is found in the eclipses of the sun or moon. When the moon happens to be interposed between the sun and the earth, in such a way that its shadow falls wholly or partly on the earth, the sun is wholly or partly obscured by the moon and there is a total or partial eclipse of the sun. If the earth is sufficiently near to the moon and passes through the complete shadow represented by the cone *FEN* (Fig. 352) the eclipse is total. If, however, the earth is further away from the moon and only passes through the partial shadow, the eclipse is said to be annular.

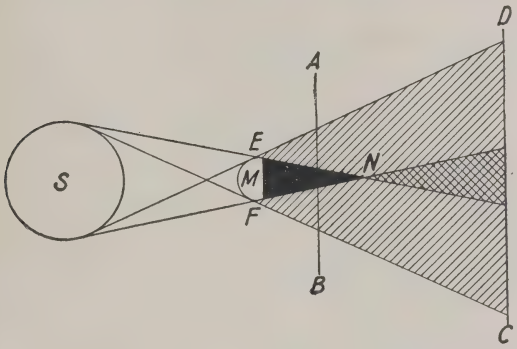


Fig. 352.—Shadows by objects smaller than the source of light.

When the earth, moon and sun are not in exactly the same straight line, there may be a partial eclipse. Sometimes the moon, when it is full, passes through the shadow of the earth. There is then an eclipse of the moon.

CHAPTER XXXVI

ILLUMINATION AND PHOTOMETRY

445. Intensity of Illumination.—The intensity of illumination is measured by the amount of light which falls on unit area of a surface. The amount of energy received by unit surface can not be determined in absolute measure. The eye is the most sensitive means of detecting light but it does not give a quantitative measure of it, for the eye is not equally sensitive to all kinds of radiations. By means of the eye it is, however, possible to make an accurate comparison of two intensities of illumination.

446. Standards of Illumination.—To compare sources of illumination a standard source of illumination is necessary. The choice of these standards is more or less arbitrary. There are a number of such standards in use. The **British standard candle** is defined to be a candle made of spermaceti, weighing six to the pound and burning at the rate of 120 grams per hour. This standard does not have a sufficiently constant illuminating power to make it of scientific value. Its illuminating power changes with atmospheric conditions and with the conditions under which it is burned. For this reason it has been replaced by the **Harcourt pentane lamp** in which air is drawn over pentane and the mixture burned in a standard burner in which the flame is adjusted to a definite height. Corrections must be made for atmospheric conditions. The lamp has an illuminating power which is equal to that of ten standard candles. **From it an international candle is defined to be a light with an illuminating power equal to one-tenth of that of a Harcourt pentane lamp.**

An electric incandescent lamp when run at a definite voltage is the most convenient standard of illumination. Such standards must be accompanied by a certificate giving their candle power when run under definite conditions.

447. Candle Power.—The candle power of a lamp is a specification of its illuminating power in terms of some standard candle. For example, the illuminating power of an incandescent lamp may be 30 times that of a standard candle and it is then said to be a 30 candle power lamp. But the illuminating power of a light varies according to the direction from which it is observed. Such a variation of the intensity of the light in different directions is shown in Fig. 353 where the light from an incandescent gas mantle has been reflected by a glass shade. It becomes therefore necessary to measure the average candle power in a given plane. The average illuminating power in the horizontal plane is called the **mean horizontal candle power.** The mean spherical candle power denotes the average illumination of a source of light from all directions in space. Fig. 353 shows how the illumination from the gas mantle varies in a vertical plane with and without the glass shade. The dotted curves show the distribution without the shade.

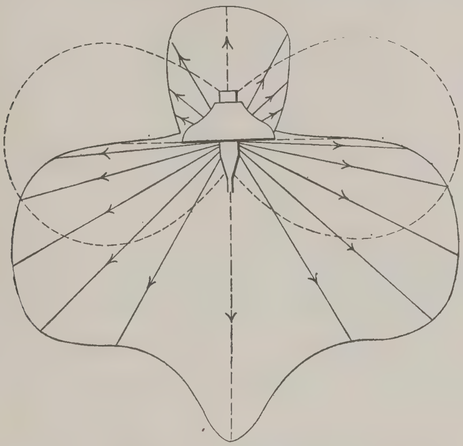


Fig. 353.—Distribution of light from gas mantle.

448. Law of Inverse Squares.—Consider the light waves proceeding from a small source L (Fig. 354). As the spherical wave advances, the energy in the surface of the wave is distributed over a larger and larger area. Assuming that the medium through which the wave travels does not absorb any of the energy, the total energy distributed over any of these concentric spherical surfaces is the same.

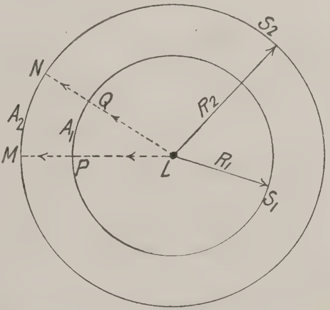


Fig. 354.—Law of inverse squares.

Let Q = the total energy sent out by the source in one wave,

Let R_1 and R_2 be the radii of two concentric spheres surrounding the source. The area of the smaller sphere is $4\pi R_1^2$ and that of the larger one is $4\pi R_2^2$. The amount of energy per unit area of the smaller sphere is,

$$I_1 = \frac{Q}{4\pi R_1^2}$$

and the amount per unit area of the larger sphere is,

$$I_2 = \frac{Q}{4\pi R_2^2}$$

Hence

$$\frac{I_1}{I_2} = \frac{R_2^2}{R_1^2}$$

Therefore **the intensity of illumination from a point source of light varies inversely as the square of the distance from the source.**

Example.—A small spherical lamp sends light in all directions. A screen 1 cm. square is placed 200 cm. from it and a second screen of the same size is placed 400 cm. from it. Find the ratio of the energies received by the two screens.

$$\frac{I_1}{I_2} = \frac{R_2^2}{R_1^2} = \frac{(400)^2}{(200)^2} = \frac{16}{4} = \frac{4}{1} = 4$$

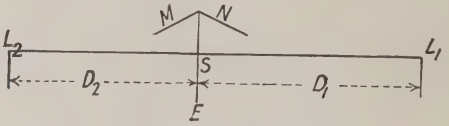


FIG. 355.—Bunsen photometer.

449. Photometer.—A photometer is a device for comparing the intensities of two different sources of light. To make this comparison the distances of the lights from a screen are adjusted until they produce the same intensity of illumination on the screen (Fig. 355). Thus, if I_1 is the illuminating power of one source, I_2 that of the other source and D_1 and D_2 their respective distances from the screen when they produce equal illumination on it, then

$$\frac{I_1}{D_1^2} = \frac{I_2}{D_2^2}$$

or

$$\frac{I_1}{I_2} = \frac{D_1^2}{D_2^2}$$

By observing the distances D_1 and D_2 the candle power of one light can be determined in terms of the other.

There are many devices for aiding the eye in determining when the intensties of illumination from the two sources are the same. Usually provision is made for varying the distance of one or the other of the sources until the eye can detect no difference in the intensities of the lights on the screen.

450. Bunsen Grease Spot Photometer.—If a spot of grease or oil is made on a piece of plain paper, the spot is more translucent than the rest of the paper. When the paper is held up to the light the grease spot will appear brighter than the rest of the paper because light passes through it more readily than it does through the remainder of the paper. If this same paper is viewed by reflected light by placing the eye between the light and the paper, the grease spot appears darker than the remainder of the paper because the grease spot transmits the greater part of the light falling on it while the paper reflects most of the light falling on it. This difference in the behavior of the spot and the paper gives a means of comparing the intensities of two lights.

A screen *S* consisting of a piece of card board having a hole in it covered with oiled paper is placed between two lights L_1 and L_2 (Fig. 355). The screen is adjusted until the appearance of both sides is the same. It is then considered that both sources produce the same intensity of illumination on the screen. To make it possible to see both sides of the screen at the same time, two mirrors *M* and *N* are placed on either side of the screen and the images of the spot in these mirrors are observed. By measuring the distances from the lights to the screen, the relative candle power of the lights can be determined.

Let I_1 = the candle power of L_1,
I_2 = the candle power of L_2,
D_1 = the distance from the screen to L_1,
D_2 = the distance from the screen to L_2.

$\frac{I_1}{D_1^2}$ = the intensity of the light from L_1 on the screen.

$\frac{I_2}{D_2^2}$ = the intensity of the light from L_2 on the screen. Since these two intensities are the same,

$$\frac{I_1}{D_1^2} = \frac{I_1}{D_2^2}$$

$$\frac{I_1}{I_2} = \frac{D_1^2}{D_2^2}$$

Example.—In comparing the intensities of two lights by means of a Bunsen grease spot photometer, it was found that when one light was 40 cm. from the screen the other must be 60 cm. from it to produce the same intensity of illumination on the screen. If the weaker light has a candle power of 16, what is the candle power of the more intense light?

$$\frac{\text{Candle power of } L_1}{\text{Candle power of } L_2} = \frac{\text{square of distance of } L_1 \text{ from the screen}}{\text{square of distance of } L_2 \text{ from the screen}}$$

$$\frac{I_1}{I_2} = \frac{D_1^2}{D_2^2} = \frac{(60)^2}{(40)^2} = \frac{36}{16}$$

$$\frac{I_1}{16} = \frac{36}{16}.$$

Hence $I_1 = 36$ candle power.

451. Lummer-Brodhun Photometer.—The adjustments with a Bunsen photometer can not be made with great accuracy and

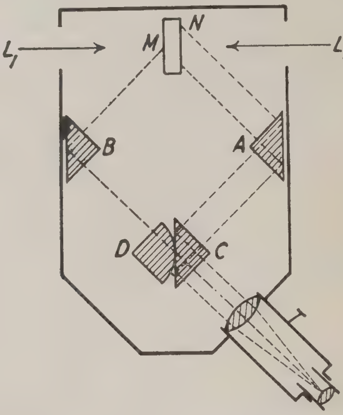


FIG. 356.—Lummer-Brohdum photometer.

where greater accuracy is required a Lummer-Brodhun photometer can be used. In this photometer (Fig. 356) light from two sources L_1 and L_2 is received on the two sides M and N of a milk white screen. Light from L_1 after irregular reflection from the surface M of the screen is received by the prism B where it is totally reflected and passes through the prisms C and D where they are in intimate contact at the middle. On the other hand the light from L_2 after irregular reflection at the surface N is totally reflected by the prism A and then enters the prism C. That part of the light from L_2 which falls on that part of the rear surface of the prism C which is in contact with the prism D passes on through the prism D. But that light from L_2 which strikes that part of the rear surface of the prism C which is not in contact with the prism D is totally reflected and enters the telescope T. On looking into the telescope it is found that the central part of the field of view is illuminated by light from L_1 after reflection by the prism B. The outer portion of the field of view is illuminated by light which came from L_2 after reflection at A and the rear surface of the prism C.

By moving the photometer box with respect to the lights and observing the two parts of the field of view in the telescope, the intensities of the two parts of the field of view can be made the same. When the two parts of the field of view are equally illuminated, an accurate measure of the candle power of one of the lights in terms of the other can be obtained by measuring the distances of the sources of light from the screen MN. The relative candle powers are calculated as in the Bunsen photometer from the equation,

$$\frac{I_1}{I_2} = \frac{D_1^2}{D_2^2}$$

Problems

1. Two lamps of 40 and 60 candle power are placed 120 cm. apart. Find the position of the screen in the Bunsen photometer when its two sides are equally illuminated.

2. The distance between a 32 candle power standard lamp and the lamp which is being tested is 400 cm. The screen of a Lummer-Brodhun photometer is equally illuminated when it is 120 cm. from the standard lamp. What is the candle power of the lamp under test?

3. How far from a screen must a 50 candle power lamp be placed to give the same illumination as a 16 candle power lamp which is 500 cm. away?

4. The intensity of illumination of an arc lamp on a screen 30 m. from it is the same as the intensity of illumination produced on the screen by a lamp of 25 candle power when it is 200 cm. from the screen. What is the candle power of the arc lamp?

5. Two sources of light each of 50 candle power are placed on the same side of a photometer screen. One of these lights is at a distance of 100 cm. from the screen and the other at a distance of 200 cm. from it. Where must a third source of light with an intensity of 25 candle power be placed in order that the two sides of the photometer screen may be equally illuminated?

CHAPTER XXXVII

REFLECTION OF LIGHT

452. Laws of Reflection.—A beam of light in a homogeneous medium travels forward in one direction but when it comes to a second medium some of the light is bent back or reflected. At a polished or silvered surface nearly all of the light is reflected. At the surface of clear glass only a small part of it is reflected. The greater part of it enters the glass and passes through it. In Fig. 357 let AB represent the reflecting surface, MP the perpendicular or normal to this surface, OP the incident ray and PN the reflected ray. The angle OPM between the incident ray and the normal to the surface is called **the angle of incidence.** The angle MPN between the reflected ray and the normal to the surface is called the **angle of reflection.** Reflection at such a surface occurs according to the following two laws:

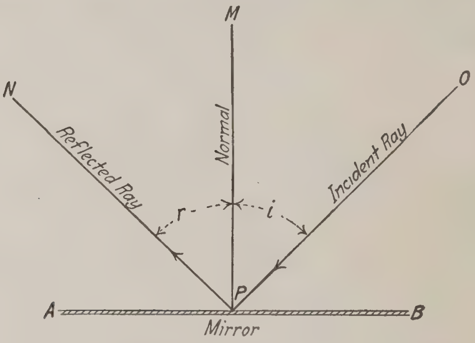


FIG. 357.—Reflection of light from plane mirror.

First Law of Reflection.—The incident ray, the reflected ray and the normal to the surface lie in the same plane.

Second Law of Reflection.—The angle of incidence is equal to the angle of reflection.

453. Image in a Plane Mirror.—When a luminous object like a small candle is placed in front of a plane mirror MN (Fig.

358) any point L of the source sends light in all directions. Two of these rays of light LA and LB after reflection by the mirror travel in the directions AD and DE respectively. To an observer in front of the mirror these reflected rays seem to diverge from a point behind the mirror. By drawing other rays it can be shown that they all seem to diverge after reflection from the same point behind the mirror. The observer will therefore see a bright spot behind the mirror and for every other spot in the luminous source there will be a corresponding bright spot behind the mirror. There will thus be built up behind the mirror an image of the flame or any other luminous body which is in front of the mirror.

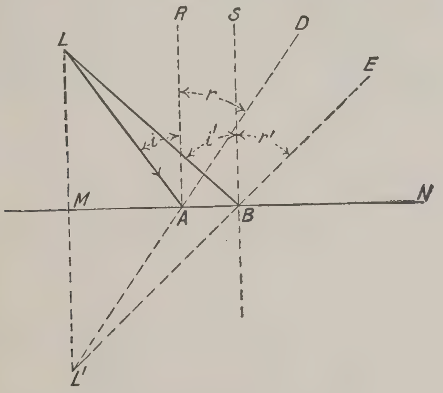


FIG. 358.—Image in a plane mirror.

From L drop a perpendicular to the mirror MN and produce this perpendicular until it intersects AD. The angles LAM, DAN and MAL' are equal. The angle at M is a right angle. Hence the triangle LAM and MAL' are similar. Since the side MA is common to both triangles, LM is equal to ML'. Next draw the line BL'. The two triangles LBM and MBL' have $LM = L'M$, the side MB in common and the angles at M right angles. Hence these triangles are equal in all their parts. Hence $LBM = MBL'$, $LBS = L'BS' = SBE$ and the line EBL' is a straight line. Consequently it is possible to say that the line joining the object and the image is perpendicular to the mirror and that the image is as far behind the mirror as the object is in front of it.

454. Reflection and Absorption of Light at an Opaque Surface.—When light falls on a rough opaque surface (Fig. 359), the incident light is broken up and scattered in all directions. When the surface is so smooth that the distance between the successive elevations on the surface are less than about one-quarter wave length of light, there will be very little scattering of light and the surface is said to be polished. Thus a surface which may be considered to be polished for light of long wave length is not polished for light of short wave length. Such a surface which reflects light with little scattering is called **a mirror.**

455. Selective Reflection and Absorption.—When light is incident on the surface of a body part of the light is reflected, part transmitted and the remainder absorbed. The fraction which is reflected depends on the wave length of the light, the angle of incidence and the medium surrounding the body. The amount which is absorbed depends on the wave length of the light and the nature of the substance. A substance which absorbs all of the light except the red, transmits and reflects only the red. It therefore appears red to the eye both in reflected and transmitted light. Selective reflection and absorption of light is the most common cause of body color of objects. No substance has been found which reflects light of only one wave length. Consequently no substance shows a single color when viewed by reflected light. A substance which reflects completely light of all wave-lengths is white when it is illuminated by white light. When a substance reflects some wave-lengths to a greater extent than others and is viewed in white light it appears gray. When a body absorbs all the light which falls on it, none of the incident light reaches the eye and the body appears black. If a body is therefore illuminated by light of frequencies other than those which it reflects, it will appear dark. Thus a piece of red glass when viewed by reflected yellow light is dark. There are no red radiations present for it to reflect and so it sends no light back to the eye. It absorbs the yellow radiations.

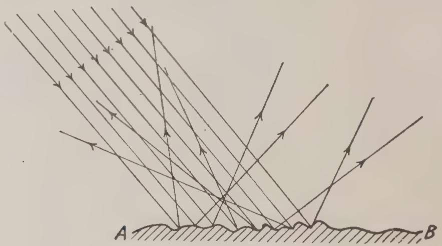


Fig. 359.—Irregular reflection.

456. Concave Spherical Mirrors.—A concave spherical mirror is part of a spherical shell with its inner surface polished. The center of the sphere from which the mirror was taken is called the **centre of curvature** of the mirror and the radius of the sphere is called the **radius of curvature** of the mirror. The middle point M of the mirror (Fig. 360) is known as the **vertex of the mirror** and the straight line MN through the vertex of the mirror and its center of curvature is **the principal axis of the mirror.** From the property of the mirror it is evident that any line from C, the center of curvature, to the mirror is perpendicular to the surface of the mirror at the point to which it is drawn. Such a line is the radius of the sphere of which the mirror is a part and the radius is perpendicular to the sphere.

Suppose that a luminous point U is placed on the principal axis and that this point sends a number of rays to the mirror. Consider two of these rays, UM and UP. The first of these rays after passing through the center of curvature, strikes the mirror normally and is reflected back over its former path. The other ray UP will be reflected by the mirror in such a way that the angle of incidence is equal to the angle of reflection. Hence the incident ray PU and the reflected ray PV will make equal angles with the radius of curvature CP. The reflected ray crosses the principal axis at some point V. The ray UM is also reflected through V. The point V is the image of U and all the rays from U will pass through it.

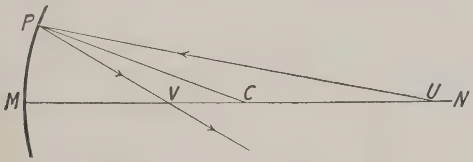


FIG. 360.—Spherical mirror.

457. Principal Focus.—If the luminous point U (Fig. 361) is removed very far from the mirror so that the rays reaching the mirror from it are nearly parallel, after reflection they will come to a single point F. This point at which all the rays parallel to the principal axis meet after reflection is known as **the principal focus.** It is half-way between the mirror and its center of curvature.

Let the ray AB (Fig. 361) parallel to the principal axis strike the mirror at B and be reflected in the direction BF, such that the angle of reflection r is equal to the angle of incidence i.

Since *AB* is parallel to the axis, the angles *i* and *x* are equal and the triangle *FBC* is isosceles and *BF* is equal to *FC*. If *B* is not too far from *M*, *BF* and *MF* are nearly equal. Hence *MF* is nearly equal to *FC* and therefore *F* is half-way between *M* and *C*.

Where a spherical mirror of large aperture is used, the rays parallel to the axis do not all meet at the principal focus. This gives rise to an imperfection known as **spherical aberration.** This imperfection is small when the concave mirror is only a small part of the sphere. This imperfection is overcome where large mirrors are necessary by using parabolic mirrors.

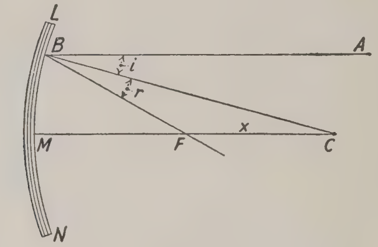


FIG. 361.—Definition of principal focus.

458. Construction of Images in a Concave Mirror.—If in a concave mirror the center of curvature and the principal focus are given, it is not difficult to construct the image of an object *AB* placed in front of the mirror. There are three cases which arise (1) the object is beyond the center of curvature, (2) the object is between the center of curvature and the principal focus, (3) the object is between the principal focus and the mirror.

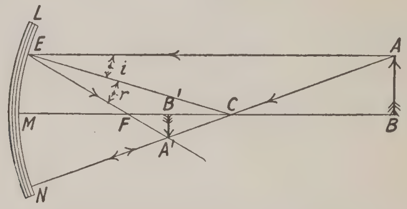


FIG. 362.—Location of images in concave mirror.

Suppose that *AB* (Fig. 362) is outside the center of curvature of the mirror. From *A* draw a ray parallel to the principal axis. After reflection this ray will pass through the principal focus *F*.

Now from *A* draw a second ray through the center of curvature. This ray will be reflected back on itself and pass again through the center of curvature. Where these two rays intersect there will be formed an image of the point *A*. The image of any other point in the arrow *AB* may be located in the same way so that the image of *AB* is found to be *A'B'*.

If the object lies between the center of curvature and the principal focus (Fig. 363) the rays are drawn as in the preceding case but the image now lies back of the center of curvature and is magnified. Let *AB* be the object lying between the principal focus and the center of curvature. From *A* draw ths ray *AE* parallel to the principal axis. After reflection it passee through the principal focus *F*. Though the center of curvature draw the ray *AC* which is reflected back on itself and intersects the ray reflected from *E* at *A'*, forming an image of *A* at that point. In a similar manner the images of the other points on the arrow *AB* are found, giving *A'B'* as the magnified and inverted image of *AB*.

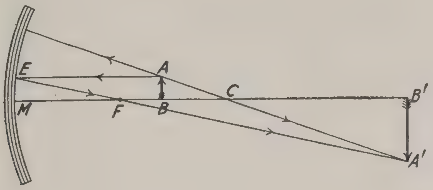


FIG. 363.—Object between center of curvature and focus.

When the object is inside the principal focus (Fig. 364) the ray from *A* parallel to the principal axis is drawn as before and as before it passes through the principal focus after reflection. The ray *AEC* through the center of curvature strikes the mirror normally and is reflected back through *C*. These reflected rays *EC* and *DF* diverge and seem to have come from a point *A'*

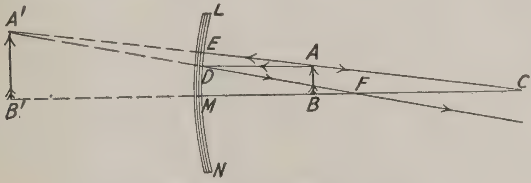


FIG. 364.—Object inside the focus.

behind the mirror. There is thus formed behind the mirror an image of the point A at the point A' where these two rays seem to intersect. Locating the images of the other points of AB in the same way, we have behind the mirror the **magnified** and **erect image** $A'B'$ of the arrow AB. In this case the image appears to be behind the mirror. It is a **virtual image** because light only appears to come from it. To an eye in front of the mirror the effect is the same as if light actually came from the image $A'B'$ but an eye behind the mirror receives no light from $A'B'$.

459. Construction of Images in a Convex Mirror.—In case of a convex mirror (Fig. 365) the center of curvature and the principal focus are both behind the mirror. Let AB be the object as before and draw form A the ray AE parallel to the principal axis BC. After reflection at E this ray leaves the mirror as if it came from F the principal focus behind the mirror. Now draw AD perpendicular to the surface of the mirror. It is reflected from the mirror at D and appears to have come from C, the center of curvature behind the mirror. At A' where these two rays appear to intersect there is formed a **virtual image** of the point A. Locating the images of the other points in AB in the same way, it is found that the object AB forms the image $A'B'$ in the mirror LN. This image is behind the mirror. **It is virtual, erect and diminished in size.**

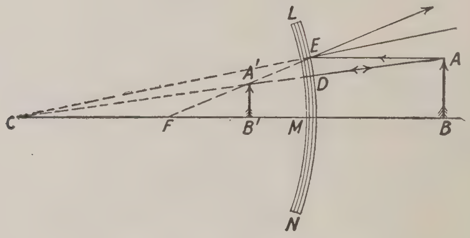


FIG. 365.—Image in a convex mirror.

460. Formula for a Spherical Mirror.—A simple formula which expresses the relation between the radius of curvature of the mirror, the object distance and the image distance can be obtained from Fig. 366 as follows:

By similar triangles,

$$AB : A'B' :: BC : CB'$$

$$AB : A'B' :: BM : B'M$$

Hence $$BC : B'C :: BM : B'M.$$

Let u and v denote the object distance and the image distance respectively, that is the distance of B and B' from the mirror, and r denote the radius of curvature of the mirror, then

$$BC = u - r$$
$$B'C = r - v$$
$$BM = u$$
$$B'M = v$$
$$u - r : r - v :: u' : v$$

multiplying means and extremes together.

$$uv - vr = ur - vu$$

$$2uv = vr + ur$$

Dividing by uvr $$\frac{2}{r} = \frac{1}{u} + \frac{1}{v}$$

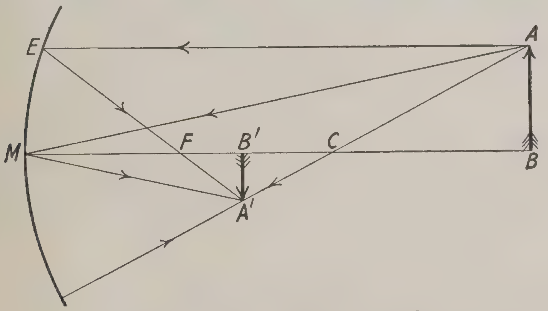


FIG. 366.—Proof of mirror formula.

If AB is a great distance from the mirror, that is, if u is infinitely large, $1/u = 0$ and therefore

$v = r/2 = f =$ the principal focal length of the mirror.

When $u = r$,

$1/r + 1/v = 2/r$ and $v = r$.

This means that when the object is at the center of curvature the image is also at the center of curvature. When $u = f$ we have

$$1/f + 1/v = 1/f$$

and $v = \infty$. Hence the image lies at infinity when the object is at the principal focus.

When u is less than f, the focal length, the image is virtual and lies behind the mirror. The image distance is therefore negative and this negative value must be substituted in the formula.

This formula can be also applied in the case of a convex mirror if account is taken of the fact that in this case the radius of curvature is negative. In convex mirrors the image is always **virtual** and behind the mirror so that the image distance is also always **negative.** Hence in the use of this formula for a convex mirror a negative sign must always be prefixed to both the radius of curvature and the image distance. As in the case of the concave mirror when

$$u = \infty, v = f$$

and the image is at the principal focus but in this case the image is virtual and behind the mirror and both f and v are negative.

Example.—An object is situated at a distance of 80 cm. from a concave mirror of radius of curvature of 60 cm. Find the position of the image.

$$u = 80 \quad f = 60/2 = 30$$
$$1/u + 1/v = 1/f$$
$$1/80 + 1/v = 1/30$$
$$v = \frac{240}{50} = 4.8 \text{ cm.}$$

Example.—A bright spot situated at a distance of 40 cm. in front of a convex mirror forms an image 20 cm. behind the mirror. Find the focal length of the mirror.

$$1/u + 1/v = 1/f$$
$$u = 40, \quad v = -20$$
$$1/40 - 1/20 = 1/f$$
$$f = -40$$

461. Size of Object and Image.—In the case of either the convex or the concave mirror there is a simple relation between the size of the image and the size of the object. In Fig. 366 the triangles ABC and $A'B'C$ are equiangular and therefore similar. Their corresponding sides are therefore proportional.

$$\text{Hence} \quad \frac{\text{Size of image}}{\text{Size of object}} = \frac{\text{distance of image from center}}{\text{distance of object from center.}}$$

Or from triangles ABM and $A'B'M$

$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{\text{distance of image from mirror}}{\text{distance of object from mirror}} = v/u.$$

Example.—Find the size of the image of a body 2.5 cm. high when placed 50 cm. in front of a concave mirror whose focal length is 20 cm.

$$u = 50 \quad f = 20$$
$$1/u + 1/v = 1/f$$
$$1/v = 1/20 - 1/50$$
$$v = 33.3 \text{ cm.}$$
$$\frac{\text{Size of image}}{\text{Size of object}} = \frac{33.3}{50} = 0.66$$
$$\therefore \text{Size of image} = 0.66 \times 2.5 = 1.66 \text{ cm.}$$

462. Parabolic Mirror.—If the width of a mirror (Fig. 367) is comparable to its radius of curvature, parallel rays after reflection do not all meet at the single point F, the principal focus. Rays which are reflected from a limited region of the mirror in the neighborhood of its vertex M are brought to a focus at F but the rays which strike the mirror at points distant from the vertex of the mirror cross the axis at points nearer to the mirror than F and are not brought to a focus. The effect of this is to destroy the sharpness of the image which would otherwise be formed by the mirror.

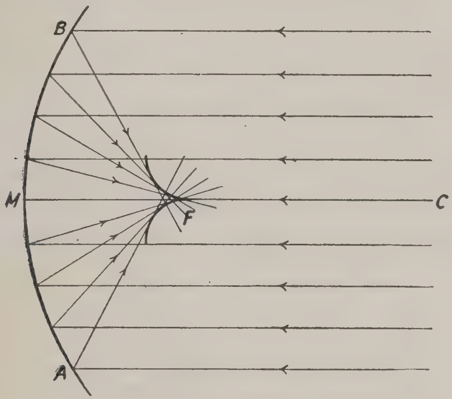


FIG. 367.—Spherical aberration.

If the section of the mirror is a parabola instead of a circle, parallel rays after reflection are all focused at a single point (Fig. 368). On the other hand a point source of light at the focus will send out parallel rays after reflection from the surface of the mirror. The chief property of a parabola is the fact that a line FB through the focus and a line BS parallel to the axis make equal angles with the normal BN. From this it follows that all rays parallel to the axis CA will pass through the point F after reflection and the whole beam, however large, will be brought to a common focus. The distortion found in the spherical mirror is thus avoided.

Such mirrors can be as large as desired without destroying the definition of the image. Mirrors of large aperture are needed for search lights and parabolic mirrors are used for such purposes. If the source of light is placed inside the focus of a parabolic mirror (Fig. 369) the rays of light diverge after reflection. If it is outside the focus, the rays converge after reflection. This fact is of importance in the adjustment of the headlights on an automobile.

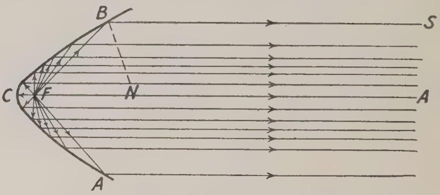


FIG. 368.—Parabolic mirror.

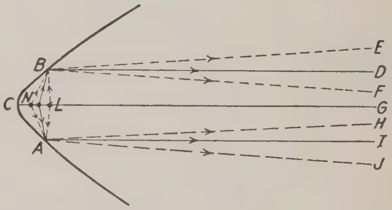


FIG. 369.—Parabolic mirror, light not at the focus.

Problems

1. An incandescent lamp is placed 60 ft. in front of a concave mirror whose radius of curvature is 10 ft. Find the distance of the image from the mirror.

2. An object when placed 12 cm. in front of a concave mirror produces an image 32 cm. behind the mirror. Find the focal length of the mirror.

3. An object is placed 12 ft. in front of a convex mirror whose focal length is 3 ft. Find the position and size of the image.

4. The image formed by a convex mirror is one-tenth the size of the object. The object is 5 m. from the mirror. What is the focal length of the mirror?

5. An incandescent lamp is 2 m. from a wall. It is desired to throw on the wall an image of the lamp which is three times the original linear dimen-

sions of the lamp. Where must the mirror be placed and what must be its radius of curvature?

6. An object is placed inside the principal focus of a concave mirror. The image is twice the size of the object. If the object is 10 m. from the mirror find the radius of curvature.

7. If the image formed by a concave mirror which has a focal length of 12 cm. is located 20 cm. behind the mirror, find the position of the object.

CHAPTER XXXVIII

REFRACTION OF LIGHT

463. Refraction.—Suppose that a regiment of soldiers marching with a speed of 4 miles per hour on smooth ground reaches a plowed field in which they march with a speed of only 3 miles per hour. As each man in turn reaches the boundary between the smooth and the plowed ground, his speed is decreased to 3 miles per hour while his neighbor will proceed with a speed of 4 miles per hour until he also reaches the plowed ground where he will be slowed down. This slowing down will happen to each man in turn until the whole column has entered the plowed field. Due to this unequal slowing up of the men in the column, the line of march of the whole column will be deflected from its original course. This deflection will be equivalent to a rotation of the line of march. After all the men in any particular rank have entered the plowed field, that rank will again proceed in a straight line. It is to be observed that the line of march in passing from a smooth field to a plowed field in which the speed of the men is less, has been bent toward the perpendicular to the line separating the smooth from the plowed ground. Such a bending also occurs whenever a wave motion passes from one medium to another in which it has a different velocity.

Experiments have shown that light travels with the greatest speed in a vacuum and that it travels with different speeds in different media. It will therefore be bent when it passes from one medium to another in which it has a different velocity. This bending of a ray of light when passing from one medium to another in which the velocity is different is known as **refraction.**

Refraction can be illustrated by taking a cup which is opaque (Fig. 370) and placing a coin on the bottom of it at the point B so that the edge of the coin can just be seen when the eye is at E. If now without moving the eye water is poured into the cup, the coin will come into view again. The ray BA as it leaves the water is bent away from the normal NA. Other rays are bent in a similar manner and there is formed an image of the coin at C

so that the depth of the coin below the surface of the water seems to have been lessened. Here it is seen that rays coming from the water to the air are bent away from the normal. The rays are always bent away from the normal when they enter a medium in which their velocity is greater than in that from which they came.

464. Laws of Refraction.—Let *RS* (Fig. 371) be a boundary surface separating two media as air and glass. Take a point *O* on the surface and draw *MN* perpendicular to the surface at *O*. If now *AO* represents the incident ray, *OB* represents the refracted ray. If, on the other hand, *BO* represents the incident ray, *OA* represents the refracted ray. In the first case the ray goes from a medium where the velocity is greater to a medium in which the velocity is less and is bent toward the normal. In the second case the ray goes from a medium where the velocity is less to one in which the velocity is greater and is bent away

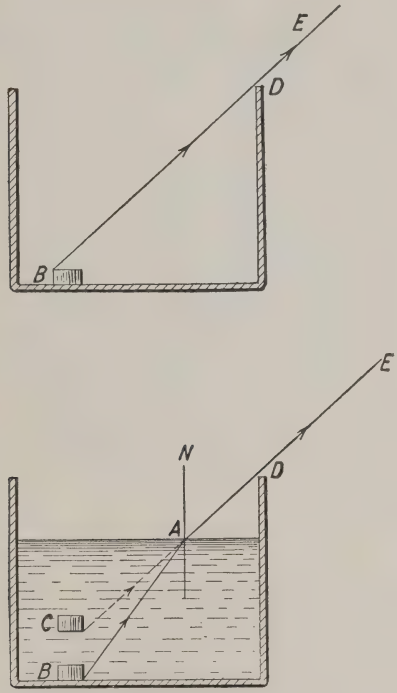


FIG. 370.—Refraction of light.

from the normal. The angle AON is called the **angle of incidence** and the angle MOB, the **angle of refraction.**

Index of Refraction.—The ratio of the velocity in the medium from which the wave came to the velocity in the medium into which the wave enters is called the **index of refraction.**

Let $V =$ the velocity in air,
$V' =$ the velocity in glass,
$V/V' = n =$ **the index of refraction of glass.**

First Law of Refraction.—The refracted ray, the incident ray and the normal to the surface lie in the same plane.

Second Law of Refraction.—Snell's Law. Let AB (Fig. 372) represent a surface separating one medium such as glass in which the velocity is less, from a second medium like air in which the velocity is greater. As soon as a ray enters the glass at D it is retarded, while the ray CL continues to advance with the same velocity as before. Consequently the direction of all the rays between D and L will be changed on entering the glass. If CL is the distance the wave travels in the upper medium in the time t, and DH the distance it travels in the lower medium in the same time and if V is the velocity in the upper medium and V' that in the lower medium, then

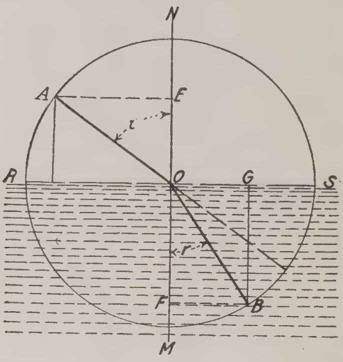


FIG. 371.—Laws of refraction.

$$CL = Vt = DL \sin i$$
$$DH = V't = DL \sin r$$
$$\frac{CL}{DH} = \frac{V}{V'} = \frac{\text{Sin } i}{\text{Sin } r} = \text{index of refraction.}$$

It thus appears that the ratio of the **sine of the angle of incidence to the sine of the angle of refraction is equal to the ratio of the velocities of the light in the two media and is therefore a constant for all angles of incidence.** This constant is the index of refraction of the medium. This law is known as **Snell's Law.**

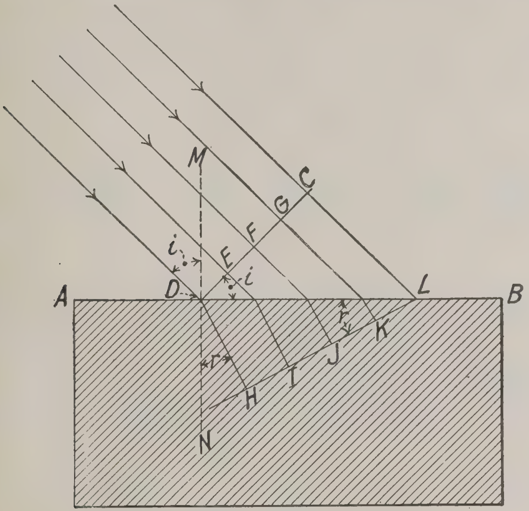


FIG. 372.—Proof of law of refraction.

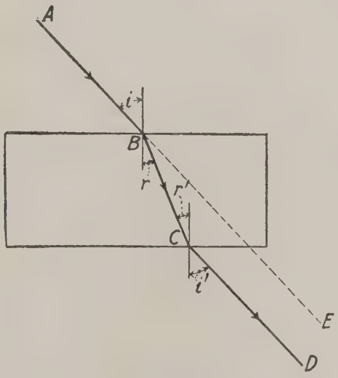


FIG. 373.—Refraction through a thin slab.

465. Refraction through a Slab with Parallel Faces.—Suppose that a ray of light *AB* (Fig. 373) falls on a slab of glass with parallel faces. As the ray enters the glass it is bent toward the normal according to Snell's law so that

$$\frac{\text{Sin } i}{\text{Sin } r} = n.$$

The ray now travels to the opposite face and as it emerges at C it is deflected away from the normal in such a way that

$$\frac{\text{Sin } i'}{\text{Sin } r'} = n$$

where n is the index of refraction of glass with respect to air. The ray of light is therefore deflected once toward the normal to the surface and once away from this normal. Since the faces of the slab are parallel to each other, the normal to the front and back faces are parallel to each other and therefore $r = r'$. Since the index of refraction is the same whether the light goes from air to glass or from glass to air, we have

$$\frac{\text{Sin } i}{\text{Sin } r} = \frac{\text{Sin } i'}{\text{Sin } r'} \text{ and since } r = r',$$

$$\text{Sin } i = \text{Sin } i' \text{ and } i = i'.$$

Consequently the direction of the ray on emerging from the glass is the same as its direction on entering the glass. It has, however, been displaced laterally.

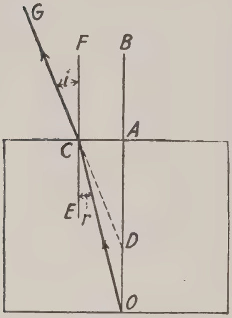


Fig. 374.—Apparent depth of a substance.

466. Apparent Thickness of a Transparent Body.—One of the consequences of the refraction of light is found in the fact that on looking through a transparent material the body under observation appears nearer to the observer than it actually is. Let CA (Fig. 374) be a surface separating air from some other denser medium like glass or water. A ray of light coming from a luminous point O in the denser medium and striking the surface of separation at A where OA is normal to the surface AC, will emerge without change of direction. Any other ray striking the surface at some point C, at any angle less than 90 will be bent away from the normal to the surface on entering the air. By prolonging this ray backward it will intersect the normal ray at D and to an eye situated in the air above the surface AC the light will appear to come from D instead of from O. By a similar construction for the other rays coming from O and striking the surface in the neighborhood of A, it will be found that these rays also appear to come from D. Hence the object O from which the rays actually come appears to be at D which is above O. The apparent depth of O below the surface is therefore less than its true depth. From Fig. 376 it is seen that

$$\angle GCF = \angle DCE = \angle ADC$$

and

$$\angle OCE = \angle AOC$$

$$n = \frac{\text{Sin } GCF}{\text{Sin } OCE} = \frac{\text{Sin } ADC}{\text{Sin } AOC} = \frac{AC}{DC} \div \frac{AC}{OC} = \frac{OC}{DC}$$

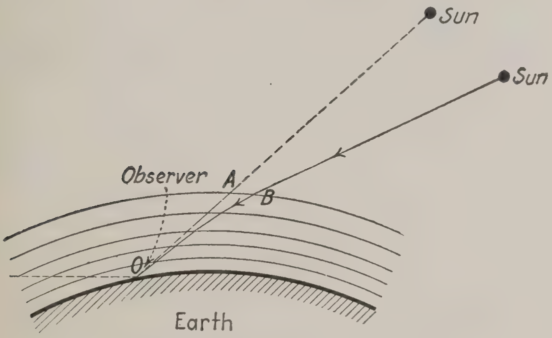


FIG. 375.—Atmospheric refraction.

The eye only receives rays very near the normal to the surface so that without sensible error we may write AO for CO and AD for CD.

$$n = \frac{AO}{AD} = \frac{\text{real depth}}{\text{apparent depth}}.$$

To spear or shoot a fish under water it is necessary to aim below where the fish appears to be. For if a fish were at O it would appear to be at D.

The greater the inclination at which the object is viewed, the greater is the distance it appears to be elevated. By looking down into clear water it is easily observed that it appears to be less deep than it really is. The cause of this shallowing effect is the refraction of the light at the surface of separation of the water and air.

467. Atmospheric Refraction.—Light is refracted in going from a vacuum to air. Let a ray of light from the sun (Fig. 375) enter the earth's atmosphere at B. This ray will be bent toward the radius of the earth and will reach an observer at O as if it had come in the direction OA instead of in its true direction. For this reason the observed altitude of the sun appears too great so that it must be corrected to get the true altitude. On account of the gradual change in the density of the atmosphere this correction is not easy to calculate. When the sun is at the zenith no correction is necessary. A body which is already below the horizon may appear to be above the horizon. Consequently the sun really sets before its last rays disappear. On the other hand the rays of the sun become visible in the morning a little while before the sun is above the horizon. Because the bending of the rays is greater the more nearly horizontal the ray becomes, the rays from the lower edge of the sun are bent more than those from the upper edge. This produces a shortening of the vertical diameter while the horizontal diameter is unchanged. The result is that the sun or moon appears elliptical when near the horizon.

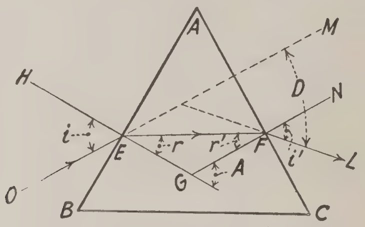


Fig. 376.—Refraction by a prism.

468. Refraction through a Prism.—A portion of a refracting medium bounded by two plane surfaces is called a prism. If the medium of which the prism is composed is optically denser than the surrounding medium, a ray of light incident on one of the faces will be bent toward the normal to that face on entering the prism. On emerging from the opposite face the ray will be going from a denser to a rarer medium and will be bent away from the normal at that face. The angle D (Fig. 376) through which the ray has been deflected in passing through the prism is called the *angle of deviation.* When the angle at which the ray enters one face is equal to the angle at which it leaves the opposite face, the angle of deviation has its least value and is known as the *angle of minimum deviation.*

469. Determination of Index of Refraction of a Prism.—By observing the angle of minimum deviation and the angle between the faces of the prism it is possible to find the index of refraction of the material out of which the prism is made.

Let A = the angle of the prism, (Fig. 376)
D = the angle of minimum deviation,
n = the index of refraction of the material out of which the prism is made.

It can then be shown (see Appendix F) that,

$$n = \frac{\text{Sin } \frac{1}{2}(D + A)}{\text{Sin } \frac{1}{2} A}.$$

Example.—In a glass prism the angle of minimum deviation was found to be 58° and the angle of the prism was 60°. What is the index of refraction of the prism?

$$\text{Index of refraction} = \frac{\text{Sin } \frac{1}{2}(A + D)}{\text{Sin } \frac{1}{2} A}$$

Angle of prism = $A = 60°$
$\text{Sin } \frac{1}{2}A = \text{Sin } \frac{1}{2}(60) = \text{Sin } 30° = 0.5$

$$\text{Sin } \tfrac{1}{2}(A + D) = \text{Sin } \tfrac{1}{2}(60 + 58) = \text{Sin } 59$$

$$n = \frac{\text{Sin } 59}{\text{Sin } 30} = \frac{.8572}{.5} = 1.714 = \text{index of refraction of glass.}$$

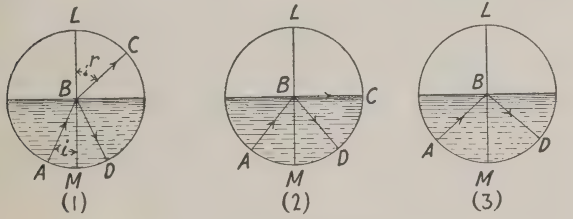


Fig. 377.—Total reflection.

470. Critical Angle.—When a ray of light passes from a dense medium such as water to a rarer medium such as air, it is bent away from the normal so that the angle of refraction is greater than the angle of incidence (Fig. 377). Now if the angle of incidence is made larger and larger, the angle of refraction will also become larger and larger and will always be greater than the corresponding angle of incidence. When the angle of incidence is increased sufficiently, the angle of refraction becomes 90° and the refracted ray travels along the surface of separation between the two media (Fig. 377). **That angle of incidence for which the angle of refraction is 90° is called the critical angle.** In order to have a critical angle, the ray of light must always pass from a more dense medium to a less dense medium. Thus there may be a critical angle when a ray passes from water to air but there can not be a critical angle when the ray passes from air to water.

471. Total Reflection.—If the angle of incidence is made larger than the critical angle as in Fig. 377, light no longer enters the rarer medium since the angle of refraction can not exceed 90°. The ray in this case is reflected back into the medium from which it came. In this reflection the ordinary laws of reflection are obeyed so that the angle of incidence is equal to the angle of reflection. Since none of the light in this case enters the second medium this type of reflection is known as **total reflection.** It always takes place at a surface separating a rarer from a denser medium, when the light comes from the denser to the rarer medium and the angle of incidence exceeds the critical angle.

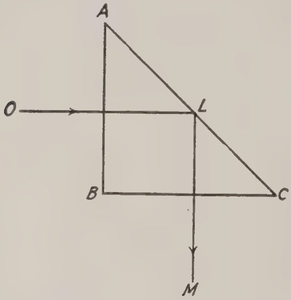


Fig. 378.—Total reflection by a right angled prism.

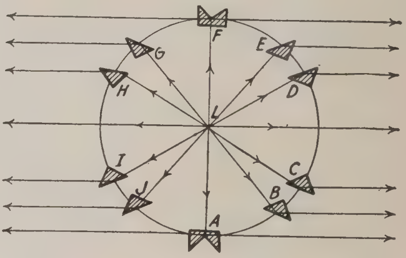


Fig. 379.—Lighthouse reflector.

The prism *ABC* (Fig. 378) produces total reflection of the ray *OL*.

472. The Lighthouse Reflector.—The lighthouse reflector is an application of total reflection. Right angled prisms are placed around the light (Fig. 379) so that they form an enclosed sphere. These prisms are so close together that light does not get out between them. The rays of light coming from the lamp at the center of the sphere strike one leg of the right angled prism, enter the glass, strike the hypotenuse at an angle greater than the critical angle and are then totally reflected in such a way that the rays of light are parallel to each other. The light is reflected without loss because it is a case of total internal reflection.

473. Prismatic Window Glass.—Where it is impossible to get sunlight into a room by means of ordinary windows, prismatic window glass may be used. The light coming almost straight down (Fig. 380) strikes the prismatic window glass in such a way that it is totally reflected into the room. This type of window glass is useful in the illumination of basements below the level of the street.

474. Lenses.—Lenses are bodies made of transparent material and bounded by faces having a cylindrical or spherical form. Although lenses differ much in form they may be divided into two classes according to the way in which they act on a parallel beam of light. Consider the lens in Fig. 381 on which parallel rays are incident. Each ray is bent toward the normal to the surface on entering the lens and away from the normal on emerging from the lens. In this way the rays above the principal axis *PO* are bent downward and those below it are bent upward. After leaving the lens the rays converge to a point *F* called the principal focus. Such a lens is **a converging lens.** When the bounding surfaces of the lens are very convex, the lens converges the rays rapidly. This gives the lens a short focal length. When the bounding surfaces are only slightly convex the lens has a long focal length. In order that all the rays may come to a point after leaving the lens, the beam of light must be restricted to a narrow bundle near the principal axis of the lens. For an extended beam the outer rays will not pass through the same point as the rays near the axis.

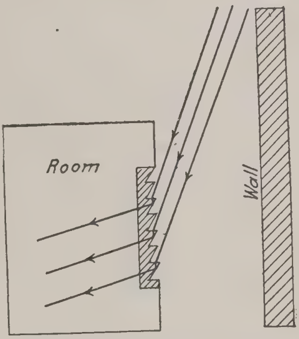


Fig. 380.—Prismatic window glass.

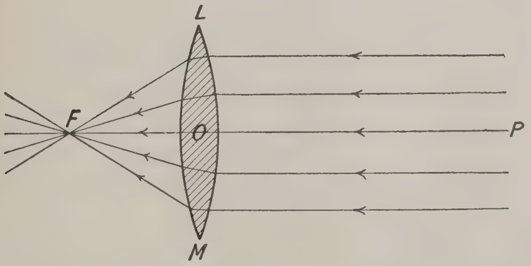


Fig. 381.—Principal focus of a converging lens.

When the surface of the lens is concave instead of convex, the lens makes the rays which pass through it more divergent and for this reason it is known as a **divergent lens.** In Fig. 382 parallel rays are incident on a concave lens. On entering the lens the rays are bent toward the normal as before and on leaving they are bent away from the normal. In this case, however, on account of the way in which the surfaces are made, the emerging rays are bent away from the principal axis. They appear on leaving the lens to come from a point F behind the lens. When the incident rays are parallel, that point from which the rays appear to come on leaving the lens is **the principal focus.** This is only an apparent focus because the light does not really come from it but the effect on the left hand side of the lens is the same as if the light actually came from this point behind the lens. This kind of focus from which the light appears to come is a **virtual focus.** It is to be carefully distinguished from a real focus from which the light actually comes.

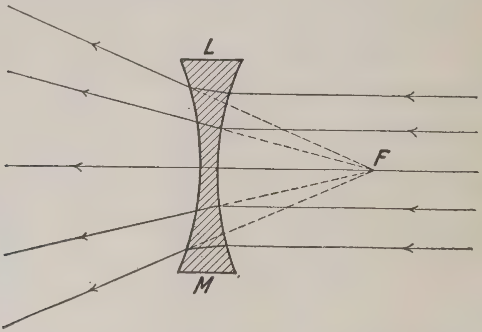


Fig. 382.—Principal focus of a diverging lens.

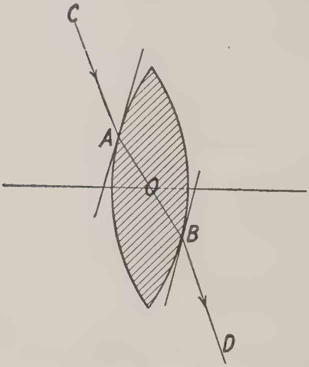


Fig. 383.—Optical center.

475. Optical Center.—For each lens there is some point such that the rays passing through it are not deviated by the lens. For example a ray of light $CABD$ (Fig. 383) emerges from the lens in the same direction as that in which it entered. This is seen by drawing a tangent plane at A and another at B parallel to the one at A. The direction of the ray in the lens is the line joining these two points of tangency. The lens behaves for rays which enter at A and leave at B as a plate of glass with parallel faces. Such a plate of glass (Fig. 373) caused only a displacement of the ray without a change in its direction. **That point where**

an undeviated ray in passing through the lens crosses the axis is known as the optical center of the lens. In a thin lens the displacement of the undeviated ray in passing through the lens is negligible.

476. Graphical Construction of Images.—In order to obtain the position and size of the image formed by a lens, select a point *A* of the object *AB* (Fig. 384). Of the rays going out from this point select two, one which is parallel to the principal axis and the other through the optical center of the lens. The ray parallel to the principal axis will pass through the principal focus of the lens and the other ray will pass through the optical center *O* without deviation. Where these two rays intersect there will be formed an image *A′* of the point *A* from which these rays came. In like manner other points on the object *AB* can be located, giving the image *A′B′*.

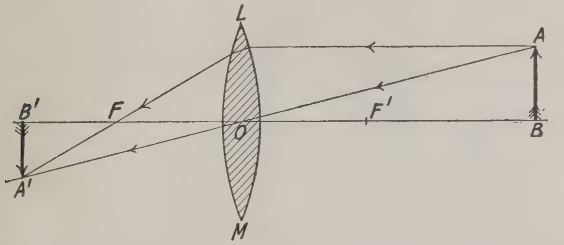


FIG. 384.—Image by convex lens—object removed from lens.

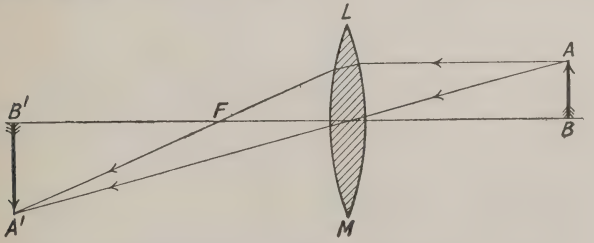


FIG. 385.—Image by convex lens—object near the lens.

In a convex lens three different cases will arise. Suppose the object is some distance from the lens, then the location of the image by this method will give a **real and diminished image** as in Fig. 384. In case the object is near the lens but still farther from it than its focus, the lens will form a **real and magnified image** (Fig. 385). When the object lies between the principal focus and the lens (Fig. 386) the rays of light on leaving the lens seem to diverge from a point A' on the same side of the lens as the object A. There is thus formed at $A'B'$ **a virtual and magnified image** of the object AB.

For the concave lens there is only one possible case, that in which the object lies outside of the principal focus (Fig. 387). The rays from A on leaving the lens diverge as if they had come from a point A' behind the lens. There is formed an image $A'B'$ inside of the principal focus of the lens. This image is **virtual, erect and diminished.** If the object is taken further away from the lens, the image $A'B'$ approaches the focus of the lens. Whatever the position of AB, the image always lies between the focus and the lens.

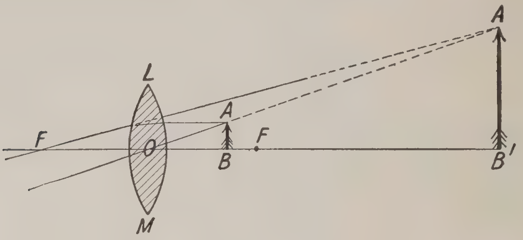


FIG. 386.—Image by convex lens—object inside focus.

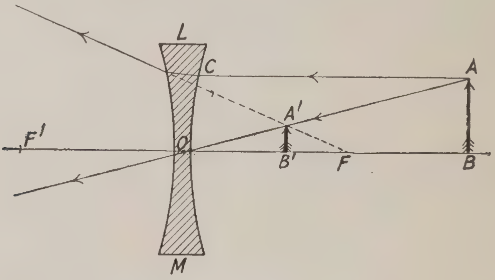


FIG. 387.—Image by concave lens.

477. Magnifying Power of a Lens.—The magnifying power of a lens is the ratio of the linear dimensions of the image to the linear dimensions of the object.

In Fig. 386 by similar triangles,

$$\frac{A'B'}{AB} = \frac{OB'}{OB} = \frac{\text{image distance}}{\text{object distance}}$$

In general,

$$\frac{\text{Length of image}}{\text{Length of object}} = \frac{\text{image distance}}{\text{object distance}}.$$

478. Formulae for Lenses.—The formula connecting the object distance, the image distance and the focal length of a lens is similar in form to the corresponding formula for mirrors. The derivation of this formula is given in Appendix E.

Let U = the distance of the object from the lens,
V = the distance of the image from the lens,
F = the focal length of the lens.

For a converging lens in which the object is farther away from the lens than the principal focus (Figs. 384 and 385),

$$\frac{1}{U} + \frac{1}{V} = \frac{1}{F}$$

If the object is very far from the lens, $U = \infty$ and

$$\frac{1}{\infty} + \frac{1}{V} = \frac{1}{F}$$

$$V = F.$$

Hence the image is at the principal focus for very distant objects. If the object lies at the principal focus,

$$U = F \text{ and } V = \infty$$

so that no image is formed or the rays which leave the lens are parallel to each other.

In the case of a converging lens when the object lies between the principal focus and the lens (Fig. 386), the image lies behind the lens and the image distance is **negative.** This fact must be taken account of by prefixing a **negative** sign to the image distance when the object is inside the principal focus of the lens.

For a diverging lens (Fig. 387) both the image distance and the focal length are **negative** and a **negative** sign must be prefixed to each of them whenever the lens formula is used for a diverging lens.

When the object is far from the lens,

$$U = \infty \text{ and } \frac{1}{V} = \frac{1}{F}$$

$$V = F$$

Hence the image is at the principal focus. Here both V and F are inherently negative.

Problems

1. What is the index of refraction of a substance for which the angle of incidence is 50° and the angle of refraction 35°?

2. If the angle of incidence is 45° and the index of refraction is 4/3, what is the angle of refraction?

3. Find the index of refraction of a medium for which the critical angle was found to be 48°.

4. Find the position and magnification of an image formed by a double convex lens whose focal length is 20 cm. when the object is placed 50 cm. away from the lens.

5. A double convex lens has a focal length of 2 cm. Find the size and position of the image formed by it when the object is 0.5 cm. inside of the focus.

6. A prism is made of glass having an index of refraction 1.65. Its angle of minimum deviation for yellow light is 30°. What is the angle of the prism?

7. A concave lens has a focal length of 14 cm. Find the position and size of the image of an object 1.5 cm. high when the object is 35 cm. from the lens.

8. A microscope is focused on a mark below it. When a plate of glass 1 cm. thick is placed over the mark, the microscope must be raised 0.5 cm. in order to be again in focus. Find the index of refraction of the glass.

9. An object is placed 8 ft. from a lens. The image is formed on the same side of the lens and is located 1.5 ft. from the lens. What is the focal length of the lens?

10. The index of refraction of carbon disulphide is 1.62. What is its critical angle?

11. An object and a screen are 200 cm. apart. Where must a lens having a focal length of 25 cm. be placed to form a clear image on the screen?

12. The focal length of a concave lens is 10 cm. An object 4 cm. in length is placed 20 cm. in front of the lens. Find the position and size of the image.

CHAPTER XXXIX

SPECTRA AND COLOR

479. Separation of Light by a Prism.—When a beam of light passes through a prism it is bent through a certain angle from the direction in which it was originally traveling. This bending arises out of the fact that the velocity of the light in air is not the same as it is in the material out of which the prism is made. If white light, which is composed of a number of colors, passes through a prism, each color will be bent by the prism. Now it happens that lights of different colors do not travel with the same velocity in the glass. Hence these different colors will be bent different amounts on entering and leaving the prism, and although all of the colors come to the prism in the same direction, they will leave it in different directions because of the unequal bending which they suffer. White light in passing through a prism (Fig. 388) is thus spread out into a band of colors known as a **spectrum.** The violet light which has the shortest wave length is bent most and the red which has the longer wave length is bent least. Light with wave lengths between the red and the violet occupies intermediate positions in the spectrum.

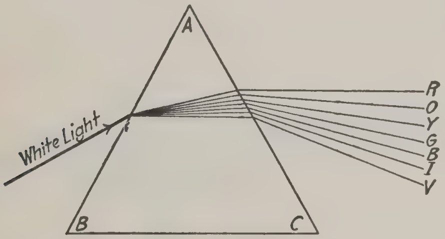


FIG. 388.—Dispersion of light by a prism.

480. The Spectrometer.—For the study of spectra a spectroscope or a spectrometer is used. One of the essential parts of a spectrometer (Fig. 389) is a collimator which consists of a tube with a convex lens *B* at one end and a slit at the other. This slit is located at the principal focus of the lens so that the light which emerges from the lens when the slit is illuminated is parallel light. There is also a telescope with an eye piece *L* and an objective *A*. This telescope is mounted so that it can rotate about the vertical axis of the instrument. The angle through which the telescope is rotated is read on a divided circle. At the center of this divided circle *MN*, is placed a prism *P* which disperses the incident light into a spectrum which is observed through the telescope.

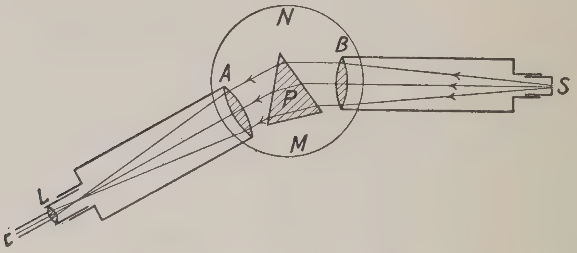


Fig. 389.—Diagram of spectrometer.

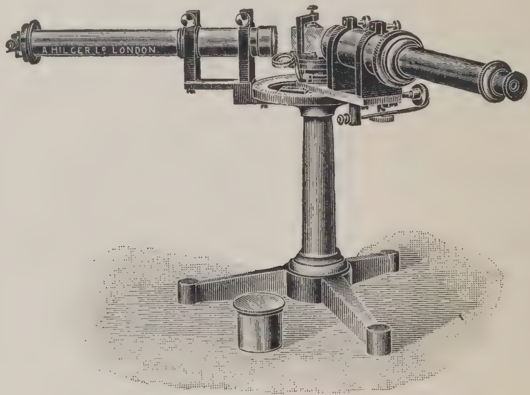


Fig. 390.—Constant deviation spectrometer.

481. Achromatic Prism.—When light of more than one color falls on a prism, the emerging beam is not only deviated but spread out into a variety of colors called a spectrum. If a second prism is placed in front of the first one with its vertex to the base of the first one, the second prism tends to gather the different colors together again and combine them into white light.

The deviation produced by the second prism is in the opposite direction to that produced by the first prism and the net deviation is the difference between the deviation produced by the first prism and the deviation produced by the second prism. In this way it is possible to construct a prism which deviates the light without spreading it out into a spectrum. Such a prism is called an **achromatic prism.**

Such prisms are made by combining a prism of crown glass with a prism of flint glass (Fig. 391). The angles of the two prisms are so chosen that the dispersion produced by the crown glass is just equal and opposite to that produced by the flint glass. This is possible because in prisms of equal angles flint glass produces greater dispersion than crown glass does. When the two prisms are placed as in Fig. 391, the net dispersion is zero. On the other hand the deviation produced by the crown glass is greater than that produced by the flint glass so that there is a net **deviation without any dispersion.** All the rays emerge parallel to each other and combine to form white light. The rays, however, have all been bent from their original direction.

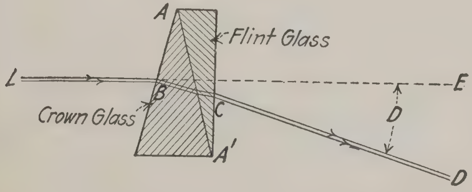


FIG. 391.—Deviation without dispersion.

482. Dispersion without Deviation.—If the angle of the flint glass prism (Fig. 392) is made larger, the deviation which it produces is increased and its dispersion is also increased. When its angle is made large enough to make its deviation equal to that produced by the crown glass prism, the dispersion of the flint glass prism exceeds that produced by the crown glass prism. When two such prisms are put together (Fig. 392) the combination produces **dispersion without deviation.** When white light travels through such a prism it will be split up into a spectrum but its direction will be unchanged.

This principle is made use of in the construction of a **direct vision spectroscope.** Such a spectroscope (Fig. 393) is made up of a number of prisms of crown and flint glass, arranged alternately.

The angles of the prisms are so chosen that the deviation of the combination is zero, allowing the light to emerge in the direction in which it entered. Since the dispersion of the flint glass exceeds the dispersion of the crown glass, the light is spread out into a spectrum. This type of spectroscope is very convenient in spectrum analysis. Fig. 394 shows one type of direct vision spectroscope.

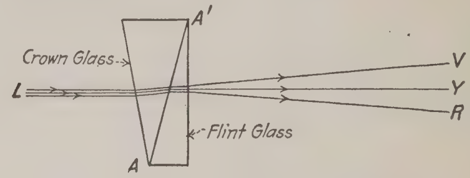


FIG. 392.—Dispersion without deviation.

483. Achromatic Lenses.—When white light passes through an ordinary lens it is not only refracted and brought to a focus as in Fig. 395 but it is separated into colors just as in the prism. Thus the red rays and the violet rays are bent unequal amounts by the lens and will not therefore come to the same focus. There will be a focus for the red rays, another for the violet rays and a focus for each of the other colors present in the light. Since the violet rays are bent more than the red rays, the focus for the violet light will be nearer the lens than the focus for the red light. Such a lens can not give a sharp image of an object illuminated

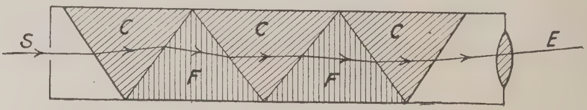


FIG. 393.—Direct vision spectroscope.

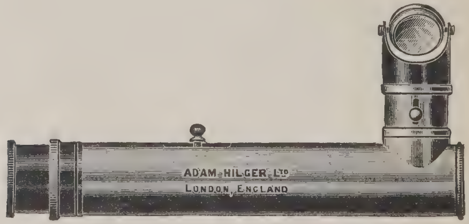


FIG. 394.—Direct vision spectroscope.

by white light. Because of the unequal bending of the different colors, the image will not be white but will be more or less colored. This defect in a lens is called **chromatic aberration.**

It may be corrected by combining a convex lens of crown glass with a concave lens of flint glass (Fig. 396). These lenses are so chosen that the power of one to separate the light is just equal to the power of the other to recombine it. In this way no separation of the light into colors occurs. On the other hand the bending or refractive power of one lens exceeds the refractive power of the other so there is a resulting bending of the rays, in spite of the fact that one of the lenses bends the rays in one direction and the other bends them in the opposite direction. Of course the bending power is not as much as it would be for a single lens by itself. Such lenses which give bending without dispersion are called **achromatic lenses.**

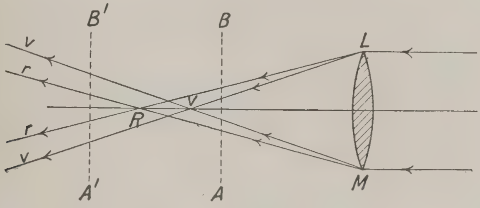
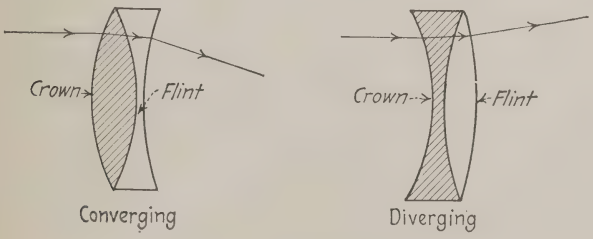


FIG. 395.—Chromatic aberration.



Converging

Diverging

FIG. 396.—Achromatic lens.

484. Bright Line Spectra.—When light from a luminous gas or vapor is examined by means of a spectroscope, the spectrum is found to be made up of bright colored lines separated by dark regions. These colored lines are just images of the slit from which the light comes. These lines are always the same for the same gas or vapor. If common salt is introduced into a colorless Bunsen flame and the flame examined with a spectroscope, a single yellow image of the slit is seen. If the power of the spectroscope to disperse the light is large and the slit narrow, two very fine lines close together are seen. The spectrum of lithium under corresponding conditions consists of a single line in the deep red. The spectrum of iron when burned in an electric arc consists of a large number of fine lines scattered throughout the visible spectrum. When an electric discharge is passed through hydrogen at a suitable pressure it gives a spectrum consisting of a bright line in the red, one in the blue and two faint ones in the violet.

Such spectra are known as **bright line spectra.** Each substance has its own characteristic spectrum which is not exactly like that of any other substance. By observing the spectrum of a substance it is possible to identify it.

485. Spectrum Analysis.—Since each substance has a spectrum which depends entirely on the nature of the substance, an examination of the light emitted by a substance gives direct evidence of the composition of the substance. When the bright line spectrum of a substance is definitely known, it is possible to conclude that the substance is present whenever its spectrum is found. Thus suppose that the spectrum of calcium is known and that there is now introduced into the colorless flame of a Bunsen burner an unknown salt. If the spectrum of calcium is now found in the spectroscope, it follows at once that calcium was present in the salt. This method is very useful in detecting small quantities of a substance. It is a very rapid and a very sensitive method. The least trace of sodium in a Bunsen flame is sufficient to give the sodium lines. Because of its great sensitiveness this method of analysis has made possible the discovery of a number of elements which were present in such small quantities that they could not be detected by other methods. Caesium, rubidium, thallium, indium and gallium were discovered in this way.

486. Continuous Spectra.—When light from an incandescent solid or liquid is examined, it is found to contain all colors from red to violet. It shows no regions of darkness such as were found in the line spectrum. Spectra from the filament of an incandescent lamp or from an arc light is a spectrum of this kind. Such spectra are known as **continuous spectra** to distinguish them from line spectra. They contain light of every wave length and therefore light of every color.

487. Ultra-violet and Infra-red Spectra.—Beside the visible part of the spectrum which affects the retina of the eye, there are wave lengths which are too short and others which are too long to affect the eye. The former is known as ultra-violet light and the latter as infra-red radiation. The ultra-violet is most easily detected by its action on a photographic plate. It also produces a number of chemical effects. The infra-red or heat rays are studied by allowing them to be absorbed and noting the heat effects which they produce.

488. Dark Line Spectra. Absorption Spectra.—Absorption spectra are produced when light from an incandescent solid or liquid passes through a layer of some medium which allows some of the colors to pass through but absorbs others. Such a medium is transparent to light in different degrees according to the color or wave length of the light. If the light which has passed through such a medium is examined, it is found to contain dark regions or bands, showing that in these regions the light was not transmitted. The light energy was absorbed by the medium in which it was traveling and became heat energy which went to increase the temperature of the medium. The remainder of the light was transmitted.

When white light falls on a piece of cobalt glass all the colors except a small amount of the red and blue are absorbed. Chlorophyl which is so important for plant growth, shows a very dark absorption line in the red indicating that wave lengths in that region have been absorbed by the chlorophyl. In case the absorbing substance is a gas, the absorption lines are sharply defined and correspond to the bright lines which the gas emits when it is luminous. This type of absorption is sometimes referred to as a reversal of the spectrum because the bright lines of the emission spectrum correspond to the dark lines of the absorption spectrum.

489. Fraunhofer Lines.—An examination of the spectrum of the sun shows that it is crossed by a large number of fine dark lines. These lines are absorption lines of unusual interest. They are known as **Fraunhofer lines** from their discoverer. These lines are produced by absorption in the atmosphere of the sun. The luminous part of the sun consisting of incandescent solids or liquids emits white light. Surrounding this central part is an atmosphere made up of vapors and gases which come from the central part. These vapors and gases constituting the atmosphere of the sun are at a much lower temperature than its central part from which the light comes. These vapors came originally from the substances of which the sun is composed. When light comes from the sun and passes through this atmosphere, the part corresponding to the light which would be emitted by these luminous vapors is absorbed by the vapors. Thus if sodium, iron and copper are present in the sun, vapors of these elements are present in its atmosphere. White light passing through the atmosphere containing these vapors will loose those wave lengths or colors which sodium, iron and copper emit when luminous. These wave lengths will therefore be missing when the solar spectrum is examined. In this way it has been possible to get a good idea of the elements which compose the sun and it has been found that there are present in the sun the same elements which are present on the earth. Indeed one element, helium, was discovered on the sun by this method before it was found on the earth.

490. Interference.—Thomas Young showed that when two trains of waves having the same wave length and amplitude of vibration and traveling in the same direction are superposed, they do not always produce increased illumination but may neutralize each other. The elements of this phenomenon known as **interference** are best understood from the original experiment of Young.

Behind a screen containing a small pin hole S (Fig. 397) is placed a source of light. A second screen having two small holes A and B is placed in front of the first screen in such a way that the openings A and B are equally illuminated by the light from S. If light is allowed to pass through either A or B there is produced a bright spot on a third screen at D. If, however, light passes through both holes A and B at the same time, there is formed on the third screen a series of bright and dark bands. To see the meaning of this fact, consider in detail Fig. 397. Let DM be a perpendicular to AB at a point M midway between A and B. Since D is equidistant from A and B, light leaving A and B at the same time will reach D at the same time and be in phase. The crests of the waves will coincide and therefore reinforce each other. The illumination at this point will be a maximum. If some point E above or below D is chosen, the light from A in reaching E will travel the distance AE and the light from B will travel the distance BE. Since BE is greater than AE, the light from B will arrive later than the light from A. If E is chosen far enough from D that BC, the difference between BE and AE, is equal to one-half wave length of light, then the light from A will be ahead of the light from B by one-half wave length or one-half period of vibration. When the waves coming from A have a maximum, those coming from B will have a minimum. The two waves will thus neutralize each other and so produce no light at E. If E is chosen still further away from D so that BC is equal to one whole wave length of light, the light from both sources A and B on reaching the screen will again be in phase, and so produce an illumination of the screen at that point. By choosing points still further away from D alternate dark and light regions will be found according to whether BC is equal to an odd number or an even number of half wave lengths of light.

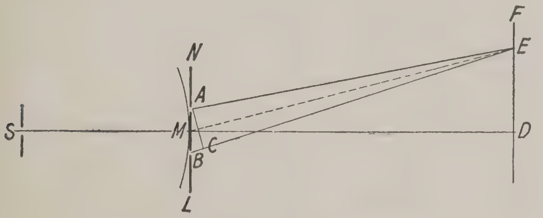


FIG. 397.—Interference experiment.

When

$$\begin{aligned} BE - AE = BC &= \tfrac{1}{2}\lambda \\ &= \tfrac{3}{2}\lambda \\ &= \tfrac{5}{2}\lambda \end{aligned}$$

the screen is dark.

When

$$\begin{aligned} BE - AE = BC &= \lambda, \\ &= 2\lambda, \\ &= 3\lambda, \text{etc.} \end{aligned}$$

the screen is illuminated.

If therefore the pin hole S be illuminated with sodium light, the screen DF which is normal to the plane of the paper will be covered by a series of alternate yellow and black fringes. These fringes start with D which is yellow and extend above and below the point D. If white light is used the central fringe at D will be white and on either side of it there will be a set of rainbow colored fringes. These colors arise out of the fact that the different colors in the white light have different wave lengths and they reinforce or destroy each other at different points on the screen. In these rainbow fringes the edges nearest *D* are violet and the edges furthest from *D* are red. This distribution of color is due to the fact that the wave length of violet light is less than that of red light so that the violet rays reinforce or destroy each other before the red rays do.

Because of the importance of this experiment in showing that light is a wave motion, many modifications of it have been devised. These were designed to remove any criticism of the experiment and make the reality of the interference phenomenon as certain as possible. The simplest of these modifications is one in which Fresnel used a biprism having a very obtuse angle in place of the screen containing the two pin holes. By means of this prism he avoided any disturbance of the light which might arise when it passed through the small apertures *A* and *B*. He also replaced the biprism by two mirrors inclined at a small angle to each other. The results were precisely like those observed by Young and the reality of the interference of light was established.

491. Diffraction Grating.—If a series of very fine equidistant, parallel slits be ruled on a plate of glass with a fine diamond point, we have what is known as a **diffraction grating.** Where the diamond point has made a furrow on the glass the light can not pass through. In between the furrows where the surface of the glass has been undisturbed, the glass is still transparent and light can pass through. The plate of glass is then somewhat like a picket fence. There is a strip through which the light can pass followed by a strip through which it can not pass. The furrows are very close together whereas the pickets are far apart.

Let *AB* (Fig. 398) represent such a grating on which is falling parallel light so that the direction of the rays is perpendicular to the plane of the grating. Through the slits in the grating will come beams of parallel light. If these rays coming in the direction perpendicular to the plane of the grating are brought to a focus on the screen *PS* by the lens *ML*, there will be formed a bright line at *K*. If the light is viewed in a direction making an angle x with the normal to the grating, parallel rays of light emerging from the slits in this direction will travel unequal distances to reach the screen and when they are brought to a focus at *N* by the lens *LM*, they may either reinforce or destroy each other. If the angle x is made such that a ray of light from one slit is one-half wave length behind the corresponding ray from the neighboring slit, then the rays from one slit will be out of phase with the rays from the neighboring slit and will destroy each other. Hence at N there will be darkness because the rays interfere in such a way that they neutralize each other. If the angle between the normal to the grating and the ray is increased, the rays from the lower slits will be more and more behind the rays from the upper slits. When the difference in path between corresponding rays from neighboring slits amounts to one wave length of light, the light from the slits will again be in phase and reinforce. When these rays are focused by the lens LM, another bright image of the slit will be formed on the screen.

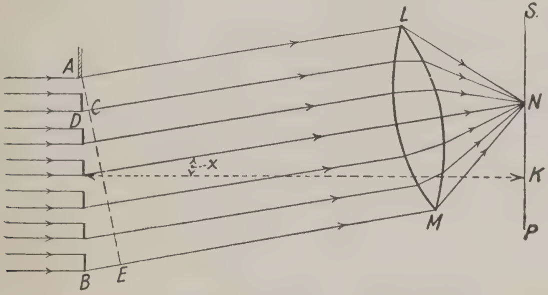


Fig. 398.—Diffraction grating.

From the geometry of the Fig. 398 it is seen that

$$DC = AD \sin x$$

When

$$\begin{aligned} DC = AD \sin x &= \lambda/2, \\ &= 3\lambda/2, \\ &= 5\lambda/2, \text{ etc.} \end{aligned}$$

the rays from different slits destroy each other, and the screen is dark.

When

$$\begin{aligned} DC = AD \sin x &= \lambda, \\ &= 2\lambda, \\ &= 3\lambda, \text{ etc.} \end{aligned}$$

the rays from different slits reinforce each other and the screen is illuminated.

The angle x though which the telescope of the spectrometer must be turned before reinforcement is again obtained after leaving the central band K can be easily measured on the divided circle of a spectrometer. The number of rulings per centimeter of the grating is given by the maker. From this the distance s between the slits on the grating can be computed. For the first reinforcement,

$$\lambda = s \text{ Sin } x.$$

Since all the quantities in this equation are known except λ, the wave length of the light can be calculated. This is a very accurate method of measuring the wave length of light.

CHAPTER XL

OPTICAL INSTRUMENTS

492. The Photographic Camera.—The simplest application of lenses for optical purposes is in an ordinary photographic camera (Fig. 399). A lens or combination of lenses in one end of the camera produces a real image of an external object on the photographic plate or film at the other end of the camera. The distance between the lens and the photographic plate or film can be altered so as to focus the image on the plate. Sometimes the photographic plate is replaced by a ground glass screen on which the image is cast and then the lens is moved into such a position that the image is sharply focused on the ground glass.

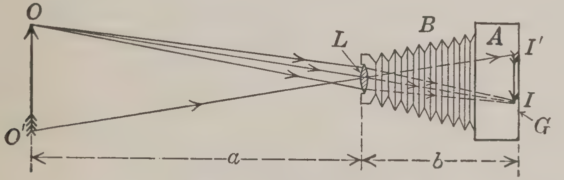


FIG. 399.—Photographic camera.

The plate or film is then substituted for the ground glass plate. The bellows *B* serves to exclude all the light except that which comes from the object. It also makes possible the to and fro motion of the lens necessary in the focusing.

To avoid spherical aberration, that is, the distortion of the image because all the rays do not come to the same focus, and to limit the quantity of light, a diaphragm is placed in front of the lens. This diaphragm restricts the rays to those which pass through the central portion of the lens. The smaller the diameter of the hole in the diaphragm, the better is the definition. On the other hand the smaller this opening, the less the intensity of the light forming the image on the plate. Where it is desired to have the image as bright as possible, the choice of the size of the opening in the diaphragm becomes a compromise between diminished brightness on the one hand and definition on the other hand.

A photographic plate consists of a silver compound spread on a glass plate or celluloid film. The image formed by the lens acts on this sensitive film but the image is not visible until the plate or film is placed in a mixture of chemicals, known as the developer.

Example.—The focal length of the lens in a camera is 20 cm. How far back must the plate be placed to take an object which is 10 m. away? If the object is 1 m. high, how high is the image?

$$1/u + 1/v = 1/f$$

$$f = 20 \text{ cm.}$$

$$u = 10 \text{ m.}$$

$$\frac{1}{1{,}000} + \frac{1}{v} = \frac{1}{20}$$

$$v = \frac{20{,}000}{980} = 20.4 \text{ cm.}$$

$$\text{Magnification} = \frac{\text{length of image}}{\text{length of object}} = \frac{u}{v} = \frac{20.4}{1{,}000} = 0.0204$$

$$\frac{\text{Length of image}}{100} = 0.0204$$

$$\text{Length of image} = 2.04 \text{ cm.}$$

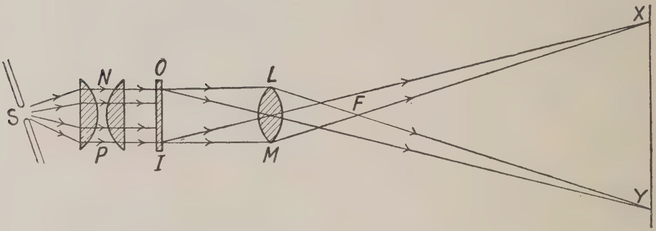


Fig. 400.—Projection lantern.

493. Projection Lantern.—The projection lantern, which is used to throw an image of a brilliantly illuminated object on a screen, consists of a powerful source of light *S* (Fig. 400), a large condensing lens *NP* and a front or objective lens *LM*. The condensing lens *NP* collects the light from the source *S* and sends it through the slide *OI* so that this slide is brilliantly illuminated. The objective *LM* then produces a real image of the slide *OI* on the screen *XY*. Since the slide *OI* is near the principal focus of the lens *LM*, the image on the screen is enlarged. This image is also real and inverted. The magnification produced by the lantern is obtained from an application of law for the magnification of a lens.

Let $u =$ the distance of the slide from the lens LM.
$v =$ the distance between the screen XY and the lens LM.

$$\text{Magnification} = \frac{v}{u}$$

If the distance from the slide to the screen and the magnification to be produced by the lantern are given, it is possible to find the focal length of the lens LM which must be used to produce an image on the screen. It is only necessary to apply the lens equation,

$$1/u + 1/v = 1/f,$$

together with the fact that the distance from the slide to the screen is $u + v$.

Example.—Find the focal length of a lens which must be used in a lantern to produce the image of a slide 8 cm. square upon a screen 3 m. square at a distance of 10 m. from the lantern. Assume the image occupies the entire screen.

$$\text{Magnification} = \frac{v}{u} = \frac{300}{8}$$

Since $v = 1{,}000$ cm. nearly

$$\frac{v}{u} = \frac{1{,}000}{u} = \frac{300}{8} \text{ and } u = 26.6$$

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f};$$

$$\frac{1}{26.6} + \frac{1}{1{,}000} = \frac{1}{f}; \qquad f = 25.9 \text{ cm.}$$

494. The Eye.—Like the camera the eye can be looked upon as a light-tight enclosure having a lens at one end and a sensitive film made of nerve fibers at the other end. The lens produces on this sensitive screen a real and inverted image of external objects. The eye is provided with means of adjustment by which it is possible for it to produce distinct images in the proper position for distinct vision. The thick tough outer coating is kept distended and in a somewhat spherical form by the transparent vitreous humor. The iris is a diaphragm corresponding to the stop of a photographic camera. In the center of this diaphragm is the circular opening known as the pupil. At the back of the eye are the terminal nerves of the optic nerve. These form the retina which serves as the screen to receive the images formed by the lens.

With a fixed lens system, only objects at a definite distance from the eye could be sharply focused on the retina. For any other distance the image would be blurred. In the camera and projection lantern the necessary focusing is done by moving the lens forward or backward. In the eye instead of altering the position of the lens the lens has been provided with muscles by which its convexity can be changed in such a way that the desired focus is obtained. In this way the eye adapts itself to different conditions not by varying the position of the lens with respect to the retina but by changing the focal length of the lens.

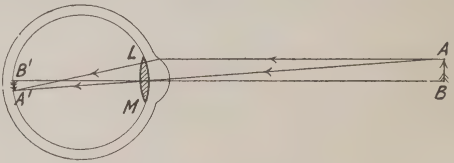


FIG. 401.—Normal eye.

When the eye receives light from nearby objects, the muscles contract in such a way as to make the lens more convex. When distant objects are viewed, the muscles relax, making the crystalline lens less convex and thus allowing the image to fall on the retina as before.

495. Defects of Vision.—The lens of an eye may have too short a focal length or the eyeball may be too long so that the image formed by the lens falls in front of the retina. Such a condition is represented in Fig. 402. The eye in this case converges the light too rapidly so that the image is formed too soon. If the object is brought closer to the eye this causes the image to move back toward the retina and it may be thus made to fall in its proper place. In this way near-sighted persons find it possible to see distinctly. To correct for this defect, a concave lens (Fig. 403) is placed in front of the eye. By choosing a concave lens of suitable focal length the image is made to fall on the retina when the object is at the normal distance from the eye.

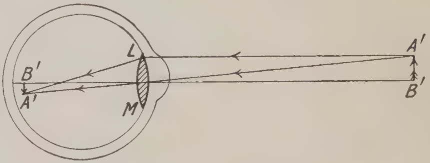


FIG. 402.—Near-sighted eye.

In the farsighted eye the image from an object at a normal distance from the eye (Fig. 404) is formed behind the retina because the crystalline lens is too flat or the eye ball is too short. As the object is moved further away the image moves nearer to the retina and may be made to fall on it. Hence distant objects can be seen when nearby objects are indistinct. In order to correct for this defect, a convex lens (Fig. 405) is placed in front of the eye. It makes the equivalent focal length of the eye shorter and thus hastens the formation of the image. By choosing a lens of suitable convexity, the image can be brought on the retina of the eye for objects at a normal distance from the eye.

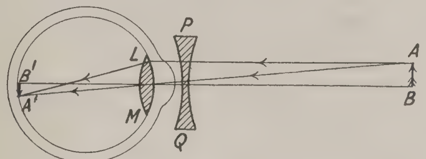


Fig. 403.—Correction for near-sighted eye.

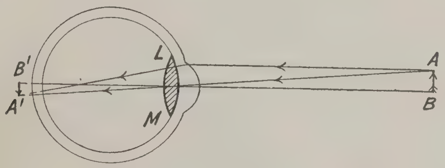


Fig. 404.—Far-sighted eye.

The distance at which the normal eye sees most distinctly is called the **distance of distinct vision.** This distance is about 25 cm. For greater or smaller distances there is an excessive strain on the eye.

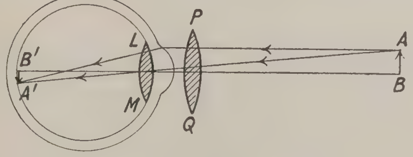


Fig. 405.—Correction for far-sighted eye.

496. Astigmatism.—Another common defect of the eye is known as **astigmatism.** This defect occurs when the lens of the eye does not have a truly spherical surface. In such a case the curvature of the crystalline lens is not the same in different planes through the axis of the eye. The focal length of the lens will not be the same in different planes. Rays of light from a vertical luminous wire will not have the same focus in the eye as rays from a horizontal luminous wire. If light from a luminous area is received by the eye, there will be an attempt to adjust the eye so that the light from all parts of the object will form a sharp image. With such an eye it is not possible to make such an adjustment because the lens has different focal lengths for light in different planes. Consequently the image formed will be indistinct and the eye will be strained in forming this image.

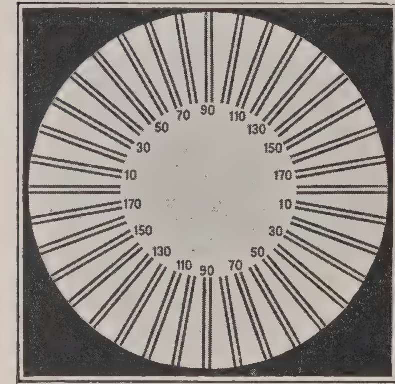


FIG. 406.—Chart viewed by normal eye.

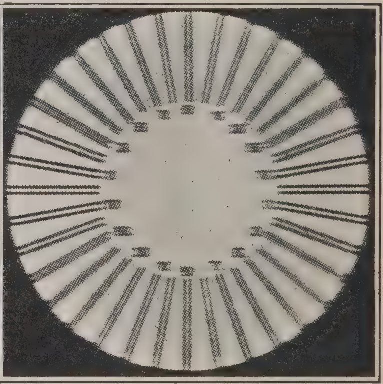


FIG. 407.—Chart viewed by astigmatic eye.

This defect is overcome by using lenses which have cylindrical surfaces. If these lenses are properly placed with respect to the eye, they combine with the lens in the eye to make a nearly normal lens.

497. Simple Microscope.—When an object is placed slightly nearer to a converging lens than its focus, an eye brought up to the lens sees a virtual, erect and magnified image $A'B'$ (Fig. 408). A lens used in this way constitutes a simple microscope. In order to obtain the greatest advantage the eye should be as near as possible to the lens. In this way the field of view is made as large as possible and the distance of the virtual image $A'B'$ from the eye is made as small as possible.

In order to find the magnification of a lens used in this way it is necessary to consider that the apparent size of an object is determined by the angle which it subtends at the eye. Now for distinct vision an object must be about 10 in. from a normal eye. If an object is placed at a greater distance than this, the image on the retina is smaller and its details are not seen so distinctly. When the object is placed nearer than 10 in., the image on the retina is blurred. When an object is examined with the aid of a magnifying glass, the object is brought nearer to the eye than would be possible for distinct vision without the magnifying glass. In Fig. 408 the angles subtended by the object *AB* and the image *A′B′* are the same. But *OD* is the distance of distinct vision and if the lens were absent the eye could not see *AB* distinctly until it was removed to *ab*. Hence by using the lens the visual angle has been increased from α to β and consequently the magnification is,

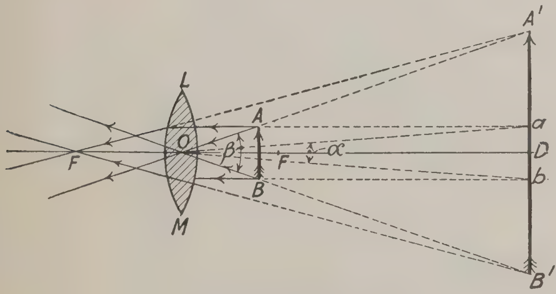


FIG. 408.—Simple microscope.

$$\frac{A'B'}{AB} = v/u$$

For distinct vision $v = 25$ cms.

$$\frac{A'B'}{AB} = \frac{25}{u}$$

Since $v = -25$ and

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

$$\frac{1}{u} = \frac{1}{f} + \frac{1}{25}$$

$$\frac{25}{u} = \frac{25}{f} + 1$$

Hence

$$\frac{A'B'}{AB} = 1 + 25/f$$

Example.—Find the greatest magnification that can be produced by a converging lens of focal length 2.5 cm.

$$\text{Magnification} = \frac{A'B'}{AB} = u/v = 1 + \frac{25}{f} = 1 + \frac{25}{2.5} = 11$$

498. Compound Microscope.—In order to obtain more powerful magnification, a compound microscope is used. It consists of a converging lens *LM* of short focal length (Fig. 409). This lens produces a real and magnified image of a small object placed just outside of its focus. This lens is called **the objective.** The image *A′B′* produced by it is viewed through the eye piece *PN* which acts like a simple magnifying glass. The eye piece produces an enlarged and virtual image *A″B″* of the real image *A′B′*. The real image *A′B′* of the object *AB* falls just inside the focus of the eye piece. The lens *PN* then acts on this real and inverted image to still further magnify it.

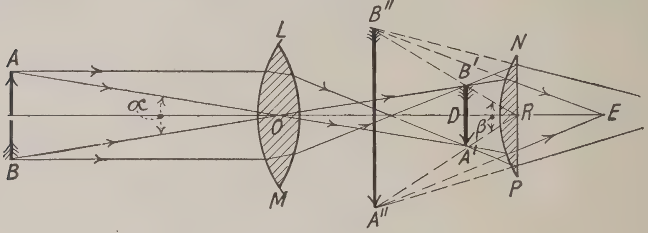


FIG. 409.—Diagram of compound microscope.

The magnification of a compound microscope depends on the focal length of both the objective and eyepiece. If the objective has a focal length of 5 mm. the real image may be at a distance of 20 cm. from the objective. Since the object is near the focus of the objective, the object distance may be taken as nearly equal to 5 mm. The magnification produced by this lens is then

$$u/v = \frac{20}{0.5} = 40$$

If now the eye piece has a focal length of 2.5 cm. it will magnify the real image *A′B′* into the image *A″B″* in accordance with the relation

$$\frac{A''B''}{A'B'} = 1 + \frac{25}{f} = 11$$

$$A'\,B' = 40 \times AB$$

$$A''B'' = 40 \times AB \times 11 = 440AB$$

$$\frac{A''B''}{AB} = 440$$

The approximate magnification may be written down as follows:

Let $L =$ the length of the microscope tube,
$F =$ the focal length of the objective,
$f =$ the focal length of the eye piece.

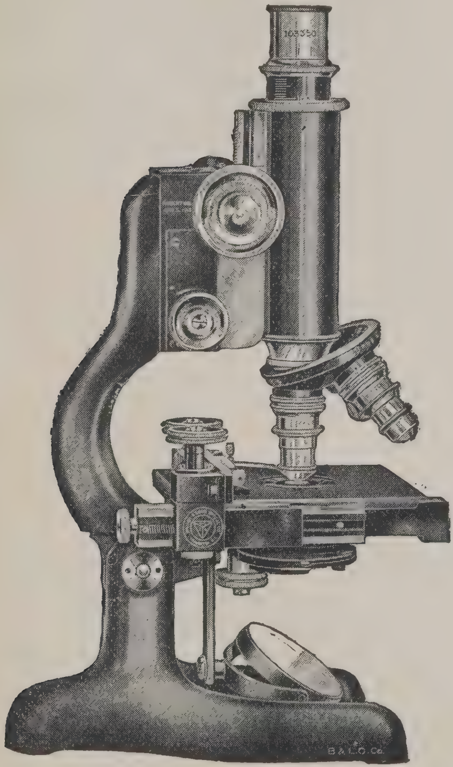


FIG. 410.—Compound microscope.

$$\frac{A'B'}{AB} = \frac{OB'}{OB} = \frac{L}{F} \text{ approximately.}$$

$$\frac{A''B''}{A'B'} = 1 + \frac{25}{f}$$

$$\frac{A''B''}{AB} = \frac{L}{F}\left(1 + \frac{25}{f}\right) = \frac{25 \times L}{F \times f} \text{ approximately.}$$

Example.—The focal length of the objective of a microscope is 0.5 cm., the distance between the objective and the eye piece is 20 cm., and the focal length of the eye piece is 1 cm. Find the magnification of the microscope.

$$\text{Magnification} = \frac{L}{F}\left(1 + \frac{25}{f}\right) = \frac{20}{0.5}\left(1 + \frac{25}{1}\right) = \frac{20}{0.5} \times 26 = 1{,}040$$

499. Astronomical Telescope.—The principle of the astronomical telescope is the same as that of the compound microscope. The objective lens *LM* is modified to meet the fact that the instrument is used to view distant objects instead of nearby objects. This objective which is a converging lens of long focal length, forms a real, inverted image *A′B′* of a distant object *AB*. This real image is formed just inside the focus of the eye piece *L′M′* by which it is viewed. By means of this eye piece an enlarged virtual image of the real image *A′B′* is produced at *A″B″*.

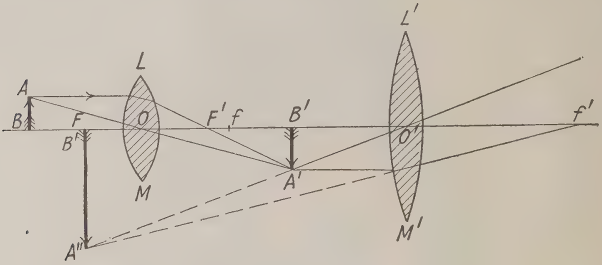


FIG. 411.—Astronomical telescope.

On account of the great distance of the object from the telescope the rays from any point of it are essentially parallel when they reach the objective. Hence the image formed by the objective lies just outside the principal focus of the objective.

The magnification of the telescope is determined by the angle which the object subtends or appears to subtend at the eye (Fig. 411). The angle which the object *AB* subtends at the eye when no lenses are present is $\angle AOB = \angle B'OA'$. The angle which the image *A″B″* subtends at the eye is $\angle B''O'A'' = A'O'B'$.

$$\text{Magnification} = \frac{\angle A'O'B'}{\angle A'OB'} = \frac{\dfrac{A'B'}{O'B'}}{\dfrac{A'B'}{OB'}} = \frac{OB'}{O'B'}$$

$$= \frac{\text{focal length of objective}}{\text{focal length of eyepiece}}$$

$$= \frac{F}{f}$$

Hence the magnification of an astronomical telescope is found approximately by dividing the focal length of the objective by the focal length of the eye piece. The image formed by such a telescope is always inverted. For terrestial uses of the telescope it is desirable that the image be erect. This is realized by inserting a third convex lens between the objective and the eye piece in such a way that the image *A′B′* formed by the objective is again inverted before it is viewed by the eye piece.

Example.—The focal length of the objective of a telescope is 150 cm. and the focal length of the eye piece is 2 cm. Find the magnifying power of the telescope for distant objects.

$$\text{Magnifying power} = \frac{\text{focal length of objective}}{\text{focal length of eye piece}} = \frac{150}{2} = 75$$

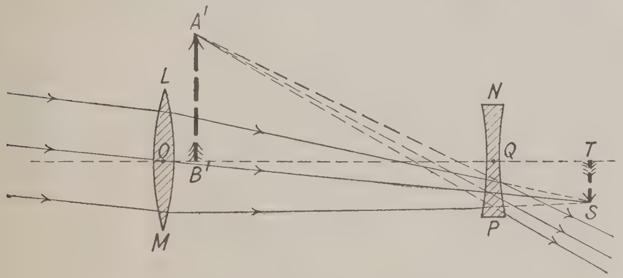


FIG. 412.—Opera or field glasses.

500. Opera or Field Glasses.—The opera glass like the telescope consists of an objective *LM* which converges the rays toward a point *S* (Fig. 412) but before they reach this point they pass through the diverging lens *NP* which replaces the eye piece of the telescope. In passing through this diverging lens the rays which were converging on entering it are made to diverge on leaving it. To an eye on the right hand side of the lens *NP* (Fig. 412) the rays seem to have come from a point *A′* behind the concave lens *NP* which thus forms the virtual and erect image *A′B′* of the object from which light was received. Unlike the astronomical telescope, the opera glass gives an erect image but its magnifying power is not so great as that of the telescope.

To have the opera glass in focus, the lens *NP* must be so placed with respect to the objective that the rays emerging from the lens *NP* are nearly parallel. The magnification is,

$$\text{Magnification} = \frac{\text{focal length of objective}}{\text{focal length of eye piece}}.$$

501. Prism Binoculars.—Field glasses of higher magnifying power and larger field of view are made of the optical parts represented in Fig. 413. The beam of light *OB* from the objective is reflected internally at *B* and *C* by a right-angled prism. In this way its direction is reversed and it travels back to a second right angled prism which is placed at right angles to the first one. Here it is again twice reflected internally, at *D* and *E* and then passes through the eye piece. The reflections at *B* and *C* interchange the two sides of the image so that it is no longer perverted like the image in an ordinary plane mirror. Reflection by the second prism at *D* and *E* makes the image upright. Hence the image formed by the object after the reflections by the two right-angled prisms is restored completely to its natural position. The eyepiece then magnifies this image without inversion. Objects viewed with such an opera glass thus appear in their natural position.

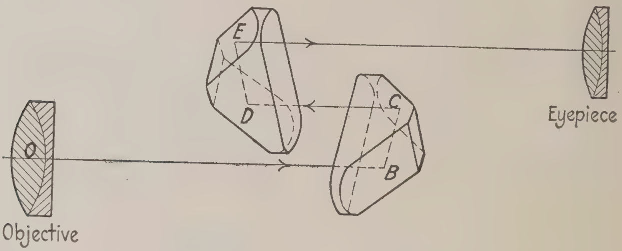


Fig. 413.—Prism binoculars.

Because the ray passes three times from the objective to the eyepiece, the focal length of the objective can be three times as great as it could be in the ordinary field glass and the magnifying power of the instrument is correspondingly increased.

502. Sextant.—The sextant is an instrument for measuring the altitude of heavenly bodies. It consists of a fixed mirror *D* (Fig. 414) which is only silvered on one-half of its surface so that an observer looking through the telescope *T* can see the horizon by means of light traveling in the direction *CDT* and passing through the unsilvered part of the mirror at *D*. The second mirror *E* is mounted on an arm which rotates about *E* as an axis. It carries a vernier at *F* by means of which its position can be read on the graduated circular scale. Light from a distant object after reflection at *E* is again reflected from the silvered part of D and passes into the telescope T. The eye observing through the telescope thus receives light simultaneously from two sources, that which has come in the direction CDT and that which has come in the direction $AEDT$. When the mirrors E and D are parallel to each other the vernier reads zero and the two sources of light lie in the same straight line.

To find the position of a star, the mirror E is rotated by means of the arm on which it is mounted until the light from the star after reflection at E and D passes down the axis of the telescope T forming an image of the star which seems to rest on the horizon. At the same time light is received in the telescope along the line CDT. This light comes from a reflecting surface which serves as an artificial horizon. By observing the angle through which it was necessary to rotate the mirror E from the position in which it was parallel to the mirror D to the position at which it reflects light from the star down the axis of the telescope while the axis of the telescope points in the direction of the horizon, the angle of elevation of the star above the horizon is at once obtained. The angle through which the mirror E is rotated is one-half the angle of elevation of the star above the horizon. The scale of the instrument can be so numbered that it reads the elevation of the star directly. To accomplish this each degree of rotation of the arm is marked on the fixed scale as 2° of rotation.

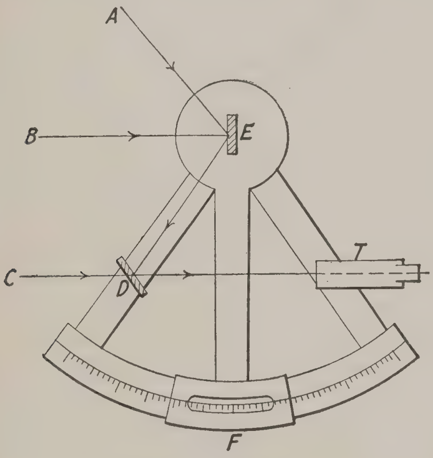


FIG. 414.—Sextant.

CHAPTER XLI

POLARIZATION AND SACCHARIMETRY

503. Polarization.—There are various phenomena which show that light is a wave motion in the ether. It is now necessary to ask whether these waves are longitudinal like sound waves or are transverse like waves in a stretched string. In the former case the displacement is in the direction in which the wave is traveling and in the latter case it is perpendicular to this direction. There are certain phenomena which offer conclusive proof that light waves are transverse waves, that is, that the vibrations which constitute light waves take place at right angles to the direction in which the wave is traveling. The following experiment with two crystals of tourmaline shows the nature of this evidence most simply.

When a crystal of tourmaline is cut parallel to the crystallographic axis and a ray of light passes through it, so far as the unaided eye can detect, the transmitted beam in no way differs from the incident beam. If the light which has passed through one tourmaline crystal is allowed to pass through another with its axis parallel to the first, the light will be almost completely transmitted by the second crystal. If now the second crystal is rotated around the ray of light as an axis so that the axes of the two crystals are inclined to each other, the intensity of the transmitted light will decrease and when the axes of the crystals are at right angles to each other, none of the light from the first crystal will pass through the second. If the rotation of the second crystal is continued until the axes of the crystals are again parallel, the light from the first crystal will be transmitted through the second. It is evident that the light in passing through the first crystal has acquired properties which ordinary light does not possess.

To see the meaning of this experiment consider a stretched string in which the particles are vibrating in a plane perpendicular to the length of the string. If a block of wood with a slot in it is placed over the string the vibrations will not be affected when the slot is parallel to the direction of vibration of the string. When, however, the slot is at right angles to the direction of vibration of the string, the vibrations will not pass beyond the slot. If the slot in the block makes various angles with the direction of vibration of the string, then that part of the vibratory motion which is parallel to the slot will pass through. If a second slot is placed over the string, the vibrations which pass the first slot will also pass the second slot when the two slots are parallel to each other. When the slots are perpendicular to each other the vibrations which pass the first slot will not pass through the second.

The action of the tourmaline is now understood if we consider ordinary light to consist of a transverse wave motion in which the vibrations take place in all directions in a plane perpendicular to the direction in which the light is traveling. When such a beam passes through the first tourmaline crystal, the crystal absorbs all the vibrations except those in a certain direction. These it transmits. Hence the emerging beam differs from the ordinary light in that all the vibrations are in one direction. Such a ray of light is said to be **polarized.** If it falls on the second crystal of tourmaline, that crystal will transmit only those vibrations which are parallel to a certain direction in the tourmaline crystal. For one position of the crystals all the vibrations are transmitted. When the crystals have been rotated 90° from this position none of the vibrations are transmitted. For intermediate positions part of the vibrations are transmitted.

504. Plane of Polarization.—When a beam of light is polarized a plane can be imagined passed through the ray in such a way that the vibrations which constitute the wave are at right angles to the plane. This plane is known as the **plane of polarization.** If a stretched string is vibrating in a vertical plane, the horizontal plane is the plane of polarization. It would seem more natural to define the plane of polarization to be the plane in which the vibrations lie rather than the plane which is perpendicular to the plane containing the vibrations but usage has determined that it be otherwise.

505. Double Refraction.—In the study of refraction it was assumed that the media had the same physical properties in all directions. In crystalline substances like Iceland spar and quartz the physical properties of the substances are quite different in different directions. When a beam of light passes through such a crystal (Fig. 415) it is split up into two beams. One of these beams obeys the ordinary laws of refraction and for that reason is known as the **ordinary ray.** The other beam is called the **extraordinary ray** because it does not obey the ordinary laws of refraction. An examination of these two rays shows that both are polarized and that their planes of polarization are perpendicular to each other. This fact is indicated in Fig. 415 by the dots and cross lines on the rays.

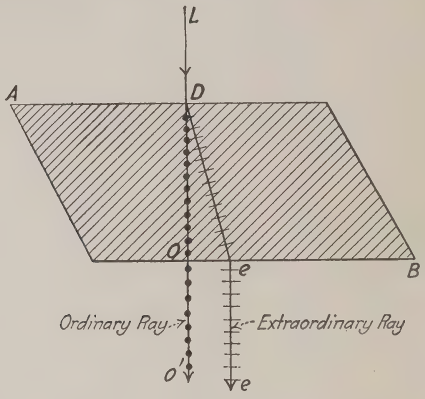


FIG. 415.—Double refraction by calcite.

Double refraction is easily observed by placing a piece of calcite on a piece of paper containing printed matter. On looking through the calcite at the printed matter, two images are seen, one produced by the ordinary ray and the other by the extraordinary ray. The image from the extraordinary ray seems nearer the observer than the image from the ordinary ray. On rotating the calcite, the extraordinary image seems to rotate about the ordinary image.

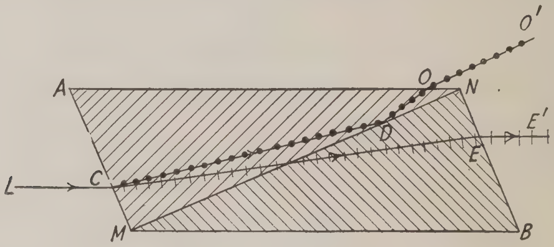


FIG. 416.—Nicol prism.

506. Nicol Prism.—One of the best methods of separating the ordinary from the extraordinary ray is by what is known as a Nicol prism (Fig. 416). A rhomb of Iceland spar *AMBN* is cut into two parts by a plane *MN* which makes an angle of about 22° with *MB*. When the cut surfaces have been polished, they are cemented together with Canada balsam. Now the index of refraction of Canada balsam is less than that of Iceland spar for the ordinary ray and greater than that of Iceland spar for the extraordinary ray. These refractive indices are:

Canada balsam = 1.55.
Iceland spar for ordinary ray = 1.658.
Iceland spar for extraordinary ray = 1.468.

The ray of light *LC* entering the face of the rhomb at *C* is broken up into the ordinary and extraordinary rays which are polarized at right angles to each other. This is indicated by the dots and cross lines in the figure. When the ordinary ray reaches the surface of separation of the Iceland spar and the Canada balsam at *D* at an angle greater than the critical angle it is totally reflected and emerges from the rhomb in the direction *OO′*. This surface of the rhomb is ordinarily painted black and this ray is absorbed in this black coating. Because the index of refraction of the Canada balsam is greater than the index of refraction of the calcite for the extraordinary ray, total reflection in this case can not occur and the extraordinary ray is transmitted in the direction *CE* and emerges from the rhomb at the polished face *BN*. In this way there is obtained a beam of light which is plane polarized, that is, all the vibrations lie in a single plane. Its intensity is only one-half that of the incident beam, the remainder of the light having been absorbed in the Nicol.

507. Rotation of the Plane of Polarization.—When a beam of plane polarized light passes through certain substances the plane of polarization is rotated. Thus if a plate of quartz which has been cut so that the faces are perpendicular to the axis of the crystal is placed between two Nicol prisms which are so oriented that the second one excludes the light transmitted by the first, the light will no longer be extinguished by the second Nicol prism. If, however, the second Nicol prism is rotated about the ray as an axis, a new position can be found at which the light is again extinguished.

This angle through which the plane of polarization has been rotated depends on the substance interposed between the Nicol prisms, on the thickness of the substance and on the wave length of light. The rotation may be either clockwise or counterclockwise. In this respect there are two kinds of quartz. One kind rotates the plane of polarization clockwise; the other rotates it counterclockwise. Some liquids and gases also cause a rotation of the plane of polarization. When two substances are mixed together, the observed rotation is the algrebraic sum of the rotations produced by the substances separately.

508. Specific Rotation.—The rotation produced by a column of substance 10 cm. in length, divided by the density of the substance is called the **specific rotation.** It varies with the wave length of light, being greater for light of short wave length than for light of long wave length. If a solution of an active substance in an inactive solvent is used, the rotation produced by a column of the substance 10 cm. in length is divided by the weight of the active substance in unit volume of the solution and this gives the specific rotation of the dissolved substance. Suppose that N grams of a substance is dissolved in an inactive solvent so that the volume of the solution is 100 c.c. If a length of 20 cm. of this solution rotates the plane of polarization θ deg., then the specific rotation of the substance is,

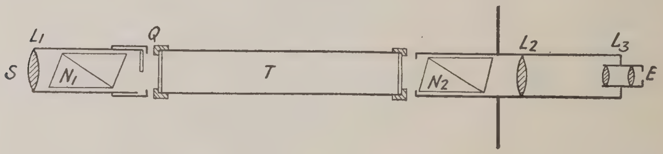


FIG. 417.—Diagram of polarimeter.

$$\alpha = \frac{100\theta}{20\,N}$$

The specific rotation depends not only on the wave length of light but also on the temperature.

509. Polarimeter.—The simplest way to measure the rotation of the plane of polarization is to examine the substance between two Nicol prisms. The Nicol prisms are first turned so that no light will pass through them. They are then said to be crossed. The substance to be examined is then inserted between the Nicols and the rotation of the plane of polarization observed by noting the amount which one of the Nicols must be rotated to again produce extinction. In practice this is not a sensitive arrangement for measuring the rotation of the plane of polarization, for it is impossible to tell exactly when the light is extinguished. The field of view appears dark while the Nicol is rotated through an appreciable angle. For this reason an auxiliary piece of apparatus is added by which the sensitiveness of the instrument is increased. One of the methods of increasing the sensitiveness is represented in Fig. 417 which shows a polarimeter.

Light from a source *S* (Fig. 417) passes through the lens L_1 and is polarized by means of the Nicol N_1. For this reason this Nicol is referred to as the **polarizer**. Between the polarizer and the tube *T* containing the solution to be studied is placed the small plate of quartz *Q* so that it covers half the field of view. Half the beam of light passes through the plate and the other half does not pass through it. This plate is cut parallel to the axis and is made of such a thickness that the ordinary ray passing through it gains in phase by one-half wave length over the extraordinary ray. In this way it is possible to make the plane of polarization of light which has passed through the quartz plate inclined at a small angle to the plane of polarization of the light which has not passed through the plate. The field of view is then divided into two parts and instead of setting the analyzer for extinction it is set so that the two parts of the field of view are equally illuminated. This very much increases the accuracy of the setting.

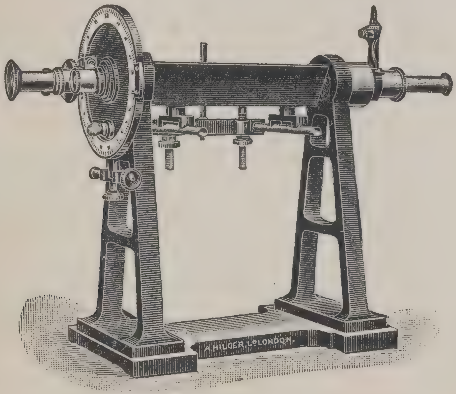

A. HILGER L^D LONDON.

FIG. 418.—Polarimeter.

After passing through the tube T, the light passes through the Nicol N_2, the **analyzer,** and is observed through the telescope L_2L_3 with the eye at E. By observing the rotation produced by the solution in the tube T, the specific rotary power is determined. This is a convenient method for determining the strength of sugar solutions or the strength of any other solution of a substance which produces a rotation of the plane of polarization.

Fig. 417 shows one type of polarimeter by Hilger and Co.

APPENDIX A

COMMON UNITS

Units of Force.—There are two sets of units of force, known as (*a*) gravitational or practical units and (*b*) absolute units. The following outline shows the relation of these units to each other:

- Units of Force
  - Gravitational
    - English = pound of force
      = weight of pound
    - Metric = gram of force
      = weight of gram
  - Absolute
    - English = poundal
    - Metric = dyne

A force of one pound or a pound of force is a force equal to the attraction of gravity on a mass of one pound at sea level and 45° north latitude.

A force of one gram or a gram of force is a force equal to the attraction of gravity on a mass of one gram at sea level and 45° north latitude. A force of one kilogram is equal to 1,000 grams of force.

A poundal is that force which will give a mass of one pound an acceleration of one foot per sec. per sec.

A dyne is that force which will give a mass of one gram an acceleration of one centimeter per sec. per sec.

1 pound of force = 32.2 poundals
1 gram of force = 980 dynes

Units of Work.—Units of work must be divided into two classes according to the units of force used in defining them.

- Units of Work
  - Gravitational
    - English = foot-pound
    - Metric = gram-centimeter
  - Absolute
    - English = foot-poundal
    - Metric = erg

A foot-pound of work is the work done by a force of one pound acting through a distance of one foot.

Foot-pounds = pounds of force × feet

A gram-centimeter of work is the work done by a force of one gram acting through a distance of one centimeter.

Gram-centimeters = grams of force × centimeters

A kilogram-meter of work is the work done by a force of one kilogram acting through a distance of one meter.

Kilogram-meters = kilograms of force × meters

A foot-poundal of work is the work done by a force of one poundal acting through one foot.

Foot-poundals = poundals × feet

An erg of work is the work done by a force of one dyne acting through a distance of one centimeter.

Ergs = dynes × centimeters

A joule of work is defined to be equal to ten million ergs of work.

One foot-pound = 32.2 foot-poundals

One gram-centimeter = 980 ergs

One joule = 10^7 ergs

Units of Power.—In the English system the unit of power is the *horsepower.* The absolute unit of power is the *watt.*

One horsepower is defined to be 33,000 foot-pounds of work per minute or 550 foot-pounds of work per second.

$$\text{Horsepower} = \frac{\text{work in foot-pounds}}{33{,}000 \times \text{time in minutes}}$$

A watt is defined to be one joule of work per second or ten million ergs of work per second.

$$\text{Watts} = \frac{\text{work in ergs}}{10{,}000{,}000 \times \text{time in seconds}}$$

One horsepower = 746 watts

APPENDIX B

CIRCULAR AND SIMPLE HARMONIC MOTIONS

Uniform Circular Motion.—In uniform circular motion (Fig. 419) there is at every instant an acceleration toward the center. To calculate this acceleration suppose that a particle passes over the arc AB with a constant speed v in the time t. In this time it has been deflected from the straight line AD, through a distance AE because of the action of a constant force toward the center.

Let $a =$ the acceleration toward the center. Then

$$AE = \frac{at^2}{2}$$

and

$$AB = vt$$

If the arc AB be made very small so that it differs little from the chord AB, it can be substituted for the chord. The triangles ABE and ABC are similar. Hence

$$\frac{AB}{AE} = \frac{AC}{AB}$$

$$\overline{AB}^2 = AE \times AC$$

If the arc AB is nearly equal to the chord AB we have by susbtitution,

$$v^2t^2 = \frac{at^2}{2} \times 2R$$

$$a = \frac{v^2}{R}$$

If T be the time of one complete revolution of the particle in the circle, and ω the angular velocity of the radius,

$$v = \omega R$$

$$a = \frac{\omega^2 R^2}{R} = \omega^2 R.$$

Simple Harmonic Motion.—Consider a point moving around a circle (Fig. 420) with uniform speed. The projection of this point on the diameter of the circle moves with varying speed and describes a complete to and fro vibration on the diameter, while the point moves completely around the circumference. If M is the projection of P on the horizontal diameter, the motion of M is called *simple harmonic* motion.

Let the radius of the circle be R and θ, the angle between OP and OA. Then $OM =$ displacement $=$ distance of M from O, the position of equilibrium.

$$OM = x = R \cos \theta$$

Let V = velocity of P, tangent to the circle
Horizontal velocity of $P = V_x = V \cos (90 - \theta) = v \sin \theta$
Velocity of M = horizontal velocity of $P = V_x = v \sin \theta$
Acceleration of P toward center $O = \frac{V^2}{R}$
Horizontal acceleration of $P = A_x = \frac{V^2}{R} \cos \theta$
Acceleration of M = horizontal accleration of $P = A_x = \frac{V^2}{R} \cos \theta$

Dividing the acceleration of M by its displacement at the same instant, we have,

$$\frac{A_x}{x} = \frac{\frac{V^2}{R} \cos \theta}{R \cos \theta} = \frac{V^2}{R^2}$$

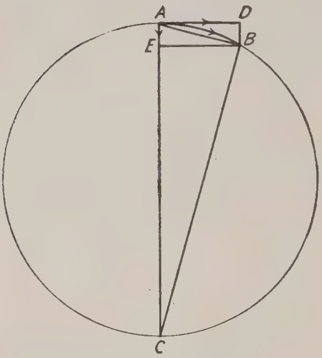


FIG. 419.—Uniform circular motion.

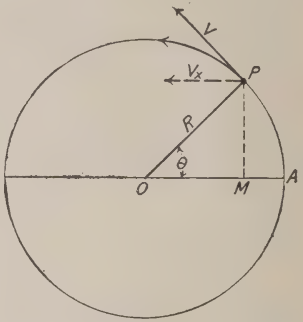


FIG. 420.—Simple harmonic motion.

Since neither V nor R change during the motion, $\frac{V^2}{R^2}$ is a constant. This means that the ratio of the acceleration of M to its displacement is always a constant whatever the position of M. Both the acceleration and the displacement change, but in such a way that their ratio remains unchanged. This then becomes the test of simple harmonic motion. Whenever a body has such a motion that the acceleration divided by the displacement is constant, it is said to move with simple harmonic motion.

If T is the time for the particle to pass once around the circle and ω the angular velocity of the radius OP, then

$$\frac{2\pi R}{T} = V, \quad \omega = \frac{2\pi}{T}, \quad V = \omega R.$$

$$\frac{V^2}{R^2} = \frac{\omega^2 R^2}{R^2} = \omega^2 = \left(\frac{2\pi}{T}\right)^2$$

$$\frac{A_x}{x} = \frac{V^2}{R^2} = \omega^2 = \left(\frac{2\pi}{T}\right)^2$$

Hence

$$\sqrt{\frac{A_x}{x}} = \frac{2\pi}{T},$$

$$T = 2\pi \sqrt{\frac{x}{A_x}}.$$

Simple Pendulum.—A simple pendulum affords an easy illustration of simple harmonic motion. When the pendulum is displaced from its position of equilibrium (Fig. 421) the force tending to bring it back to its original position is,

$$F = -Mg \sin \theta$$

The acceleration which this force produces is $a = -g \sin \theta$
The displacement of the pendulum bob from its position of equilibrium N is,

$$S = l\theta$$

Hence, $$\frac{\text{Displacement}}{\text{Acceleration}} = \frac{S}{a} = -\frac{l\theta}{g \operatorname{Sin} \theta}$$

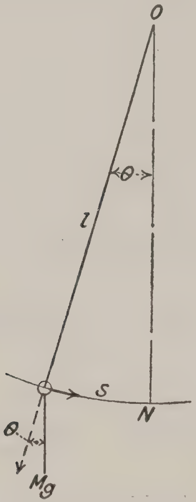


FIG. 421.—Simple pendulum.

For small angles $\theta = \operatorname{Sin} \theta$ and for small displacements of the pendulum,

$$\frac{\text{Displacement}}{\text{Acceleration}} = -\frac{l}{g} = \text{Constant}$$

Consequently the pendulum moves with simple harmonic motion. Its period is therefore,

$$T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$$

$$= 2\pi\sqrt{-\frac{l}{g}}$$

The longer the pendulum the greater is its period and the greater the acceleration of gravity the shorter the period of the pendulum.

APPENDIX C

ELECTROMAGNETIC UNITS

Unit Strength of Current.—The electromagnetic unit of current is defined to be that current which when flowing in a wire bent in the form of a circle with a radius of 1 cm. exerts a force of 2π dynes on unit magnetic pole at the center of the circle. This force is perpendicular to the plane of the circle.

Unit Difference of Potential or Electromotive Force.—An electromagnetic unit difference of potential exists between two points when it requires the expenditure of one erg of work to carry a unit of positive electricity from one point to the other against the electric force.

Unit of Resistance.—A conductor has one electromagnetic unit of resistance when one electromagnetic unit of difference of potential causes one electromagnetic unit of current to flow through it.

Unit Quantity of Electricity.—One electromagnetic unit of quantity of electricity is the quantity of electricity conveyed by one electromagnetic unit of current in one second.

Unit of Capacity.—A conductor has one electromagnetic unit of capacity when one electromagnetic unit of charge changes its potential by one electromagnetic unit.

Unit of Induction.—A circuit has one electromagnetic unit of induction when the variation of the current in it at the rate of one electromagnetic unit per second produces one electromagnetic unit of electromotive force.

PRACTICAL UNITS

Resistance.—The *Ohm* = 10^9 electromagnetic units of resistance. It is also defined as the resistance at 0°C. of a uniform column of mercury 106.3 cm. long and 14.4521 grams in mass.

Current.—The *Ampere* = 10^{-1} electromagnetic units of current. The ampere is also defined to be that unvarying current which deposits silver at the rate of 0.001118 grams per second from an aqueous solution of silver nitrate of standard concentration and at a fixed temperature.

Electromotive Force.—The *Volt* = 10^8 ergs or electromagnetic units of difference of potential. It is also defined as that electromotive force which when applied to conductor having a resistance of one ohm produces in it a current of one ampere.

Quantity.—The *Coulomb* = 10^{-1} electromagnetic unit of quantity. The coulomb is that quantity of electricity which is conveyed by a current of 1 ampere in one second.

Capacity.—The *Farad* = 10^{-9} electromagnetic unit of capacity. A conductor or a condenser has a capacity of one farad when it is charged to a potential of 1 volt by 1 coulomb of electricity. The microfarad is the millionth part of a farad.

Inductance.—The *Henry* = 10^9 electromagnetic units of inductance. It is the inductance in a circuit when an electromotive force of 1 volt is induced in the circuit by a current which is changing at the rate of 1 ampere per second.

Power.—The *Watt* = 10^7 ergs per second = 1 joule per second. It is the work done in one second by a current of 1 ampere flowing under an electrical pressure of 1 volt. *Kilowatt* = 1,000 watts. One kilowatt for one hour is called a *kilowatt-hour.* It is represented by a current of 10 amperes flowing for one hour under a pressure of 100 volts.

MAGNETIC UNITS

Unit Magnetic Pole.—A unit magnetic pole is one which when placed at a distance of 1 centimeter in air from a similar pole of equal strength repels it with a force of 1 dyne.

Unit Magnetic Field.—The intensity of a magnetic field is unity when the field acts on a unit magnetic pole with a force of one dyne. Such a field is represented by drawing one line of force for each square centimeter.

Magnetic Flux.—The maxwell is the unit of magnetic flux. One line of magnetic force is called a *maxwell.*

Magnetomotive Force or Magnetic Difference of Potential.—The *gilbert* is the unit of magnetomotive force or of magnetic difference of potential. The magnetomotive force of a circuit is one gilbert when it requires one erg of work to carry unit pole completely around the circuit. The magnetic difference of potential between two points is one gilbert when it requires one erg of work to carry unit north pole from one point to the other.

$$1 \text{ Ampere-turn} = \frac{4\pi}{10} \text{ gilberts}$$

Magnetic Reluctance.—The unit of magnetic reluctance is called the *oersted.* A magnetic circuit has a reluctance of one oersted when a magnetomotive force of 1 gilbert sets up a magnetic flux of one maxwell in it.

APPENDIX D

COMBINATION OF CONDENSERS

Condensers in Parallel.—If a number of condensers having capacities C_1, C_2, C_3, etc. (Fig. 422), are placed in parallel, the whole system will have a capacity C which is equal to the sum of the separate capacities. This result can be proved as follows:

The condensers are all charged to the same difference of potential. Let E denote this difference of potential and let Q_1, Q_2 and Q_3 be the charges on the condensers C_1, C_2 and C_3 respectively. Let Q be the total charge on all of the condensers. Then

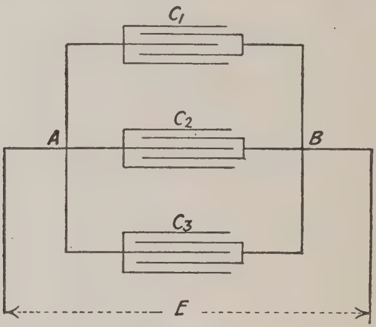


FIG. 422.—Condensers in parallel.

$$Q = Q_1 + Q_2 + Q_3, \text{ etc.}$$
$$Q_1 = EC_1, \quad Q_2 = EC_2, \quad Q_3 = EC_3, \text{ etc.}$$
$$\text{and } Q = EC$$

Substituting these values

$$EC = EC_1 + EC_2 + EC_3$$

Dividing by E

$$C = C_1 + C_2 + C_3$$

Hence to find the equivalent capacity of a number of condensers connected in parallel, it is only necessary to add together the separate capacities.

Condensers in Series.—If condensers are placed in series (Fig. 423), the same quantity of electricity is stored up on each condenser. In general, however, the difference of potential between the terminals of the different condensers will not be the same. The difference of potential over the combination will be equal to the sum of the difference of potential over the separate condensers.

Let Q be the charge on each condenser; C, the equivalent capacity of the condensers when joined in series and E_1, E_2 and E_3 the differences of potential between the terminals of C_1, C_2 and C_3 respectively. Since the total difference of potential is equal to the sum of the separate differences of potential,

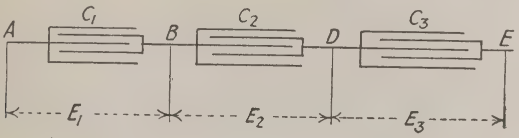


FIG. 423.—Condensers in series.

$$E = E_1 + E_2 + E_3$$

Since the charge on each condenser is the same

$$Q = E_1C_1 = E_2C_2 = E_3C_3$$

and also $Q = EC$

Whence $E = \frac{Q}{C}$, $E_1 = \frac{Q}{C_1}$, $E_2 = \frac{Q}{C_2}$, $E_3 = \frac{Q}{C_3}$

Substituting these values, we have

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

Dividing by Q,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

This equation gives the equivalent capacity of the condensers in series in terms of the capacities of the separate condensers.

APPENDIX E

LENS FORMULA

The Lens Formula.—In Fig. 424 let V be a point source of light on the principal axis. A ray VA on entering the glass will be bent toward BA, the normal to the surface at A. By the law of refraction,

$$\frac{\text{Sin } VAC}{\text{Sin } DAB} = \frac{\text{Sin } i}{\text{Sin } r} = n = \text{index of refraction of glass}$$

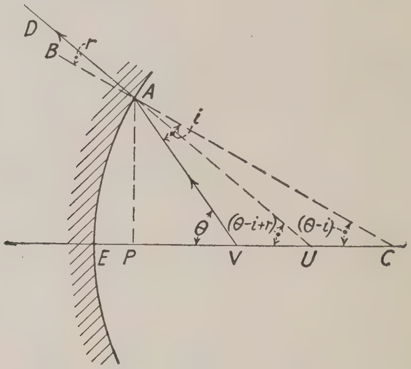


Fig. 424.—Refraction of light at a spherical surface.

If A lies near the principal axis, the following approximate relations may be written down:

$$\theta = \frac{AP}{PV} = \frac{AP}{\text{object distance}} = \frac{AP}{U}$$

$$\theta - i = \frac{AP}{CP} = \frac{AP}{\text{radius of curvature}} = \frac{AP}{R_1}$$

$$\theta - i + r = \frac{AP}{PU} = \frac{AP}{\text{image distance}} = \frac{AP}{V}$$

$$\therefore i = \frac{AP}{U} - \frac{AP}{R_1}$$

$$r = \frac{AP}{V} - \frac{AP}{R_1}$$

Since i and r are small, we may write

$$\frac{\text{Sin } i}{\text{Sin } r} = \frac{i}{r} = n, \text{ approximately}$$

and

$$i = nr$$

$$\left(\frac{AP}{U} - \frac{AP}{R_1}\right) = n\left(\frac{AP}{V} - \frac{AP}{R_1}\right)$$

Hence

$$\frac{1}{U} - \frac{1}{R_1} = n\left(\frac{1}{V} - \frac{1}{R_1}\right)$$

and

$$\frac{n}{V} - \frac{1}{U} = \frac{n-1}{R_1}$$

This gives the equation for a single spherical surface.

To get the equation for a lens consider Fig. 425.

Suppose that the ray now emerges from the glass at a convex spherical surface and that the curvature of the convex surface is greater than the curvature of the concave surface. The refraction at the concave surface produces an image of the object V and this image is located at W, a distance V' from the surface. In this case the relation between the image and the object is given by,

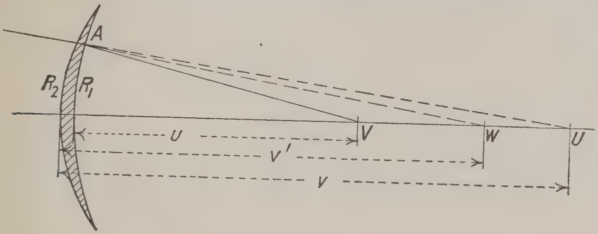


FIG. 425.—Refraction of light by two spherical surfaces.

$$\frac{n}{V'} - \frac{1}{U} = \frac{n-1}{R_1}$$

For the convex surface, W can be considered the object and V' the object distance. Refraction at the second surface of radius of curvature R_2 produces an image at U. The light now goes from glass to air and the index of refraction in such a case is the reciprocal of the index of refraction when the light goes from air to glass. For this case we have

$$\frac{\frac{1}{n}}{V} - \frac{1}{V'} = \frac{\frac{1}{n} - 1}{R_2}$$

or

$$\frac{1}{V} - \frac{n}{V'} = -\frac{n-1}{R_2}$$

Combining these equations gives,

$$\frac{1}{V} - \frac{1}{U} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

If the rays are parallel, $U = \infty$ and $\frac{1}{U} = 0$, $V = f$

Hence

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore \frac{1}{V} - \frac{1}{U} = \frac{1}{f}$$

In the use of this lens formula regard must be had for the signs of *R*' *U*, *V* and *f*. In this formula it is agreed that distances measured from the lens to the source of light are positive and those measured from the lens away from the source of light are negative. Thus in Fig. 425 the radii of curvature of both surfaces are positive. The object distance and the image distance are also positive. In a double convex lens one of the radii of curvature is negative and for real images the image distance is negative. The formula in this case becomes:

$$\frac{1}{V} + \frac{1}{U} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{1}{f}$$

APPENDIX F

Refraction by a Prism.—Let i and r denote the angle of incidence and refraction Fig. 376 at the first face of the prism and r' and i' the angle of incidence and emergence at the second face.

Then since the angle between the faces of the prism is the same as that between the normals HG and NG, we have

$$A = r + r'$$

The deviation at the first face is $i - r$ and at the second face $i' - r'$. Hence the total deviation is

$$\begin{aligned} D &= i - r + i' - r' \\ &= (i + i') - (r + r') = i + i' - A \end{aligned}$$

It can be shown by experiment and also from theoretical considerations that the angle of deviation has a minimum value when the angle of incidence is equal to the angle of emergence. In this case, $i = i'$ and $r = r'$.

$$2r = A$$

$$r = \frac{A}{2}$$

Hence

$$D = 2i - A$$

$$i = \frac{A + D}{2}$$

From the law of refraction,

$$\text{Index of refraction} = n = \frac{\text{Sin } i}{\text{Sin } r}$$

Substituting for i and r

$$\text{Index of refraction} = n = \frac{\sin \dfrac{A + D}{2}}{\sin \dfrac{A}{2}}$$

Table I.—Densities of Solids and Liquids

Substance	Density		Substance	Density	
	Per c.c.	Per cu. ft.		Per c.c.	Per cu. ft.
	grams	*pounds*		*grams*	*pounds*
Alcohol at 20°	0.789	49.3	Iron (wrought)	7.80	480.
Aluminum	2.65	164.	Leather	0.90	56.
Brass	8.60	535.	Lead	11.37	710.
Brick	2.10	131.	Mercury	13.60	840.
Copper	8.93	555.	Nickel	8.90	556.
Cork	0.24	15.	Oak	0.80	50.
Diamond	3.52	22.	Pine	0.50	31.2
Ether at 0°	0.735	45.9	Sandstone	2.25	140.
Glass (crown)	2.50	156.	Silver	10.50	655.
Glass (flint)	3.70	230.	Turpentine	0.87	54.3
Gasoline	0.79	49.4	Tin	7.29	455.
Gold	19.32	1200.	Water (fresh)	1.00	62.5
Glycerine	1.26	78.7	Water (sea)	1.03	64.4
Iron (cast)	7.20	450.	Zinc	7.15	446.2

Table II.—Densities of Gases

Gas	Density at 0°		Gas	Density at 0°	
	Per liter	Per cu. ft.		Per liter	Per cu. ft.
	grams	*pounds*		*grams*	*pounds*
Air	1.2930	0.0807	Chlorine	2.491	0.2011
Acetylene	1.1791	0.0736	Hydrogen	0.0899	0.00561
Ammonia	0.7708	0.0481	Nitrogen	1.2507	0.0781
Carbon dioxide	1.9768	0.1234	Oxygen	1.4291	0.0892

TABLE III.—ELASTIC CONSTANTS

Substance	Young's modulus		Breaking strength per sq. in.
	Per sq. cm.	Per sq. in.	
	grams	*pounds*	*pounds*
Aluminum	$7. \times 10^8$	9.8×10^6	1.3 to 2.1×10^4
Brass	10.	14.	4.4 to 5.6
Copper	12.	17.	4.0 to 4.4
Gold	8.	11.2	1.56
Cast iron	13.	18.	1.1 to 3.3
Wrought iron	20.	28.	4.1 to 6.4
Steel	22.	31.	10 to 11
Nickel	20.	28.	7.6
Silver	7.9	11.1	4.1

TABLE IV.—COMPRESSIBILITY OF LIQUIDS

Substance	Temperature	Compressibility per atmosphere per sq. cm.	Substance	Temperature	Compressibility per atmosphere per sq. cm.
	°C.			°C.	
Ethyl alcohol	14	0.0000987	Mercury	20	0.0000039
Ethyl ether	0	0.000143	Turpentine	20	0.000075
Kerosene	20	0.0000543	Water	20	0.000048

TABLE V.—COEFFICIENTS OF FRICTION

Substances	Limits of coefficient	Substances	Limits of coefficient
	$\mu = \frac{F}{P}$		$\mu = \frac{F}{P}$
Wood on drywood	0.25 to 0.50	Leather on iron	0.30 to 0.50
Metals on drywood	0.20 to 0.60	Leather on wood	0.30 to 0.50
Metals on dry metals	0.15 to 0.20	Iron on stone	0.30 to 0.70
		Wood on stone	0.40 to 0.60

TABLE VI.—SURFACE TENSION

Substance	Temperature	Surface tension per sq. cm.	Substance	Temperature	Surface tension per sq. cm.
	°C.	*grams*		°C.	*grams*
Acetic acid....	20	23.9×10^{-3}	Mercury......	18	55.6
Alcohol (ethyl).	20	22.1	Turpentine....	20	28.9
Ether (ethyl)...	20	17.2	Petroleum....	20	26.4
Glycerine......	18	64.5	Water........	20	74.3

TABLE VII.—PROPERTIES OF AQUEOUS VAPOR

Boiling points of water at pressures near standard atmospheric pressure. The pressures are given in mm. of mercury at 0°C. at the sea level in latitude 45°

Pressure	Temperature	Pressure	Temperature	Pressure	Temperature	Pressure	Temperature
mm.	°C.	*mm.*	°C.	*mm.*	°C.	*mm.*	°C.
733	98.99	745	99.44	757	99.89	769	100.33
735	99.07	747	.52	759	.96	771	.40
737	.14	749	.59	761	100.04	773	.47
739	.22	751	.67	763	.11	775	.55
741	.29	753	.74	765	.18	777	.62
743	.37	755	.82	767	.26	779	.69

Table VIII.—Properties of Saturated Steam (Centigrade Units)

Temperature	Pressure		Volume		Heat units per unit mass		
	Weight per sq. cm.	Weight per sq. in.	Per kilogram	Per pound	Of water	Latent heat	Total heat of vapor
°C.	*kg.*	*lb.*	*cu. m.*	*cu. ft.*			
0	0.0063	0.089	204.970	3283.00	0.	594.7	594.7
10	0.0125	0.178	106.620	1707.90	10.	589.4	599.4
20	0.0236	0.336	58.150	931.48	20.	584.1	604.1
30	0.0429	0.61	33.132	530.72	30.	578.8	608.8
40	0.0747	1.06	19.650	314.77	40.1	573.4	613.5
50	0.125	1.78	12.091	193.68	50.1	567.9	618.
60	0.202	2.88	7.695	123.26	60.1	562.4	622.6
70	0.317	4.51	5.050	80.89	70.2	556.8	627.
80	0.482	6.86	3.4085	54.60	80.3	551.	631.5
90	0.714	10.16	2.3592	37.79	90.4	545.2	635.6
100	1.033	14.70	1.6702	26.754	100.5	539.1	639.7
110	1.462	20.79	1.2073	19.339	110.7	532.9	643.6
120	2.027	28.83	0.8894	14.247	120.9	526.6	647.4
130	2.760	39.26	0.6664	10.675	131.1	520.	651.
140	3.695	52.56	0.5071	8.123	141.3	513.2	654.5
150	4.868	69.24	0.3917	6.274	151.6	506.2	657.8
160	6.323	89.93	0.3065	4.91	161.9	498.9	660.8
170	8.104	115.27	0.2429	3.891	172.2	491.4	663.7
180	10.258	145.90	0.1945	3.116	182.6	483.7	666.3
190	12.835	182.56	0.1575	2.523	193.1	475.7	668.8
200	15.890	226.00	0.1288	2.063	203.6	467.5	671.1
210	19.490	277.20	0.1063	1.703	214.1	459.1	673.2

Table IX.—Melting and Freezing Points of Fats and Waxes

Substance	Melting point	Freezing point	Substance	Melting point	Freezing point
	°C.	°C.		°C.	°C.
Butter	28–33	20–23	Beeswax	61–64	60–63
Lard	36–40	27–30	Paraffin (soft)	38–52	....
Tallow (mutton)	44–45	36–41	Paraffin (hard)	52–56	....

TABLE X.—HEAT CONSTANTS OF LIQUIDS

Substance	Boiling point	Cubical expansion	Specific heat	Heat of vaporization	
				Per gram	Per pound
	°C.			*calories*	*B.t.u.*
Ammonia	−34.	....	....	294.	529
Aniline	184.	....	0.514	110.	198
Alcohol (ethyl)	78.1	0.0011	0.55	205.	369
Benzine	80.3	0.00124	0.34	94.4	170
Chloroform	61.	0.00126	0.232	58.	106
Ether (ethyl)	34.5	0.00163	0.56	88.4	159
Glycerine	290.	0.00053	0.58	....	....
Mercury	358.	0.00182	0.0332	68.	122
Turpentine	159.	0.00094	0.42	70.	126
Water	100.	0.00030[1]	1.00	538.	970

[1] Between 20° and 40°C.

TABLE XI.—HEAT CONSTANTS OF SOLIDS

Substance	Melting point	Coefficient of linear expansion	Specific heat	Heat of fusion	
				Per gram	Per pound
	°C.			*calories*	*B.t.u.*
Aluminum	657.	0.0000255	0.22	76.8	140.
Bismuth	268.	0.0000157	0.0304	12.6	22.7
Brass	....	0.0000193	0.09	....	....
Copper	1084.	0.0000167	0.0909	43.	77.
Glass	....	0.0000083	0.2	....	....
Gold	1063.	0.0000139	0.0303	....	....
Ice	0.	0.000051	0.502	79.8	144.
Iron	1503.	0.0000119	0.104	30.	54.
Lead	327.	0.0000276	0.0302	5.4	9.7
Mercury	−38.8	....	0.033	2.8	5.4
Nickel	1452.	0.0000128	0.109	4.6	8.3
Platinum	1756.	0.0000089	0.0324	27.	48.6
Silver	960.	0.0000188	0.0556	22.	39.
Tungsten	3360.	0.0000044	0.034	....	....
Zinc	418.	0.0000263	0.0918	28.1	50.6

Table XII.—Heat of Combustion

Substance	Calories per gram	B.t.u. per pound
Lignite (low grade)	3,520	6,350
Lignite (high grade)	3,990	7,190
Bituminous coal (low grade)	6,090	10,960
Bituminous coal (high grade)	7,850	14,134
Anthracite coal (low grade)	6,990	12,580
Anthracite coal (high grade)	7,420	13,360
Wood (oak)	4,620	8,320
Gasoline (liquid)	11,100	20,000
Alcohol (liquid)	6,440	11,590
Acetylene (gas)	11,900	21,400
Hydrogen (gas)	34,460	62,030

Table XIII.—Thermal Conductivities

Substance	Thermal conductivity	Substance	Thermal conductivity
	c.g.s. units		*c.g.s. units*
Aluminum	0.504	Lead	0.083
Air	0.000054	Sand, white dry	0.00093
Brass	0.204	Sawdust	0.00012
Copper	0.918	Silver	0.974
Cork	0.00011	Soil, dry	0.00033
Glass	0.0015	Water	0.00143
Iron	0.161	Zinc	0.265
Ice	0.00396		

TABLE XIV.—CRITICAL DATA

Substance	Critical temperature	Critical pressure atmosphere	Critical volume
	°C.		*cu. cm.*
Ammonia	130.	115.	0.00481
Carbon dioxide	31.1	73.	0.0066
Ether	197.	35.8	0.158
Helium	−268.	2.3	0.00299
Nitrogen	−146.	33.	0.00517
Oxygen	−118.	50.	0.00426
Water	365.	194.6	0.00386

TABLE XV.—DIELECTRIC CONSTANTS

Substance	Temperature	Dielectric constant	Substance	Temperature	Dielectric constant
	°C.			°C.	
Air (at 76 cm. pressure)	0	1.000586	Dry paper		2.0 to 2.5
Hydrogen (at 76 cm. pressure)	0	1.000264	Paraffin		2.0 to 2.3
Benzene	18	2.29	Quartz		4.5
Ordinary glass		7.0 to 8.0	Shellac		3.0 to 3.7
Mica		5.7 to 7.0	Sulphur		3.6 to 4.3
			Olive oil		3.1 to 3.2

TABLE XVI.—ELECTRO-CHEMICAL EQUIVALENTS

Substance	Valency	Per coulomb	Substance	Valency	Per coulomb
		grams			*grams*
Aluminum	3	0.0009357	Mercury	1	0.002075
Copper	1	0.0006588	Mercury	2	0.001037
Copper	2	0.0003294	Nickel	2	0.0003042
Gold	3	0.000681	Oxygen	2	0.0000829
Hydrogen	1	0.0000104	Potassium	1	0.0004054
Iron	2	0.0002895	Silver	1	0.001118
Iron	3	0.000193	Sodium	1	0.0002387
Lead	2	0.001072	Zinc	2	0.0003387
Magnesium	2	0.0001261			

TABLE XVII.—SPECIFIC RESISTANCES AND TEMPERATURE COEFFICIENTS

Substance	Resistance at 0°C. per centimeter cube	Mean temperature coefficient, 0–100°C.
	ohms	
Aluminum	2.906×10^{-6}	0.00435
Bismuth	108.000	0.00450
Copper	1.584	0.00420
Iron	9.696	0.00625
Gold	2.088	0.00377
Mercury	94.340	0.00090
Nickel	12.350	0.00622
Platinum	9.035	0.00367
Silver	1.561	0.00400
Tin	10.500	0.00440
Zinc	5.750	0.00406
German silver	20.890	0.00027

TABLE XVIII.—RESISTANCE OF SOFT OR ANNEALED COPPER WIRE

B & S gauge	Diameter in inches	Ohms per 1,000 ft. at 20°C. or 68°F.	B & S gauge	Diameter in inches	Ohms per 1,000 ft. at 20°C. or 68°F.
0000	0.46	0.04893	19	0.03589	8.038
000	0.40964	0.0617	20	0.031961	10.14
00	0.3648	0.0778	21	0.028462	12.78
0	0.32486	0.09811	22	0.025347	16.12
1	0.2893	0.1237	23	0.022571	20.32
2	0.25763	0.156	24	0.0201	25.63
3	0.22942	0.1967	25	0.0179	32.31
4	0.20431	0.248	26	0.01594	40.759
5	0.18194	0.3128	27	0.014195	51.38
6	0.16202	0.3944	28	0.012641	64.79
7	0.14428	0.4973	29	0.011257	81.7
8	0.12949	0.6217	30	0.010025	103.
9	0.11443	0.7908	31	0.008928	129.9
10	0.10189	0.9972	32	0.00795	163.8
11	0.090742	1.257	33	0.00708	206.6
12	0.080808	1.586	34	0.006305	260.5
13	0.071961	1.999	35	0.005615	328.4
14	0.064084	2.521	36	0.005	414.2
15	0.057068	3.179	37	0.004453	522.2
16	0.05082	4.009	38	0.003965	658.5
17	0.045257	5.055	39	0.003531	830.4
18	0.040303	6.374	40	0.003145	1047.

TABLE XIX.—VELOCITY OF SOUND

Substance	Temperature	Velocity per second	
		Meters	Feet
	°C.		
Air	0	331.8	1,088.
Brass		3,500.	11,480.
Steel	20	4,990.	16,360.
Water	0	401.	1,315.

TABLE XX.—INDEX OF REFRACTION

Substance	Index of refraction for sodium D-line	Substance	Index of refraction for sodium D-line
Air	1.0002918	Water	1.333
Carbon dioxide	1.000334	Crown glass	1.52
Ethyl alcohol	1.362	Flint glass	1.64
Carbon bisulphide	1.632	Heavy flint glass	1.9
Glycerine	1.47		

TABLE XXI.—TRIGONOMETRIC FUNCTIONS

A	Sin	Cos	Tan	A	Sin	Cos	Tan
0	0.000	1.000	0.000	21	0.358	0.934	0.384
1	0.017	0.999	0.017	22	0.375	0.927	0.404
2	0.035	0.999	0.035	23	0.391	0.921	0.424
3	0.052	0.999	0.052	24	0.407	0.914	0.445
4	0.070	0.998	0.070	25	0.423	0.906	0.466
5	0.087	0.996	0.087	26	0.438	0.898	0.488
6	0.105	0.995	0.105	27	0.454	0.891	0.510
7	0.122	0.993	0.123	28	0.469	0.883	0.532
8	0.139	0.990	0.141	29	0.485	0.875	0.554
9	0.156	0.988	0.158	30	0.500	0.866	0.577
10	0.174	0.985	0.176				
				31	0.515	0.857	0.601
11	0.191	0.982	0.194	32	0.530	0.848	0.625
12	0.208	0.978	0.213	33	0.545	0.839	0.649
13	0.225	0.974	0.231	34	0.559	0.829	0.675
14	0.242	0.970	0.249	35	0.574	0.819	0.700
15	0.259	0.966	0.268	36	0.588	0.809	0.727
16	0.276	0.961	0.287	37	0.602	0.799	0.754
17	0.292	0.956	0.306	38	0.616	0.788	0.781
18	0.309	0.951	0.325	39	0.629	0.777	0.810
19	0.326	0.946	0.344	40	0.643	0.766	0.839
20	0.342	0.940	0.364				

TABLE XXI.—TRIGONOMETRIC FUNCTIONS—*Continued*

A	Sin	Cos	Tan	A	Sin	Cos	Tan
41	0.656	0.755	0.869	66	0.914	0.407	2.25
42	0.669	0.743	0.900	67	0.921	0.391	2.36
43	0.682	0.731	0.933	68	0.927	0.375	2.48
44	0.695	0.719	0.966	69	0.934	0.358	2.61
45	0.707	0.707	1.000	70	0.940	0.342	2.75
				71	0.946	0.326	2.90
46	0.719	0.695	1.04	72	0.951	0.309	3.08
47	0.731	0.682	1.07	73	0.956	0.292	3.27
48	0.743	0.669	1.11	74	0.961	0.276	3.49
49	0.755	0.656	1.15	75	0.966	0.259	3.73
50	0.766	0.643	1.19				
51	0.777	0.629	1.23	76	0.970	0.242	4.01
52	0.788	0.616	1.28	77	0.974	0.225	4.33
53	0.799	0.602	1.33	78	0.978	0.208	4.70
54	0.809	0.588	1.38	79	0.982	0.191	5.14
55	0.819	0.574	1.43	80	0.985	0.174	5.67
				81	0.988	0.156	6.31
56	0.829	0.559	1.48	82	0.990	0.139	7.12
57	0.839	0.545	1.54	83	0.993	0.122	8.14
58	0.848	0.530	1.60	84	0.995	0.105	9.51
59	0.857	0.515	1.66	85	0.996	0.087	11.43
60	0.866	0.500	1.73				
61	0.875	0.485	1.80	86	0.998	0.070	14.30
62	0.883	0.469	1.88	87	0.999	0.052	19.08
63	0.891	0.454	1.96	88	0.999	0.035	28.64
64	0.898	0.438	2.05	89	0.999	0.017	57.28
65	0.906	0.423	2.14	90	1.000	0.000	Infinity

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